Joint inversion of seismic traveltimes and gravity data on unstructured grids with application to mineral exploration

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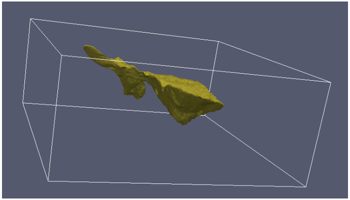
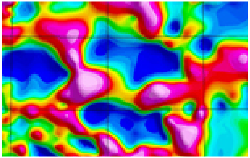
SEG Denver, MIN1, October 20, 2010

Problem statement

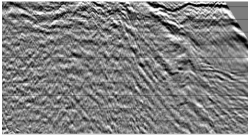
Forward problem



Earth model (density, velocity)



Gravity data



Seismic data

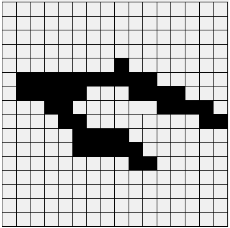
Inverse problem

Motivation: joint inversion

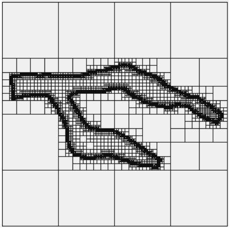
- Complicated hard-rock geology can cause difficulties with seismic data processing and interpretation
- Improve resolution (different sensitivities)
- Reduce uncertainty (limit number of acceptable models)

Motivation: unstructured grids

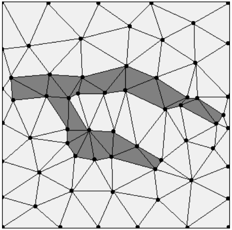
- Efficient generation of complicated subsurface geometries when known *a priori*
- Significant reduction in problem size



Rectilinear



Quadtree, Octree



Unstructured

Forward modelling: two types of data

Gravity data

- Analytic response of a triangle, tetrahedron (Okabe, 1979, Geophys.)
- Finite element solution to Poisson's equation

Seismic data

- First-arrival traveltimes
- Fast Marching Method (Sethian, 1996, P.N.A.S.; Lelièvre et al., *in review*, G.J.I.)

Joint optimization problem

Single dataset

- Objective function

$$\Phi = \beta\Phi_d + \Phi_m$$

- Data misfit

$$\Phi_d = \sum_i \left(\frac{d_i^{pred}(m) - d_i^{obs}}{\sigma_i} \right)^2$$

- Model structure (regularization)

$$\Phi_m = [\text{smallness term}] + [\text{smoothness term}]$$

Joint optimization problem

Single dataset

$$\Phi = \beta \Phi_d + \Phi_m$$

Two datasets

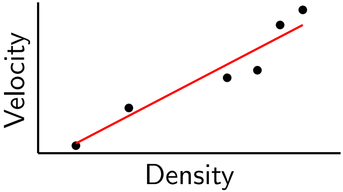
$$\Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \Phi_{joint}$$

$$\Phi_{joint} = \sum_j \rho_j \Psi_j \left(m_1, m_2 \right)$$

Measures of model similarity: compositional

Explicit analytic relationship

- From sample measurements
- Linear-Linear
- Log-Linear
- Log-Log, etc.

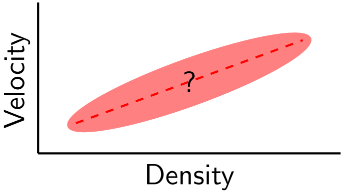


$$\Psi(m_1, m_2) = \sum_{i=1}^{M} (a m_{1,i} + b m_{2,i} + c)^2$$

Measures of model similarity: compositional

Implicit analytic relationship

- "Some" (linear) relationship expected
- Correlation from statistics
- Independent of scale of physical properties

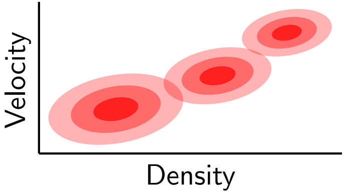


$$\Psi\left(m_1, m_2\right) = \left(\frac{\sum_{i=1}^{M}\left(m_{1,i} - \mu_1\right)\left(m_{2,i} - \mu_2\right)}{M\sigma_1\sigma_2} \pm 1\right)^2$$

Measures of model similarity: compositional

Statistical relationship

- From sample measurements
- Probability density function
  e.g. combination of Gaussians
- Fuzzy C-means clustering
  (Paasche & Tronicke, 2007,
  Geophys.)

