Refining 3D Earth models through constrained joint inversion on flexible unstructured meshes

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Motivation

- 3D Earth models typically comprise wireframe surfaces of connected triangles that represent geological contacts

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- Earth models used by most current 3D geophysical numerical modeling and inversion methods are built on rectilinear meshes

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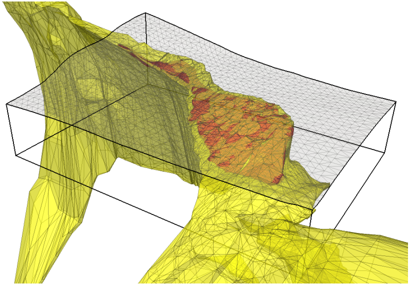
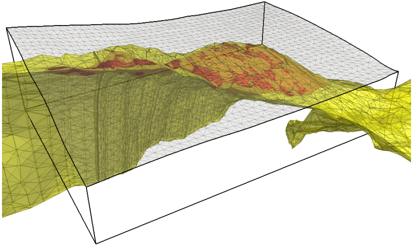
- 3D Earth models typically comprise wireframe surfaces of connected triangles that represent geological contacts

Voisey's Bay

- nickel-copper-cobalt deposit
- north-east coast of Labrador
- the "ovoid" (main sulfide ore body) is currently being mined





Geophysical Models

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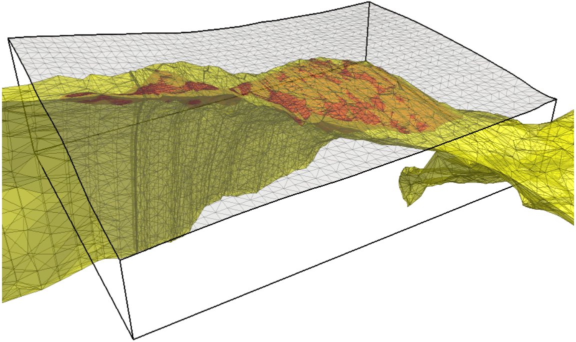
- simplify development of numerical methods
- produce pixellated representations
- can be impossible to adequately model complicated geology
- incompatible with wireframe geological models

Discretizing Voisey's Bay on Rectilinear Mesh

- some intermediary process is always required to convert from a geophysical model to a geological one, or the reverse (e.g. see Nick Williams, 2008, PhD Thesis, UBC)



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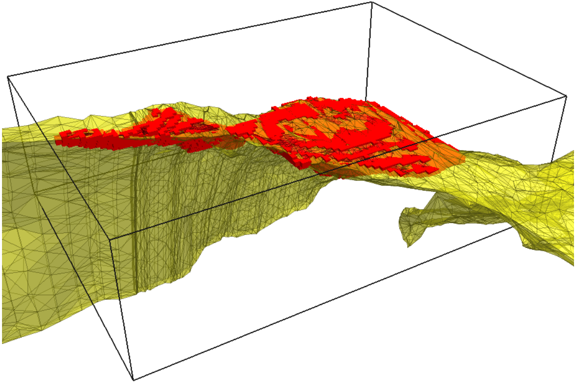

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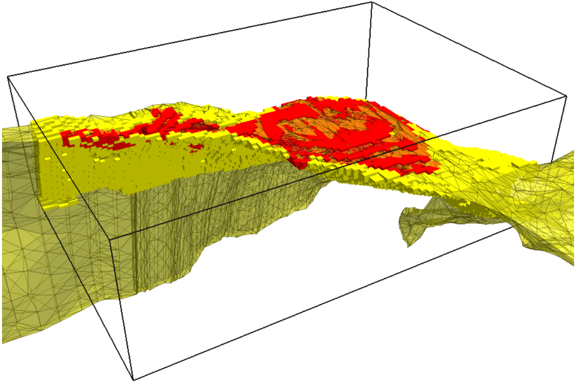

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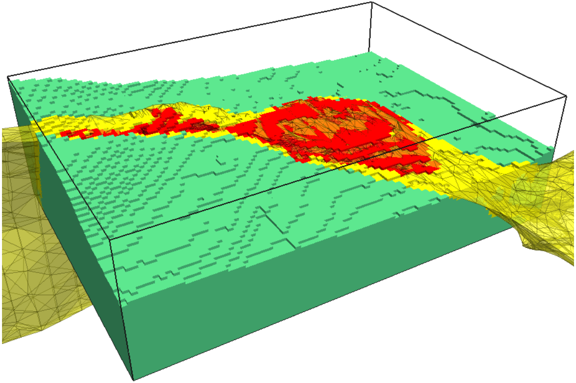

