Numerical Modeling of Geophysical Electromagnetic Inductive and Galvanic Phenomena

SeyedMasoud Ansari
Colin G. Farquharson

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Maxwell’s equations

Deriving a PDE from Maxwell’s equations, constitutive relations, Ohm’s law in the quasi-static regime.

Faraday’s law of induction

\[ \nabla \times \mathbf{E} + i\omega \mu_0 \mathbf{H} = 0, \]  
(1)

Ampère’s law

\[ \nabla \times \mathbf{H} - \mathbf{J} = \mathbf{J}^s, \]  
(2)

Ohm’s law

\[ \mathbf{J} = \sigma \mathbf{E} \]  
(3)

\( \mathbf{E}(\mathbf{r}, \omega) \): Electric field, \( \mathbf{H}(\mathbf{r}, \omega) \): Magnetic field intensity, \( \mathbf{J}^s(\mathbf{r}, \omega) \): source current density, and \( \sigma(\mathbf{r}) \): Electrical conductivity
E-field PDE

\[ \nabla \times \nabla \times \mathbf{E} + i\omega \mu_0 \sigma \mathbf{E} = -i\omega \mu_0 \mathbf{J}^s. \]  (4)

Because of the vanishing conductivity term, \( i\omega \mu_0 \sigma \mathbf{E} \) for lower frequencies it is difficult to solve the E-field equation.

- Decomposition of the electric field into potentials

\[ \mathbf{E} = -i\omega \mathbf{A} - \nabla \phi \]  (5)

\( \mathbf{A}(\mathbf{r}, \omega) \) and \( \phi(\mathbf{r}, \omega) \) are vector and scalar potentials respectively.

\[ \nabla \times \nabla \times \mathbf{A} + i\omega \mu_0 \sigma \mathbf{A} + \mu_0 \sigma \nabla \phi = \mu_0 \mathbf{J}^s. \]  (6)
Equation of Conservation of Charge

For low frequencies also the coupling between $\mathbf{E}$ and $\mathbf{H}$ in the Maxwell’s equations reduces and electric charges become important in distorting the field in the conductive medium.

\[
\nabla \cdot (\sigma \mathbf{E}) = \begin{cases} 
-\nabla \cdot \mathbf{J}^s & \text{at the source location,} \\
0 & \text{otherwise,}
\end{cases}
\]

Conservation of charge in the decomposed form

\[-i\omega \nabla \cdot (\sigma \mathbf{A}) - \nabla \cdot (\sigma \nabla \phi) = -\nabla \cdot \mathbf{J}^s.\]
Reiterating the Equations

The E-field system

\[ \nabla \times \nabla \times \mathbf{E} + i \omega \mu_0 \sigma \mathbf{E} = -i \omega \mu_0 \mathbf{J}_s. \quad (9) \]

The Decomposed system

\[ \nabla \times \nabla \times \mathbf{A} + i \omega \mu_0 \sigma \mathbf{A} + \mu_0 \sigma \nabla \phi = \mu_0 \mathbf{J}_s, \quad (10) \]

\[ -i \omega \nabla \cdot (\sigma \mathbf{A}) - \nabla \cdot (\sigma \nabla \phi) = -\nabla \cdot \mathbf{J}_s. \quad (11) \]
Introduction to discretization

Unstructured Tetrahedral meshes

Mesh generator tool: TetGen (Si, 2007)

Plotted using ParaView →
Discretization

Working with unstructured meshes

Flexibility to optimally generate curvilinear contacts with minimal staircasing effects

Only refining the mesh at the region of interest
The method of weighted residuals

\[ R = \int_{\Omega} W \cdot r \, d\Omega = 0, \quad (12) \]

with

\[ r = \nabla \times \nabla \times \tilde{A} + i \omega \mu_0 \sigma \tilde{A} + \mu_0 \sigma \nabla \tilde{\phi} - \mu_0 J^s. \]

\[ \rho = \int_{\Omega} \nu \, r \, d\Omega = 0, \quad (13) \]

with

\[ r = -i \omega \nabla \cdot (\sigma \tilde{A}) - \nabla \cdot (\sigma \nabla \tilde{\phi}) + \nabla \cdot J^s. \]
Finite-element basis functions

Vector basis functions or edge-elements for the approximate vector potential, $\tilde{A}$

$$
\tilde{A} = \sum_{j=1}^{n_A} \tilde{A}_j N_j,
$$

(14)