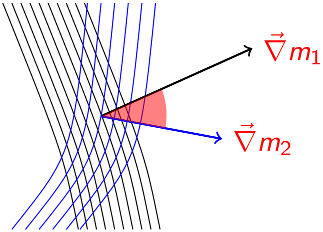


$$\Psi\left(m_1, m_2\right) = \sum_{k=1}^{C} \sum_{i=1}^{M} w_{ik}^2 \left(\left(m_{1,i} - u_{1,k}\right)^2 + \left(m_{2,i} - u_{2,k}\right)^2\right)$$

Measures of model similarity: structural

Assumed spatial correlation
(changes occur in same place)

- "Structural" similarity
  (versus "compositional")
- Curvature measure
  (Haber & Oldenburg, 1997, Inv. Probs.)
- Cross-gradients
  (Gallardo & Meju, 2004, J.G.R.)



$$\Psi(m_1, m_2) = \left\| \vec{\nabla} m_1 \times \vec{\nabla} m_2 \right\|^2$$

Measures of model similarity: key point

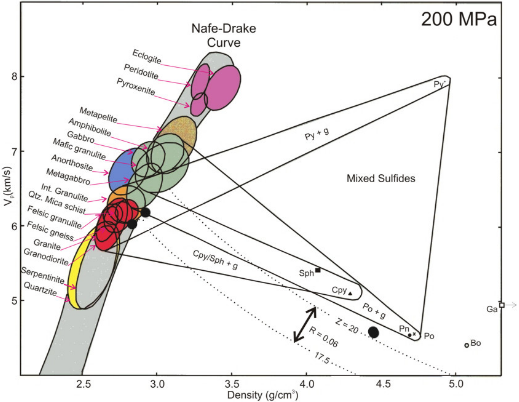
- There are many joint inversion tools available (many joint similarity measures). Those applied should depend on one's existing knowledge of the subsurface.

Voisey's Bay: the ovoid deposit

- Labrador, Canada
- Massive sulphide deposit (nickel-copper-cobalt)
- A triangulated surface model for the ovoid has been generated from drillcore logging
  - problematic to discretize on a rectilinear grid

Voisey's Bay: rock types and physical properties

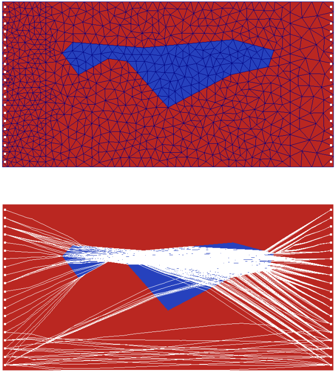
Ovoid has high density, high slowness (low velocity) compared to surrounding rock



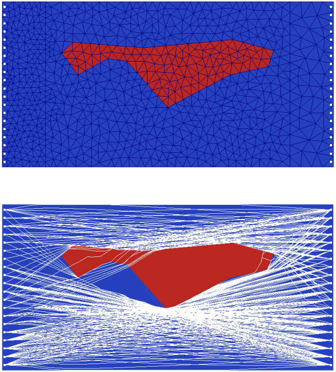
Density versus velocity (Salisbury et al, 2003)

2D scenarios: true models and rays

First arrival energy paths avoid slow (red) regions, prefer fast (blue) regions



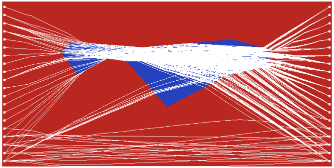
0.0, 2.0 g/cc ; 0.16, 0.11 s/km

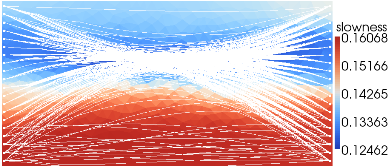


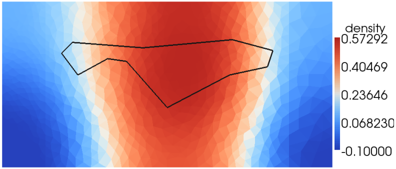
0.0, 2.0 g/cc ; 0.16, 0.23 s/km

2D scenario #1: independent inversions

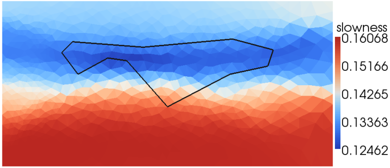
Gravity gives lateral resolution; first-arrivals give depth resolution; nonlinear seismic regime