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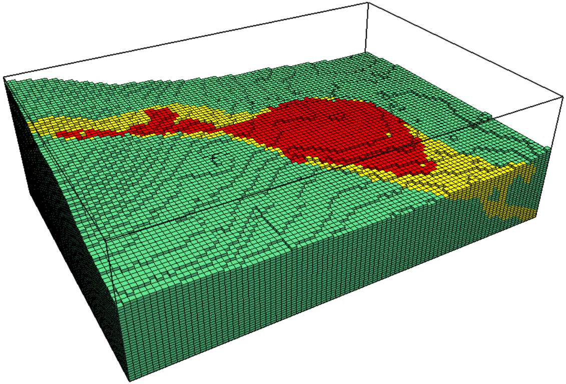
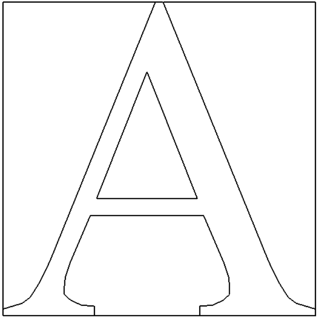
87x61x54
= 286,578
cells

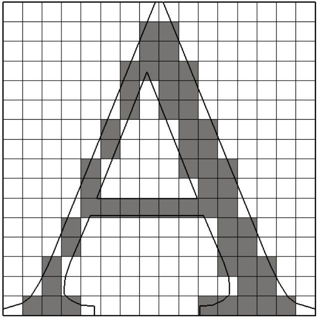


stair-casing
still evident

Discretization Options: Rectilinear

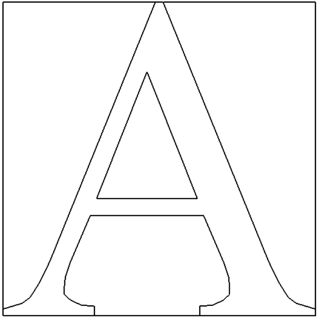
- may require infeasibly many cells for adequate representation
- pixellated representation
- 256 cells

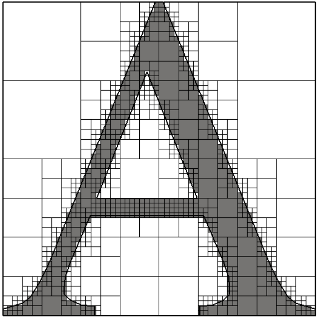




Discretization Options: Quadtree/Octree

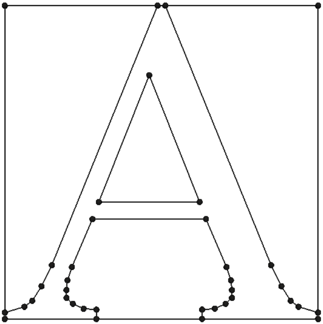
- need fewer cells and are still structured
- pixellated representation
- 946 cells (4096 in underlying regular mesh)

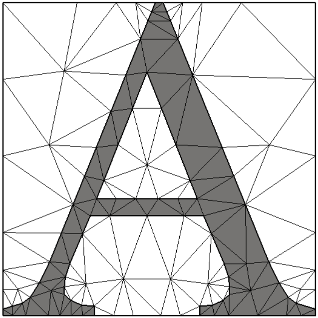




Discretization Options: Unstructured

- efficient generation of complicated geometries
- significant reduction in problem size
- 183 cells (compare to 4096 and 946; factor of 22 and 5.2)





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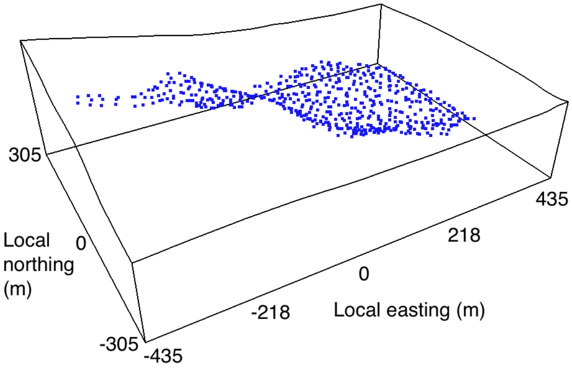
Challenges:

- mathematics of numerical modelling on tetrahedral meshes
- create, manipulate and visualize Earth models

Wireframe Reconstruction from Point Clouds

- the amount of drilling necessary to define an ore body before the advanced exploration or development stage is often substantial



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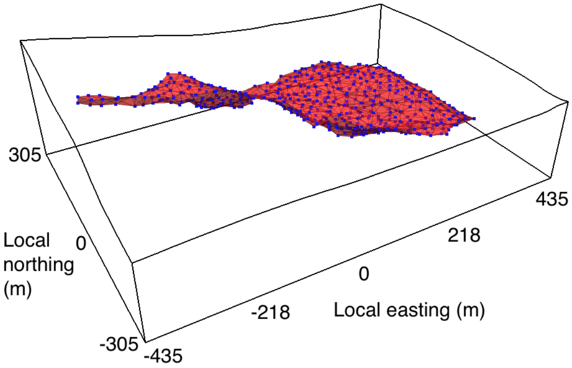
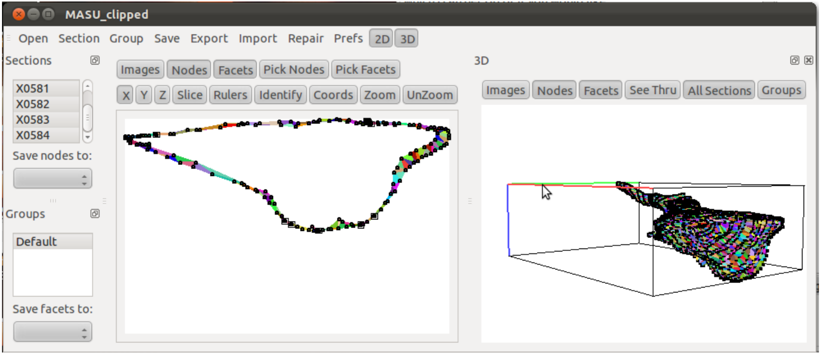


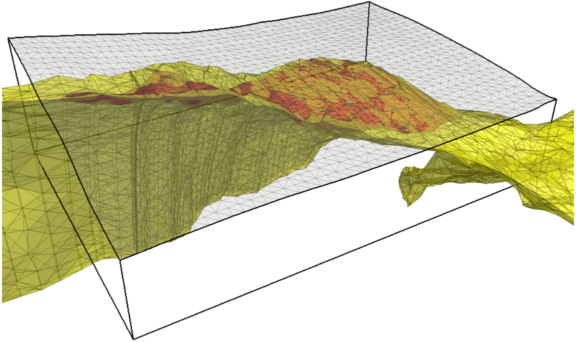
Wireframe Creation/Manipulation by Hand

- FacetModeller, Blender
- ParaView



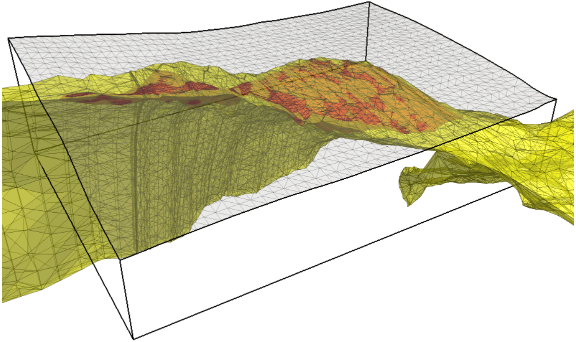
Volumetric Discretization of Wireframes

- TetGen generates 3D tetrahedral meshes from piecewise polygonal complexes (PPCs)
- interconnected planar polygonal facets (boundary, topography, contacts, etc.)



