Two effective inverse Laplace transform algorithms for computing time-domain electromagnetic responses

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Outline

- Background
- Method
- Examples
- Conclusions
- References

Background

- Forward modellings for time-domain methods: more complex
- Analytic methods: some simple models and particular configurations of source and receivers
- Time-stepping and spectral methods: general scenarios

Background

- Inverse Fourier transform

Cosine and Sine transforms with 787 coefficients (Anderson, 1983) and 201 coefficients (Key, 2012)

- Inverse Laplace transform

Gaver-Stehfest algorithm (Knight and Raiche, 1982): low accuracy for late times due to round-off errors

Other inverse Laplace transform algorithms: never be applied to geophysical domain

Method

- Gaver-Stehfest algorithm

$$f_G(t) = \frac{\ln 2}{t} \sum_{m=1}^{M} c_m^G \cdot \hat{f}\left(\frac{m \times \ln 2}{t}\right)$$

$$c_m^G = (-1)^{\frac{M}{2}+m} \sum_{k=\left[\frac{m+1}{2}\right]}^{\min\left\{m,\frac{M}{2}\right\}} \frac{k^{M/2}(2k)!}{\left(\frac{M}{2}-k\right)!k!(k-1)!(m-k)!(2k-m)!}$$

- Round-off errors: caused by multiple binomial coefficients which become large as m increases, and by alternating signs which give cancellation issues. It is difficult to provide accurate solutions in a fixed-precision computing environment.

Method

- Euler algorithm

Manipulating the original formula of inverse Laplace transform into a Fourier transform

Utilizing Euler summation to accelerate the convergence of an infinite Fourier series

$$f_E(t) = \frac{10^{M/3}}{t} \sum_{m=0}^{2M} c_m^E \cdot \mathrm{Re}\left(\hat{f}\left(\frac{\beta_m}{t} \right) \right)$$