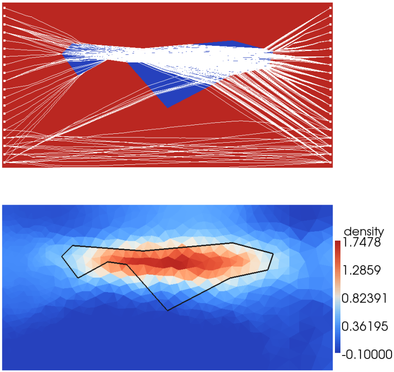
-0.10 to 0.57 g/cc



0.125 to 0.161 s/km

2D scenario #1: explicit linear relationship

Density is compacted which increases density and therefore slowness decreases

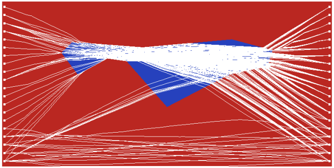


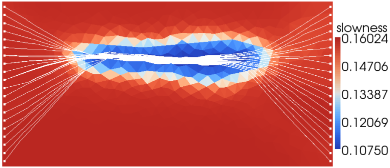
-0.10 to 1.75 g/cc

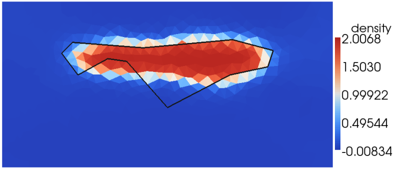
0.108 to 0.161 s/km

2D scenario #1: explicit linear relationship and clustering

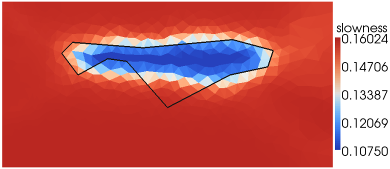
Density increased further





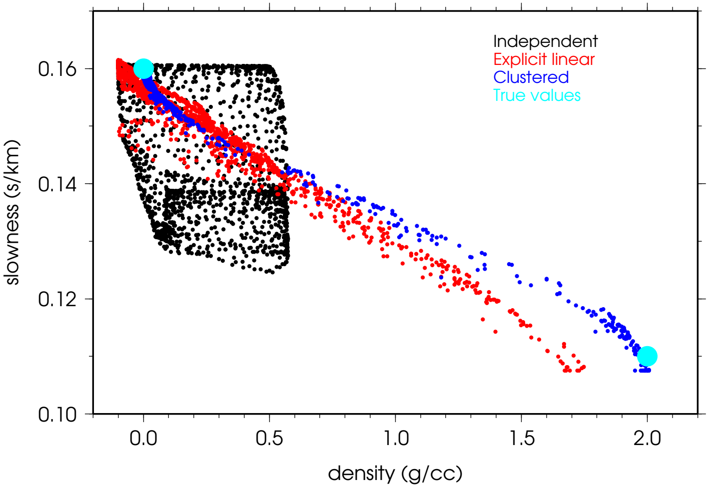


-0.01 to 2.01 g/cc



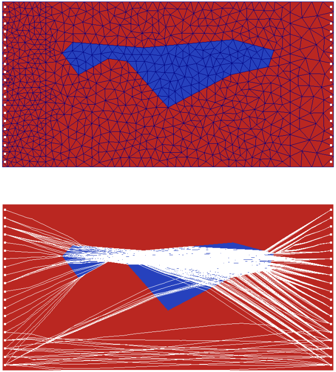
0.108 to 0.160 s/km

2D example #1: density versus slowness

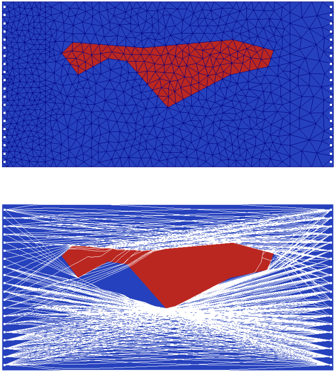


2D scenarios: true models and rays

First arrival energy paths avoid slow (red) regions, prefer fast (blue) regions



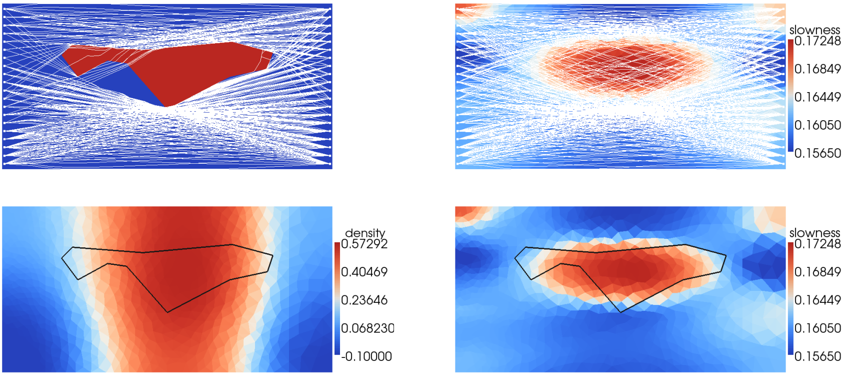