Volumetric Discretization of Wireframes

- TetGen discretizes the volume between the tessellated surfaces while maintaining those surfaces exactly
- geological and geophysical models can share the same modelling mesh; they can be the same model



Forward Modelling on Unstructured Meshes

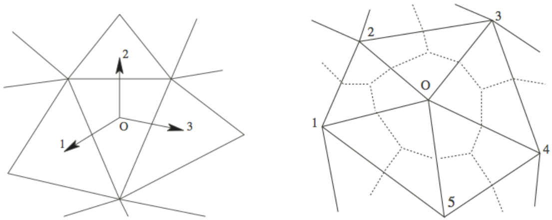
We have developed modelling methods for various data types:

- gravity
- magnetic
- seismic (first-arrivals)
- geoelectric (surfaces)
- electromagnetic

Forward Modelling on Unstructured Meshes

Gravity, Magnetics:

- closed form expression for tetrahedra (Okabe, 1979)
- finite volume or finite element solution of Poisson's equation



(Hormoz Jahandari, PhD Student)

Forward Modelling on Unstructured Meshes

Electromagnetics:

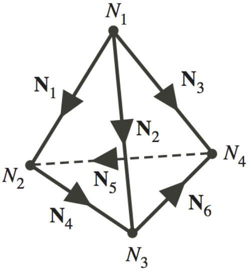
- decomposition into inductive and galvanic parts

$$\mathbf{E} = -i\omega\mathbf{A} - \nabla\phi$$

- finite-element solution using edge and nodal elements

$$\mathbf{A}(\mathbf{r}) = \sum_{j=1}^{N_{edges}} A_j \mathbf{N}_j(\mathbf{r})$$

$$\phi(\mathbf{r}) = \sum_{k=1}^{N_{nodes}} \phi_k N_k(\mathbf{r})$$

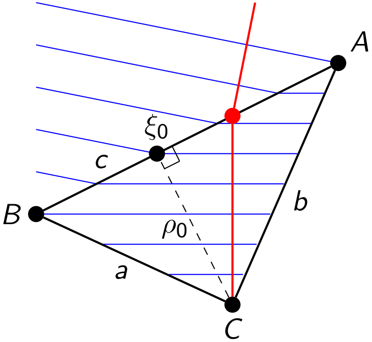


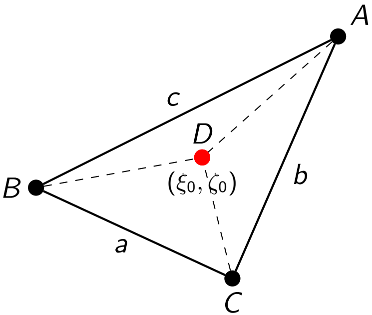
(Seyedmasoud Ansari, PhD student)

Forward Modelling on Unstructured Meshes

Seismic (First-Arrivals):

- fast marching method; assume planar wavefronts





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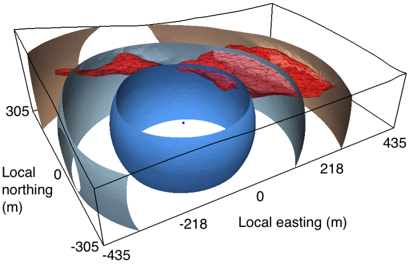
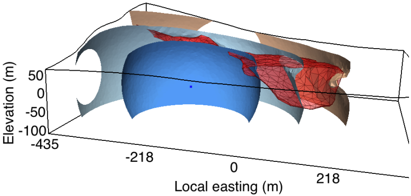




Aside: Standard Deterministic Inversion Approach

Single dataset

- Objective function

$$\Phi = \Phi_d + \beta\Phi_m$$

- Data misfit

$$\Phi_d = \sum_i \left(\frac{d_i^{pred}(m) - d_i^{obs}}{\sigma_i} \right)^2$$

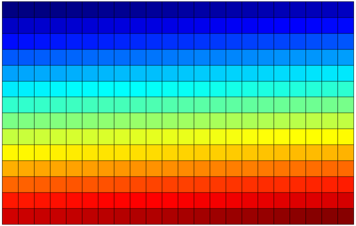
- Model structure (regularization)

$$\Phi_m = [\text{smallness term}] + [\text{smoothness term}]$$

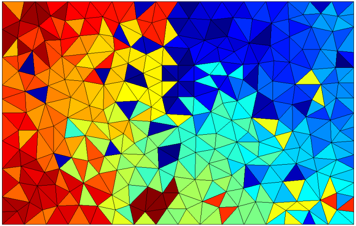
Challenges for Inversion on Unstructured Meshes

Algorithms can exploit mesh structure:

- sparsity structure of finite-difference operators
- sensitivity compression via wavelet transform