Method

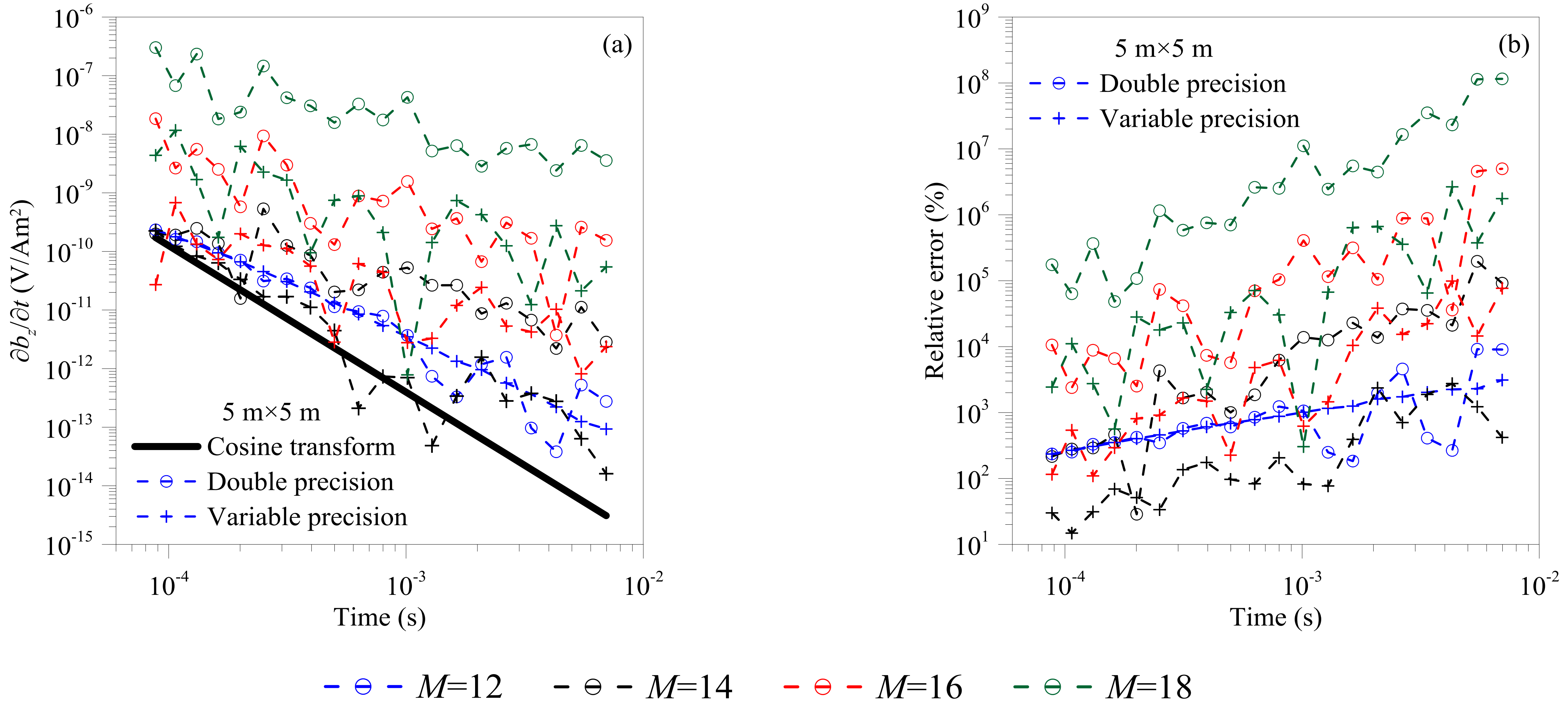
- Talbot algorithm

Applying a deformed Bromwich contour to the original integral of inverse Laplace transform

Having some improved versions, e.g. the fixed-Talbot algorithm

$$f_T(t) = \frac{2}{5t} \sum_{m=0}^{M-1} \mathrm{Re}\left(c_m^T \cdot \hat{f}\left(\frac{\delta_m}{t} \right) \right)$$

Example 1— A 1000 Ωm half-space



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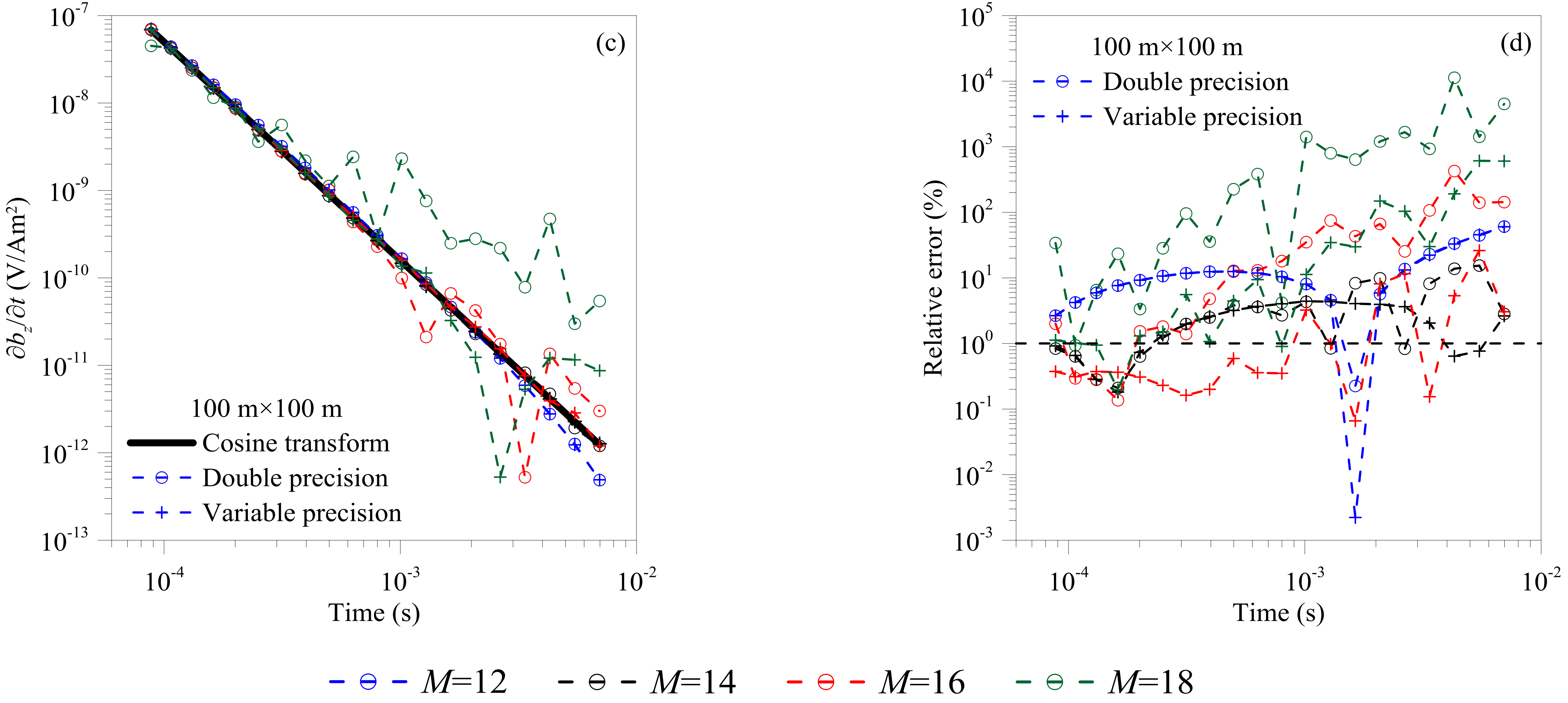