Scalar basis functions or nodal-elements for the approximate scalar potential, $\tilde{\phi}$

$$
\tilde{\phi} = \sum_{k=1}^{n_\phi} \tilde{\phi}_k N_k.
$$

(15)
Linear Basis functions

(a) Finite-Element functions
(b) Linear Basis functions

(c) $\nabla N_j$
(d) $N_{ji}$

(e) $N_{jm}$
(f) $N_{jl}$
Weighted PDEs

\[\int_{\Omega} (\nabla \times W) \cdot (\nabla \times \tilde{A}) \, d\Omega - \int_{\gamma+\Gamma} W \times (\nabla \times \tilde{A}) \cdot \hat{n} \, dS + \]
\[i\omega \mu_0 \int_{\Omega} \sigma \, W \cdot \tilde{A} \, d\Omega + \mu_0 \int_{\Omega} \sigma \, W \cdot \nabla \phi \, d\Omega = \mu_0 \int_{\Omega} W \cdot J^s \, d\Omega. \tag{16}\]

\[i \omega \int_{\Omega} \nabla v \cdot \sigma \tilde{A} \, d\Omega - i \omega \int_{\gamma+\Gamma} v \, \sigma \tilde{A} \cdot \hat{n} \, dS + \]
\[\int_{\Omega} \nabla v \cdot \sigma \nabla \phi \, d\Omega - \int_{\gamma+\Gamma} v \, \sigma \nabla \phi \cdot \hat{n} \, dS = - \int_{\Omega} v \, \nabla \cdot J^s \, d\Omega. \tag{17}\]

Applying the Galerkin Method and Boundary conditions to the above ...
The $\mathbf{A} - \phi$ system in the matrix form

$$
\begin{pmatrix}
\mathbf{C} + i\omega\mu_0\mathbf{D} & \mu_0\mathbf{F} \\
i\omega\mathbf{G} & \mathbf{H}
\end{pmatrix}
\begin{pmatrix}
\tilde{\mathbf{A}} \\
\tilde{\phi}
\end{pmatrix}
= 
\begin{pmatrix}
\mu_0\mathbf{S}_1 \\
\mathbf{S}_2
\end{pmatrix}
$$

(18)

Finite-element inner product functionals:

$\mathbf{C} = \mathcal{F}(\mathbf{N})$

$\mathbf{D} = \mathcal{F}(\sigma, \mathbf{N})$

$\mathbf{F} = \mathcal{F}(\sigma, \mathbf{N}, \nabla \mathbf{N})$

$\mathbf{G} = \mathcal{F}(\sigma, \mathbf{N}, \nabla \mathbf{N})$

$\mathbf{H} = \mathcal{F}(\sigma, \nabla \mathbf{N})$.

$\tilde{\mathbf{A}}$ and $\tilde{\phi}$ are the approximated potentials.
Source function

\[ S_1 = \int_\Omega N_i \cdot J^s \, d\Omega , \quad S_2 = -\int_\Omega N_1 \nabla \cdot J^s \, d\Omega. \]

Arbitrarily positioned in the mesh.

\[ J = I (\mathcal{H}(x_{i+1}) - \mathcal{H}(x_i)) \delta(y - y_0) \delta(z - z_0) \]  

(19)
Iterative solution and preconditioning

\[
\begin{pmatrix}
C + i\omega \mu_0 D & \mu_0 F \\
i\omega G & H
\end{pmatrix}
\begin{pmatrix}
\tilde{A} \\
\tilde{\phi}
\end{pmatrix}
= 
\begin{pmatrix}
\mu_0 S_1 \\
S_2
\end{pmatrix}
\] (20)

ILUT preconditioning (Saad, 1990) with an appropriate fill-in factor prior to solution

Iterative solver of GMRES from SPARSKIT (Saad, 1990): A generalized minimum residual method in the Krylov subspace

\[
\tilde{E} = -i\omega \tilde{A} - \nabla \tilde{\phi}
\]

Inductive part: \(-i\omega \tilde{A}\)
Galvanic part: \(-\nabla \tilde{\phi}\)
Frequency = 0.1 Hz, cells: 708796, nodes: 116058, edges: 825232. \( \sigma_{\text{air}} = 10^{-8} \text{ S/m} \), \( \sigma_{\text{Earth}} = 0.01 \text{ S/m} \)
Electric dipole source

Comparison with the analytic total field solution.
Solver parameters

GMRES solver, lfill = 3.
Dimension of the Krylov subspace: 200.
Residual norm of $10^{-12}$ after 4000 iterations.
Computation time for the solution was roughly 30 minutes on an Apple Mac Pro computer (2.4 GHz Quad-core Intel Xenon processor) with a total memory usage of 8 Gbytes.
Inductive and Galvanic components
**A − φ solution against the E-field solution**

The E-field system

\[
\nabla \times \nabla \times E + i\omega \mu_0 \sigma E = -i\omega \mu_0 J^s,
\]

\[
(C + i\omega \mu_0 D)(\tilde{E}) = (i\omega \mu_0 S_1)
\tag{21}
\]