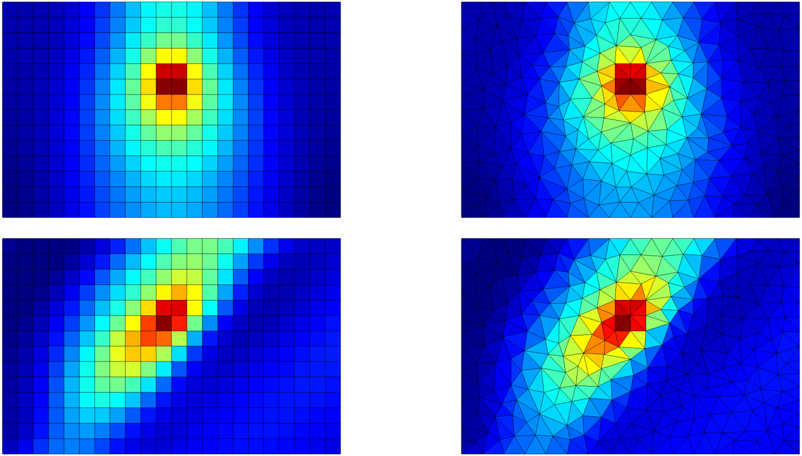


regular rectilinear



triangular unstructured

The Same Regularization is Possible



regular rectilinear

triangular unstructured

Aside: Joint Inversion

Single dataset

$$\Phi = \Phi_d + \beta\Phi_m$$

Two datasets

$$\Phi = \lambda_1\Phi_{d1} + \lambda_2\Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \Phi_{joint}$$

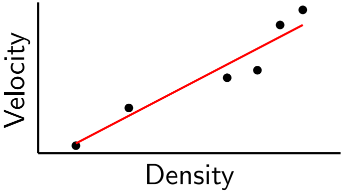
$$\Phi_{joint} = \sum_j \rho_j \Psi_j \left(m_1, m_2\right)$$

The joint similarity measure(s) applied should depend on one's existing knowledge of the subsurface.

Measures of model similarity: compositional

Explicit analytic relationship

- From sample measurements
- Linear-Linear
- Log-Linear
- Log-Log, etc.

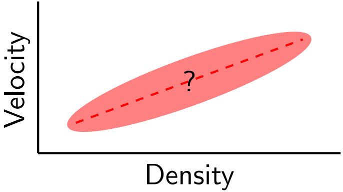


$$\Psi(m_1, m_2) = \sum_{i=1}^{M} (a m_{1,i} + b m_{2,i} + c)^2$$

Measures of model similarity: compositional

Implicit analytic relationship

- "Some" (linear) relationship expected
- Correlation from statistics
- Independent of scale of physical properties

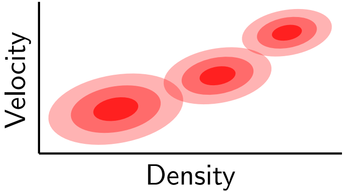


$$\Psi\left(m_1, m_2\right) = \left(\frac{\sum_{i=1}^{M}\left(m_{1,i}-\mu_1\right)\left(m_{2,i}-\mu_2\right)}{M\sigma_1\sigma_2} \pm 1\right)^2$$

Measures of model similarity: compositional

Statistical relationship

- From sample measurements
- Probability density function e.g. combination of Gaussians
- Fuzzy C-means clustering (Paasche & Tronicke, 2007, Geophys.)

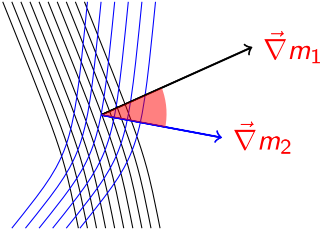


$$\Psi(m_1, m_2) = \sum_{k=1}^{C} \sum_{i=1}^{M} w_{ik}^2 \left((m_{1,i} - u_{1,k})^2 + (m_{2,i} - u_{2,k})^2 \right)$$

Measures of model similarity: structural

Assumed spatial correlation
(changes occur in same place)

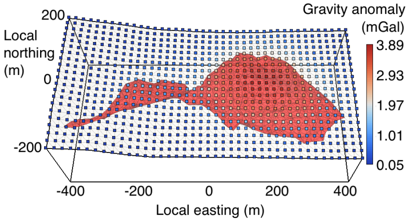
- "Structural" similarity
  (versus "compositional")
- Curvature measure
  (Haber & Oldenburg, 1997, Inv. Probs.)
- Cross-gradients
  (Gallardo & Meju, 2004, J.G.R.)