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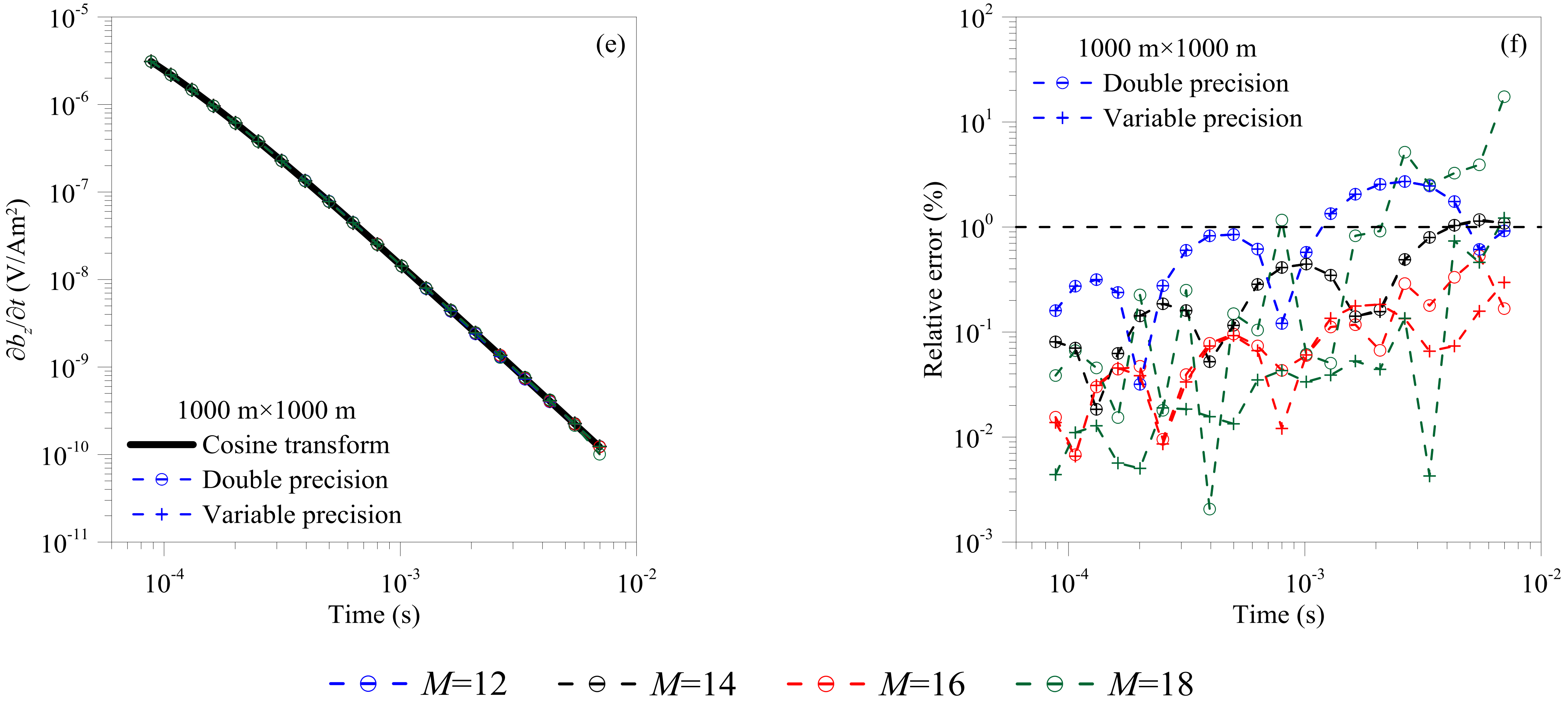