The **A − φ** system

\[
\begin{pmatrix}
C + i\omega \mu_0 D & \mu_0 F \\
i\omega G & H
\end{pmatrix}
\begin{pmatrix}
\tilde{A} \\
\tilde{\phi}
\end{pmatrix} =
\begin{pmatrix}
\mu_0 S_1 \\
S_2
\end{pmatrix}
\tag{22}
\]
Magnetic dipole source

Examples

Efficiency of the approach

Magnetic dipole source

300 Hz

Real

 Imaginary

Analytic

Decomposed FE

E-field FE

3 Hz

|H_z| (V/m)

−1000 −500 0 500 1000

a)

b)

c)

d)

−6000 −4000 −2000 0 2000 4000 6000

X (m)

|H_z| (V/m)

−1000 −500 0 500 1000

a)

b)

c)

d)
Residual norms

Rapid convergence of the $\mathbf{A} - \phi$ solution
Slow convergence of the E-field solution

![Graph showing residual norms for different frequencies](image)
Grounded wire and prism

A mineral exploration scenario done by Li, Oldenburg and Shekhtman, 1999 : DCIP3D
Farquharson and Oldenburg, 2002 : Verification of the Integral Equation code
Frequency = 3 Hz
Unstructured Mesh

cells: 613300, nodes: 99855, and edges: 713542
Electric fields

Comparison against Integral Equations and DC-resistivities.
Scattered fields

- Examples: Grounded wire and prism-in-half-space

- Scatter fields

- $10^{-10}$  $10^{-9}$  $10^{-8}$  $10^{-7}$  $10^{-6}$

- Secondary $E_x$ (V/m)

- Distance from source (m)

- Real IE
- Imag IE
- Real FE
- Imag FE
- DCIP3D

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Inductive and Galvanic fields
Inductive and Galvanic current density
Transmitter-Receiver pair and cube in brine

Comparison against the physical scale modeling of Farquharson et al., 2006 and Finite-volume solution of Jahandari and Farquharson, 2013.

Five frequencies 1, 10, 100, 200, and 400 kHz.

Free space $10^{-4}$ S/m

Brine 7.3 S/m

Graphite $6.3 \times 10^4$ S/m
Unstructured Mesh

Graphite cube in brine model

X (cm)  Z (cm) 25-10  60  16  -16  0

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Normalized magnetic fields: FE, FV, Physical Scale modeling

In phase

Quadrature

H_z (%)

X (cm)
Inductive and Galvanic fields

![Graphite cube in brine model](image)

- Inductive Real
- Inductive Imaginary
- Galvanic Real
- Galvanic Imaginary
- Total Real
- Total Imaginary

\[ \log_{10} E (\text{V/m}) \]
Inductive and Galvanic current densities

Examples

Graphite cube in brine model

- Inductive Real
- Inductive Imaginary
- Galvanic Real
- Galvanic Imaginary
- Total Real
- Total Imaginary

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3D Modeling

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 Canonical disk model in marine sediments

A hydrocarbon exploration scenario by Constable and Weiss (2006)

Frequency of 1 Hz

3.3 S/m
In-line total electric fields.

![Graph showing in-line total electric fields with log-log scale. The graph plots electric field magnitude against distance, with different symbols representing different diameter scenarios.]
In-line total electric fields.

![Graph showing in-line total electric fields with X (m) on the horizontal axis and |E_z| (V/m) on the vertical axis. The graph includes three curves representing different diameters: 2000-mm diameter, 5000-mm diameter, and half-space. The curves show the electric field decreasing with increasing X (m).]
In-line total electric fields.

![Graph showing in-line total electric fields with various diameters marked as half-space, 2000-diameter, and 5000-diameter.](image)
Inductive and Galvanic fields

- Inductive Real
- Inductive Imaginary
- Galvanic Real
- Galvanic Imaginary
- Total Real
- Total Imaginary

\[ \text{log}_{10} E (V/m) \]
Inductive and Galvanic current densities

- Inductive Real
- Inductive Imaginary
- Galvanic Real
- Galvanic Imaginary
- Total Real
- Total Imaginary

$\log_{10} J \text{ (A/m}^2\text{)}$
Conclusions

- A 3D finite-element solution for forward modeling of geophysical electromagnetic problems is presented.

- The algorithm is written for the total field approximation on unstructured tetrahedral meshes.

- The approach is based on decomposing the electric field into vector and scalar potentials in the Helmholtz equation and equation of conservation of charge.

- The decomposition is done not only from the perspective of solving the equations efficiently, but also in order to delve into the physical meaning of the inductive and galvanic components.

- We verified the method for multiple examples in different geophysical scenarios where either electric and magnetic sources are used.
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