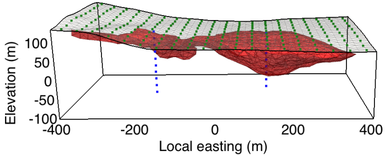
$$\Psi\left(m_1, m_2\right) = \left\|\vec{\nabla} m_1 \times \vec{\nabla} m_2\right\|^2$$

Inversion on Unstructured Meshes

- joint inversion of gravity and first-arrival traveltimes



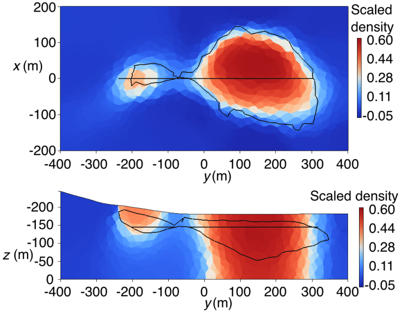
gravity data



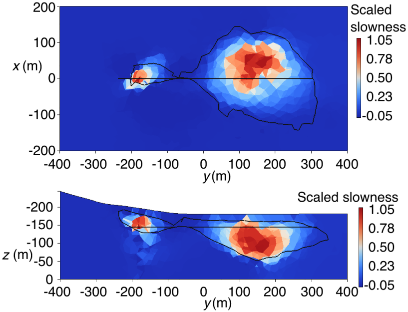
seismic sources and receivers

Inversion on Unstructured Meshes

- joint inversion of gravity and first-arrival traveltimes
- independent inversions



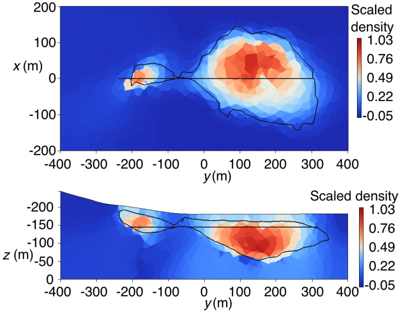
density



slowness

Inversion on Unstructured Meshes

- joint inversion of gravity and first-arrival traveltimes
- linear relationship



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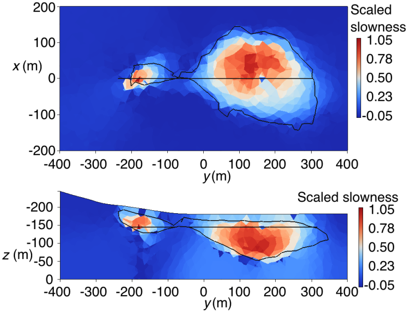
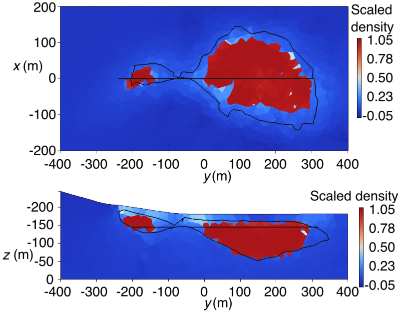


slowness

Inversion on Unstructured Meshes

- joint inversion of gravity and first-arrival traveltimes
- clustering (fuzzy c-means)



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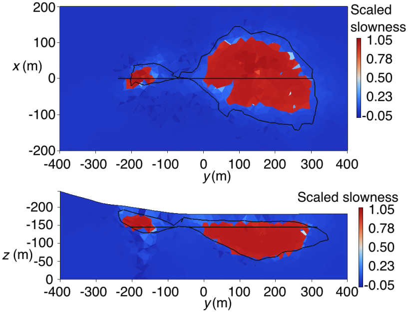
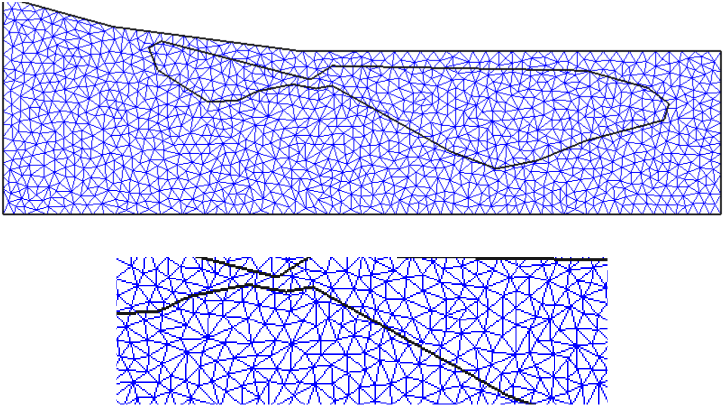


slowness

Inversion on Unstructured Meshes

- non-conforming vs. conforming mesh (the inverse "crime")