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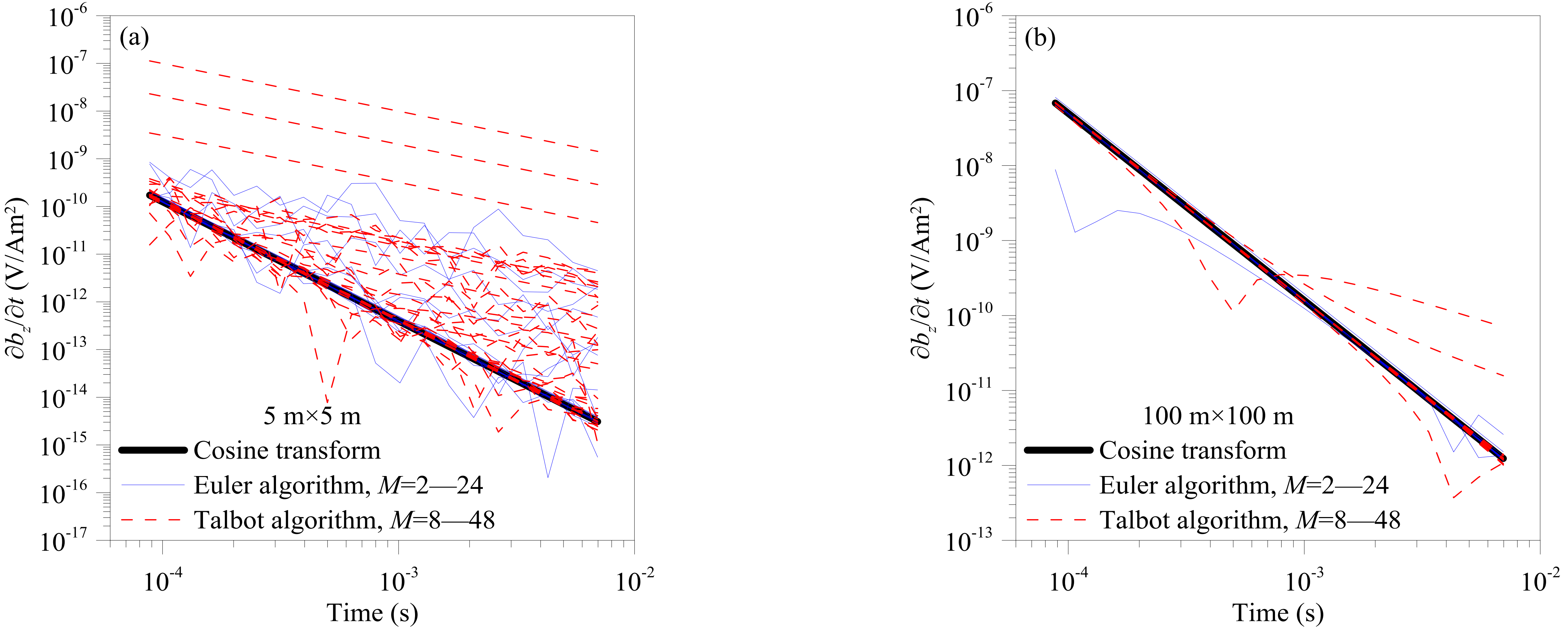