0.0, 2.0 g/cc ; 0.16, 0.11 s/km



0.0, 2.0 g/cc ; 0.16, 0.23 s/km

2D scenario #2: independent inversions

Gravity gives lateral resolution; first-arrivals give depth resolution; LINEAR SEISMIC REGIME

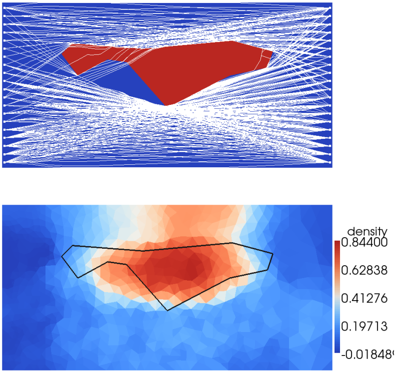


-0.10 to 0.57 g/cc

0.157 to 0.172 s/km

2D scenario #2: explicit linear relationship

Density is compacted which increases density and therefore slowness increases

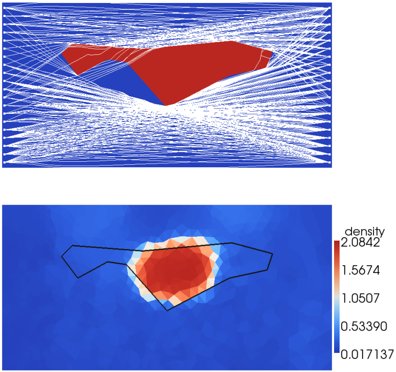


-0.02 to 0.84 g/cc

0.157 to 0.181 s/km

2D scenario #2: explicit linear relationship and clustering

Left and right extensions lost (MULTIPLE MINIMA)

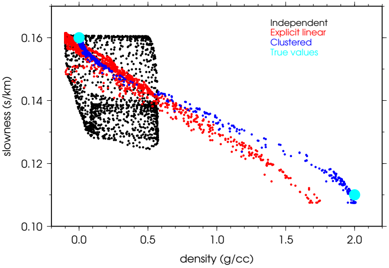


0.02 to 2.08 g/cc

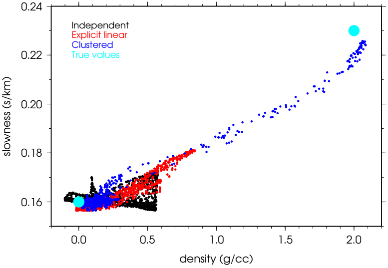
0.157 to 0.226 s/km

2D scenarios: density versus slowness

Independent
Explicit linear
Clustered
True values



Scenario #1 (fast ovoid)



Scenario #2 (slow ovoid)