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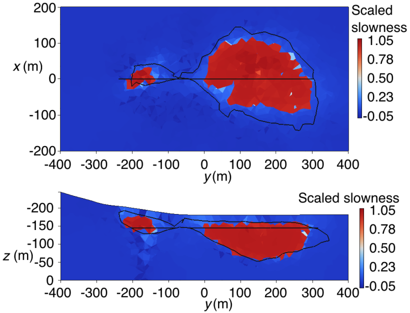
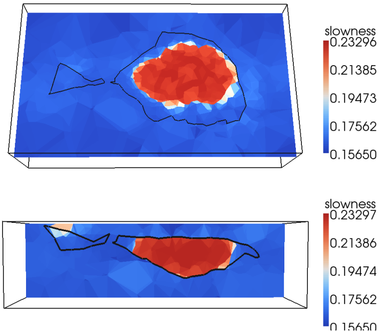


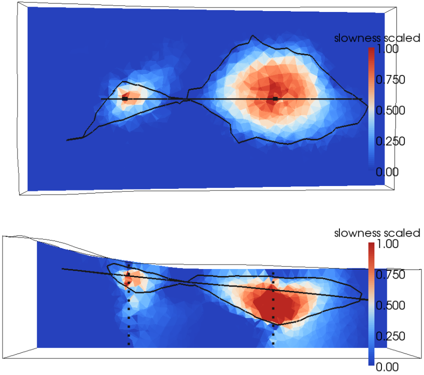
non-conforming mesh



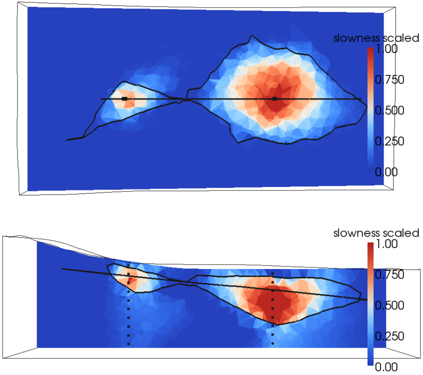
conforming mesh

Inversion on Unstructured Meshes

- first-arrival traveltime inversion



non-conforming mesh



conforming mesh

Conclusion

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- most current 3D geophysical modelling is performed on rectilinear meshes
- unstructured meshes allow for efficient incorporation of complicated *a priori* geometries
  (forward modelling; constrained inversions)

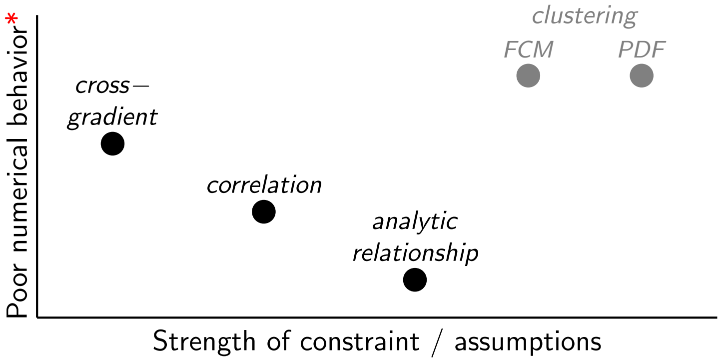
Acknowledgements

- ACOA
  (Atlantic Canada Opportunities Agency)
- NSERC
  (Natural Sciences and Engineering Research Council of Canada)
- Vale

Additional Slides Follow

Measures of model similarity: strength and behavior

The joint similarity measure(s) applied should depend on one's existing knowledge of the subsurface.



* nonlinearity, multiple minima

Inversion on Unstructured Meshes

- joint inversion of gravity and first-arrival traveltimes
- scatter plots