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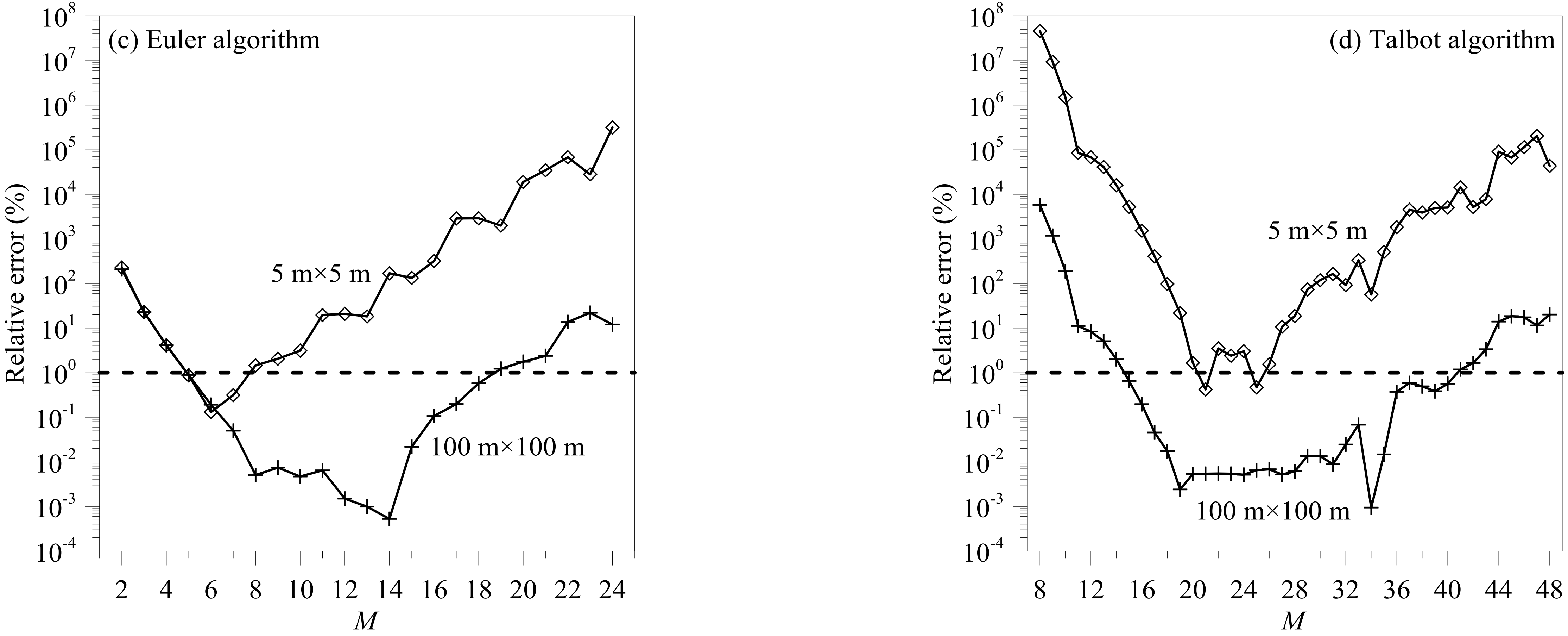
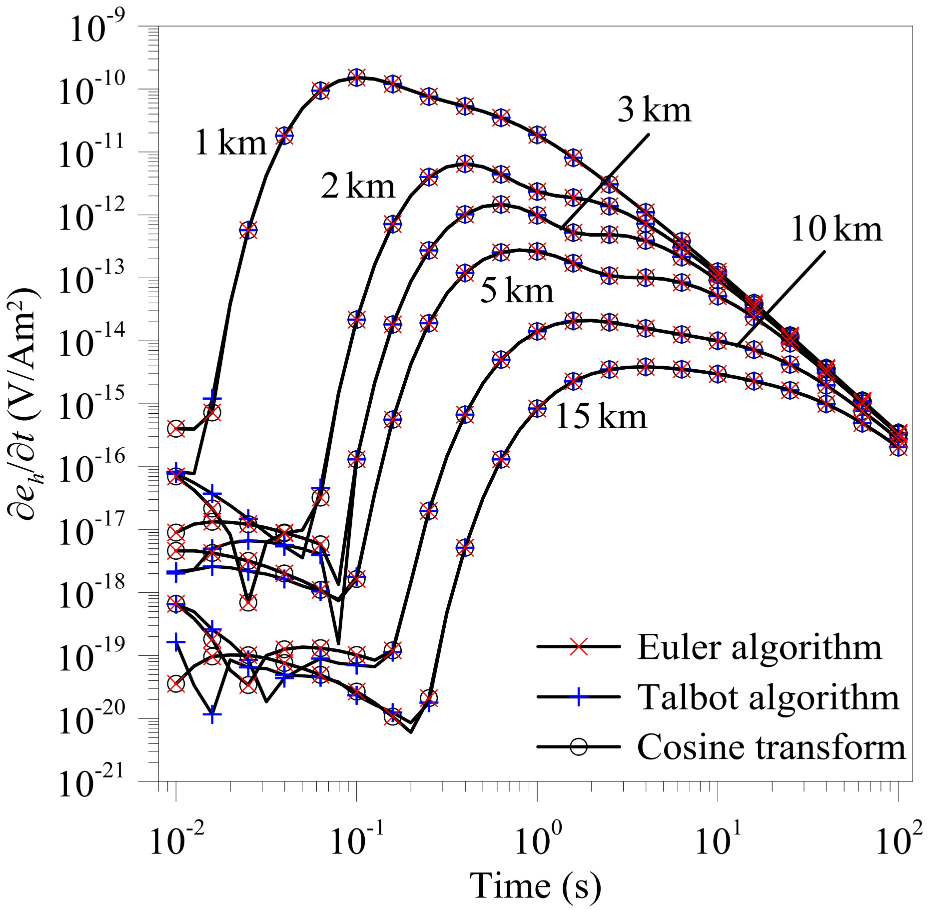