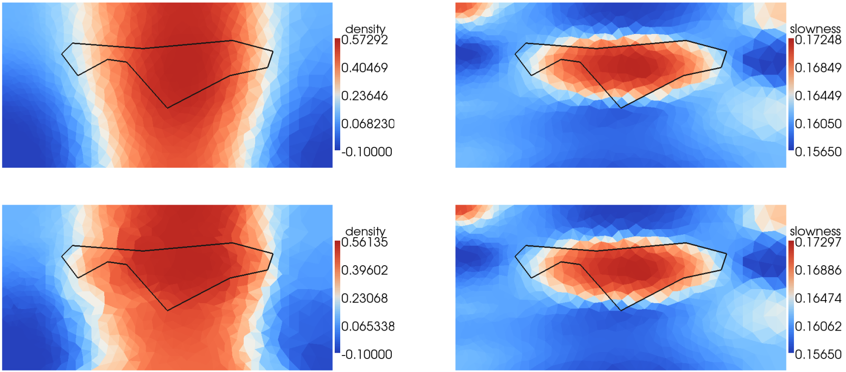
2D scenario #2: lessons learned

We need to push the seismic inversion into the nonlinear regime

- Incorporation of explicit linear relationship increases slowness but not enough
- Incorporation of cluster information increases slowness further but multiple minima are problematic

2D scenario #2: cross-gradient measure

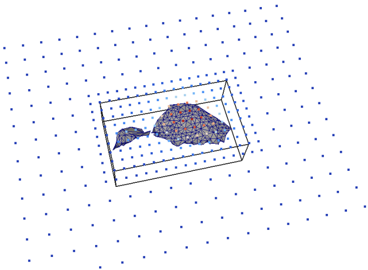
Not effective for this scenario because it does not force the anomalous slowness higher



