Three-dimensional inversion of gravity data for blocky models using a minimum-structure algorithm and general measures

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Acknowledgments

- Voisey’s Bay Nickel Company, and Inco/CVRD, for access to the gravity data over the Ovoid.

- Brian Bengert of VBNC/Inco/CVRD, and Michael Ash of MUN, for their advice and assistance.

- MeshTools3D by Roman Shekhtman of UBC–Geophysical Inversion Facility.

- Funded by IIC/AIF Project at MUN.
Outline

- Motivation.
- Previous work.
- General minimum-structure inversion strategy.
  - General measures.
  - Iterative solution procedure.
  - Measure of model structure.
- Example: 3-D gravity inversion, Voisey’s Bay Ovoid.
- Conclusions.
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Minimum-structure inversion for sharp interfaces
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“True” minimum-structure algorithms:

Farquharson & Oldenburg (1998, GJI), 1-D EM;
Portniaguine & Zhdanov (1999, Geophysics), 3-D focusing;
Loke, Acworth & Dahlin (2003, Expl. Geop.), 2-D resistivity;
Farquharson & Oldenburg (2003, SEGJ), 2-D resistivity.

Laterally constrained layered inversions:

Smith et al. (1999, Geophysics), 2-D MT;
Auken & Christiansen (2004, Geophysics), 2-D resistivity;
de Groot-Hedlin & Constable (2004, Geophysics), 2-D MT.
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General minimum-structure inversion strategy

- Mesh fixed during inversion; fine discretization.
General minimum-structure inversion strategy

- Minimize objective function:

\[ \Phi = \phi_d + \beta \phi_m, \]

where \( \phi_d \) is measure of data-misfit,

\[ \phi_d = \phi_d(u) \quad u = W_d(d^{obs} - d^{prd}), \]

and \( \phi_m \) is measure of structure in model,

\[ \phi_m = \sum_k \alpha_k \phi_k(v_k) \quad v_k = W_k(m - m^{ref}_k). \]
General measures

- A general form for $\phi_d$ and $\phi_m$ is:

$$\phi(x) = \sum_{j=1}^{N} \rho(x_j).$$

For example, the $l_2$-norm: $\rho(x) = x^2$;

the $l_p$-norm: $\rho(x) = |x|^p$;

Ekblom’s $l_p$-like measure: $\rho(x) = (x^2 + \epsilon^2)^{p/2}$;

Huber’s $M$-measure: $\rho(x) = \begin{cases} x^2 & |x| \leq c, \\ 2c|x| - c^2 & |x| > c. \end{cases}$
General measures

12

1 2 6 2 1 = 12
1 4 36 4 1 = 46

0 0 12 0 0 = 12
0 0 144 0 0 = 144
Iterative solution procedure

- Differentiate $\Phi$ with respect to model parameters and equate to zero.

Get normal system of equations:

\[
\begin{bmatrix}
G^T W_d^T R_d W_d G + \beta^n \sum_k \alpha_k W_k^T R_k W_k
\end{bmatrix} \delta m
\]

\[
= G^T W_d^T R_d W_d (d^{obs} - d^{n-1}) + \\
\beta^n \sum_k \alpha_k W_k^T R_k W_k (m_k^{ref} - m^{n-1}).
\]

Update $R_d$ and $R_k$. 
Measure of model structure

- Regularization via finite-difference matrices.

Old way:
Measure of model structure

- Regularization via finite-difference matrices.

New way:

\[
\Delta x_i \quad \Delta x_{i+1}
\]

\[
\begin{array}{c|c|c|c}
   & \text{i} & \text{i+1} \\
\hline
\text{j-1} &   &   &   \\
\text{j} &   &   &   \\
\text{j+1} &   &   &   \\
\end{array}
\]

\[
\Delta z_{j-1} \quad \Delta z_j \quad \Delta z_{j+1}
\]
Measure of model structure
Measure of model structure
Measure of model structure
Measure of model structure
Measure of model structure
Measure of model structure
Measure of model structure

- The measure of model structure becomes

\[ \phi_m = \sum_k \alpha_k \phi_k(v_k) \quad v_k = W_k(m - m_{ref}), \]

where the summation is now over 14 terms, rather than 4.
Particulars of 3-D gravity inversion program used here

- Finite-difference forward solver.
- Preconditioned CG solver for Gauss-Newton equations.
- Preconditioner is ILU decomposition with approximate Jacobian.
- Sparse matrix-vector products, and solution of forward system.
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Observed Bouguer anomaly over the Ovoid

- Bouguer anomaly relative to 2.67 g/cm³.
- Regional removal by upward continuation – Mike’s talk.
- 89 data.
- Assumed measurement uncertainties of 0.05 mGal.
Inversions

- Results for three inversion coming up.
  For all inversions . . .

  Mesh: $87 \times 61 \times 54$ cells, each cell $10 \times 10 \times 5$ m.
  Topography incorporated.
  Overburden incorporated via the reference model.
  Same depth weighting as GRAV3D.
  More smoothing in easting direction (relative to northing);
  less smoothing in vertical direction (relative to northing).
Inversions

1. Traditional $l_2$ measure of model structure:
   - only the usual $x, y, z$ finite differences in $\phi_m$.

2. $l_1$-type measure of model structure:
   - only the usual $x, y, z$ finite differences in $\phi_m$;
   - 20 iterations.

3. $l_1$-type measure of model structure:
   - all diagonal finite differences included in $\phi_m$;
   - 20 iterations.

- Sum-of-squares, $l_2$ data misfit used in all inversions.
  (Final misfits for the three inversions: 108, 103, 100.)
Inversion 1: $l_2$
Inversion 1: $l_2$

Northing = 6243137.5
Inversion 1: $l_2$
Inversion 2: $l_1$, no diagonal differences
Inversion 2: $l_1$, no diagonal differences

Northing = 6243137.5
Inversion 2: $l_1$, no diagonal differences
Inversion 3: $l_1$, diagonal differences
Inversion 3: $l_1$, diagonal differences

Northing = 6243137.5
Inversion 3: $l_1$, diagonal differences
Inversions
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Conclusions

- Minimum-structure inversions can be made to produce blocky models by using non-$l_2$ measures and iterative solution procedures.

- Explicit inclusion of diagonal differences in the measure of model structure allows dipping interfaces to be produced.
  
  - Computation time is significantly increased for linear inverse problems: not such an onerous increase for an already non-linear problem.
  
  - Interfaces not quite as sharp as I had hoped – because of CG solver of Gauss-Newton system?