Two effective inverse Laplace transform algorithms for computing time-domain electromagnetic responses

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Outline

• Background
• Method
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Background

• Forward modellings for time-domain methods: more complex
• Analytic methods: some simple models and particular configurations of source and receivers
• Time-stepping and spectral methods: general scenarios
Background

- Inverse Fourier transform
  Cosine and Sine transforms with 787 coefficients (Anderson, 1983) and 201 coefficients (Key, 2012)

- Inverse Laplace transform
  Gaver-Stehfest algorithm (Knight and Raiche, 1982): low accuracy for late times due to round-off errors
  Other inverse Laplace transform algorithms: never be applied to geophysical domain
Method

• Gaver-Stehfest algorithm

\[ f_G(t) = \ln2 \sum_{m=1}^{M} c_m^G \cdot \hat{f}\left(\frac{m \times \ln2}{t}\right) \]

\[ c_m^G = (-1)^{\frac{M}{2}+m} \sum_{k=\left[\frac{m+1}{2}\right]}^{\min\left\{ m, \frac{M}{2} \right\}} \frac{k^{M/2} (2k)!}{(M - k)! k! (m - k)! (2k - m)!} \]

• Round-off errors: caused by multiple binomial coefficients which become large as \( m \) increases, and by alternating signs which give cancellation issues. It is difficult to provide accurate solutions in a fixed-precision computing environment.
Method

- Euler algorithm

Manipulating the original formula of inverse Laplace transform into a Fourier transform

Utilizing Euler summation to accelerate the convergence of an infinite Fourier series

\[ f_E(t) = \frac{10^{M/3}}{t} \sum_{m=0}^{2M} c_m^E \cdot \text{Re}\left( \hat{f}\left(\frac{\beta_m}{t}\right) \right) \]
Method

• Talbot algorithm

Applying a deformed Bromwich contour to the original integral of inverse Laplace transform

Having some improved versions, e.g. the fixed-Talbot algorithm

\[
 f_T(t) = \frac{2}{5t} \sum_{m=0}^{M-1} \text{Re} \left( c_m^T \cdot \hat{f} \left( \frac{\delta_m}{t} \right) \right)
\]
Example 1— A 1000 Ωm half-space
Example 1— A 1000 Ωm half-space

(c) 100 m×100 m
- Cosine transform
- Double precision
- Variable precision

(d) 100 m×100 m
- M=12
- M=14
- M=16
- M=18
Example 1—A 1000 Ωm half-space
Example 1—A 1000 Ωm half-space
Example 1—A 1000 $\Omega$m half-space

(c) Euler algorithm

(d) Talbot algorithm
Example 2—The canonical 1D reservoir model

- Constable and Weiss (2006)
- Reference value: The 787-coefficient cosine transform for which all frequency-domain responses are explicitly computed
- The splined cosine and sine transforms
- The Euler ($M=7$) algorithm
- The Talbot ($M=11$) algorithm
Example 2—The canonical 1D reservoir model

(a) $r=1 \text{ km}$, 41 frequencies

(b) $r=15 \text{ km}$, 41 frequencies

- 787-coefficient splined sine transform
- 201-coefficient splined sine transform
- 787-coefficient splined cosine transform
- 201-coefficient splined cosine transform
- Euler algorithm, $M=7$
- Talbot algorithm, $M=11$
Example 2—The canonical 1D reservoir model

(c) $r=1$ km, 161 frequencies

(d) $r=15$ km, 161 frequencies

- 787-coefficient splined sine transform
- 201-coefficient splined sine transform
- Euler algorithm, $M=7$
- 787-coefficient splined cosine transform
- 201-coefficient splined cosine transform
- Talbot algorithm, $M=11$
Conclusions

• The Gaver-Stehfest algorithm with variable-precision arithmetic could provide accurate solutions, but it is lowly efficient and seriously problem-dependent.

• The Euler and Talbot algorithms with double-precision arithmetic are less problem-dependent, and have the capacity for yielding more accurate solutions than the splined cosine and sine transforms.
References


Thanks for your attention!