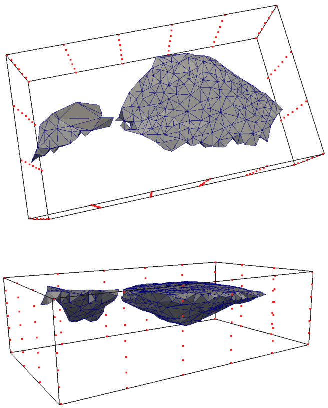
-0.10 to 0.56 g/cc

0.157 to 0.173 s/km

3D example: true model and surveys



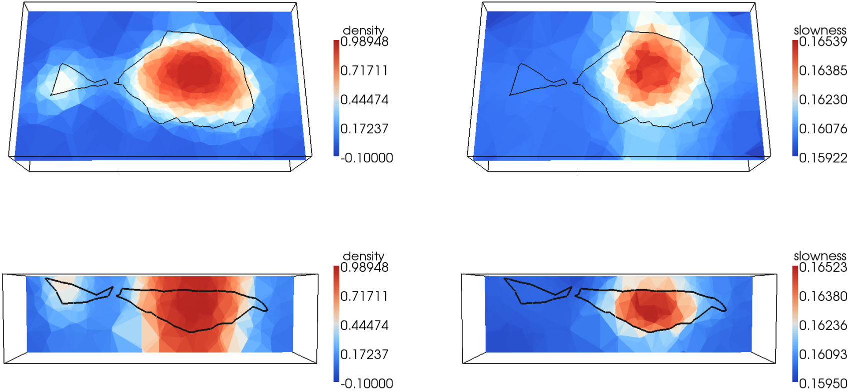
Gravity data



Sources & receivers

3D example: independent inversion results

Gravity gives lateral resolution; first-arrivals give depth resolution

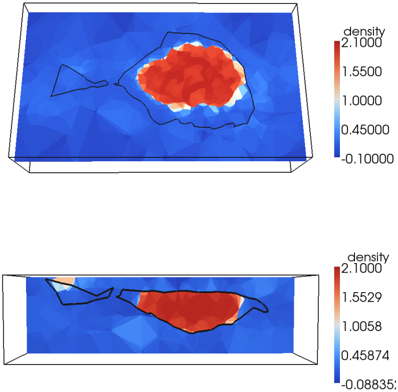


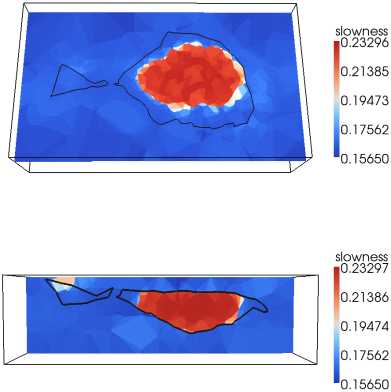
Density = -0.10 to 0.99

Slowness = 0.160 to 0.165

3D example: joint inversion results

Gravity gives lateral resolution; first-arrivals give depth resolution





Density = -0.09 to 2.10 Slowness = 0.157 to 0.233

Summary

- We consider many joint similarity measures; those applied should depend on one's existing knowledge of the subsurface
- The slow body in faster background scenario contains some significant challenges not seen in the opposite scenario
  - We have obtained promising results but ...

Future directions

- Global optimization strategy for clustering joint measures
- Alternate regularization scheme
- 3D joint inversion of survey data from Voisey's Bay

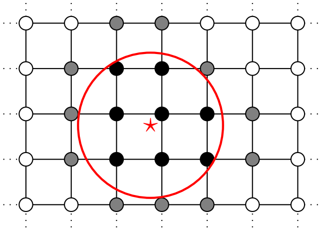
Acknowledgements

- ACOA
  (Atlantic Canada Opportunities Agency)
- NSERC
  (Natural Sciences and Engineering Research Council of Canada)
- Vale

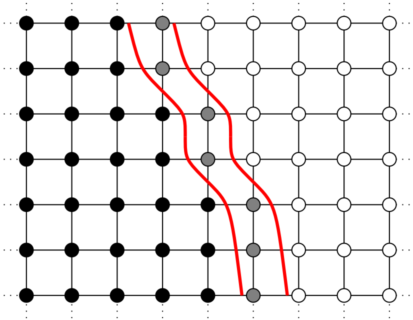
(additional slides follow)

Seismic first-arrivals: fast marching solution

1) Initialization near-source

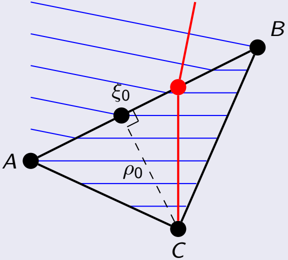


2) Solution-front marching



Seismic first-arrivals: local update via Fermat's Principle

2D triangular grid

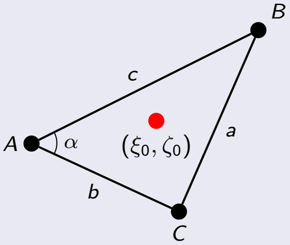


$$t_C = t_A + (t_B - t_A)\,\xi_0 + wc^{-1}\rho_0$$

$$w = \sqrt{s^2c^2 - (t_B - t_A)^2}$$

(Fomel, 2000, S.E.P.)

3D tetrahedral grid

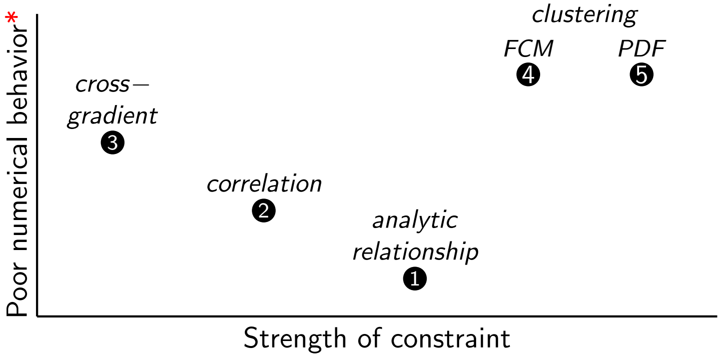


$$t_D = t_A + (t_B - t_A)\,\xi_0 + \ldots$$
$$+ (t_C - t_A)\,\zeta_0 + \tilde{w}\varphi^{-1}\rho_0$$
$$\tilde{w} = \ldots$$

(Lelièvre et al., *in review*, G.J.I.)

Measures of model similarity: strength and behavior

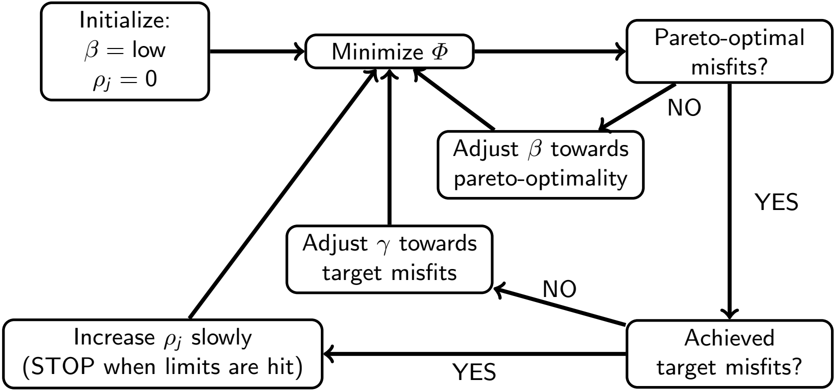
The joint similarity measure(s) applied should depend on one's existing knowledge of the subsurface.



* nonlinearity, multiple minima

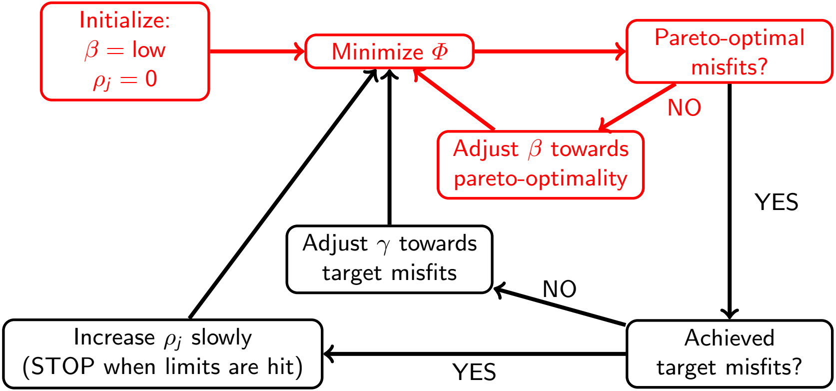
Algorithm: how to deal with two trade-off parameters?

$$\Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \sum_j \rho_j \Psi_j$$



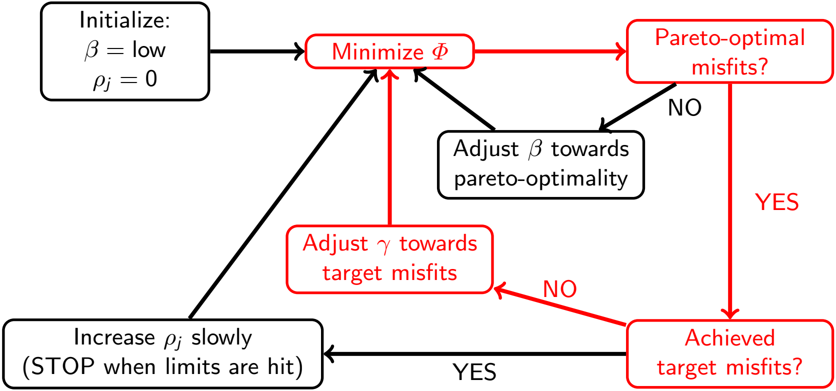
Algorithm: single beta, pareto search

$$\Phi = \beta\left(\Phi_{d1} + \gamma\Phi_{d2}\right) + \Phi_{m1} + \Phi_{m2} + \sum_j \rho_j \Psi_j$$



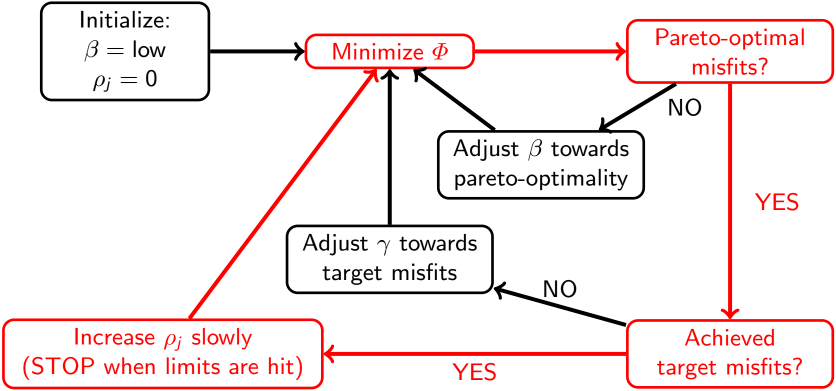
Algorithm: beta ratio, pareto search

$$\Phi = \beta\left(\Phi_{d1} + \gamma\Phi_{d2}\right) + \Phi_{m1} + \Phi_{m2} + \sum_j \rho_j \Psi_j$$



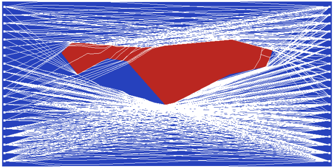
Algorithm: slow heating of joint measures

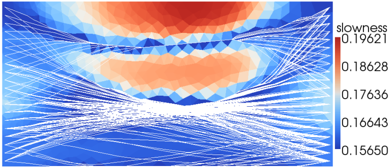
$$\Phi = \beta\left(\Phi_{d1} + \gamma\Phi_{d2}\right) + \Phi_{m1} + \Phi_{m2} + \sum_j \rho_j \Psi_j$$

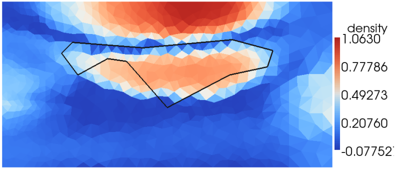


2D scenario #2: stronger explicit linear relationship

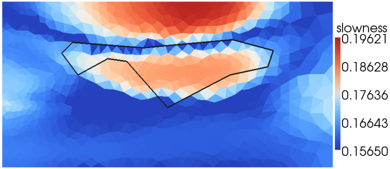
Undesired artifacts present







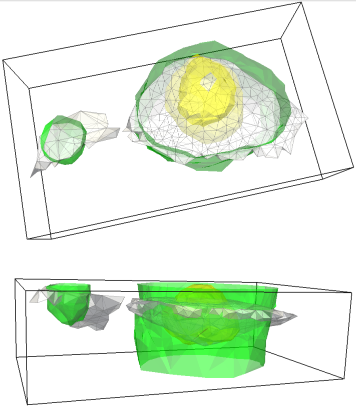
-0.08 to 1.06 (0.75) g/cc



0.157 to 0.196 (0.185) s/km

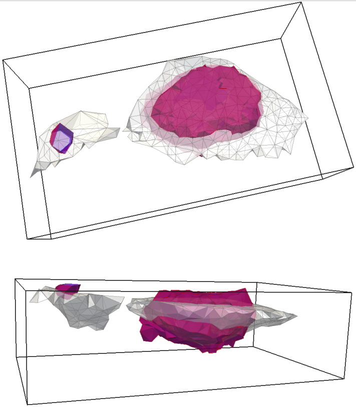
3D example: inversion results

Gravity gives lateral resolution; first-arrivals give depth resolution



Independent

density , slowness

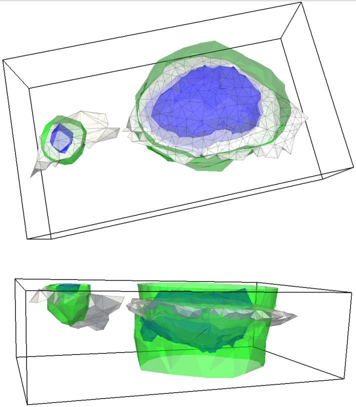


Joint: explicit linear & clustered

density , slowness

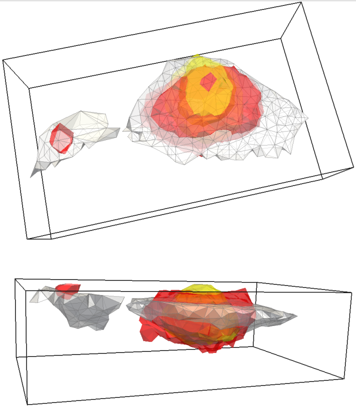
3D example: inversion results

Gravity gives lateral resolution; first-arrivals give depth resolution



Density

independent , joint



Slowness

independent , joint

3D example: density versus slowness