Example 2— The canonical 1D reservoir model

- Constable and Weiss (2006)
- Reference value: The 787-coefficient cosine transform for which all frequency-domain responses are explicitly computed
- The splined cosine and sine transforms
- The Euler (M=7) algorithm
- The Talbot (M=11) algorithm



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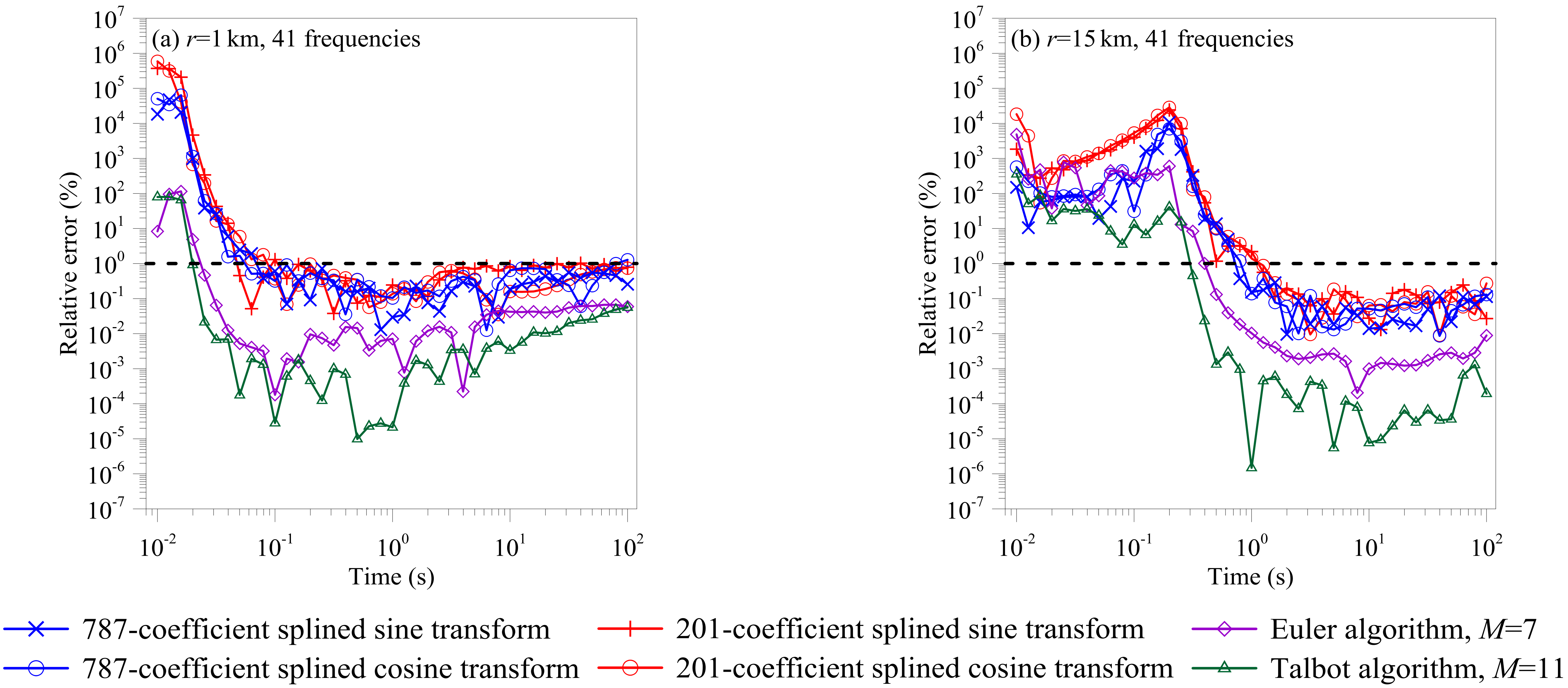

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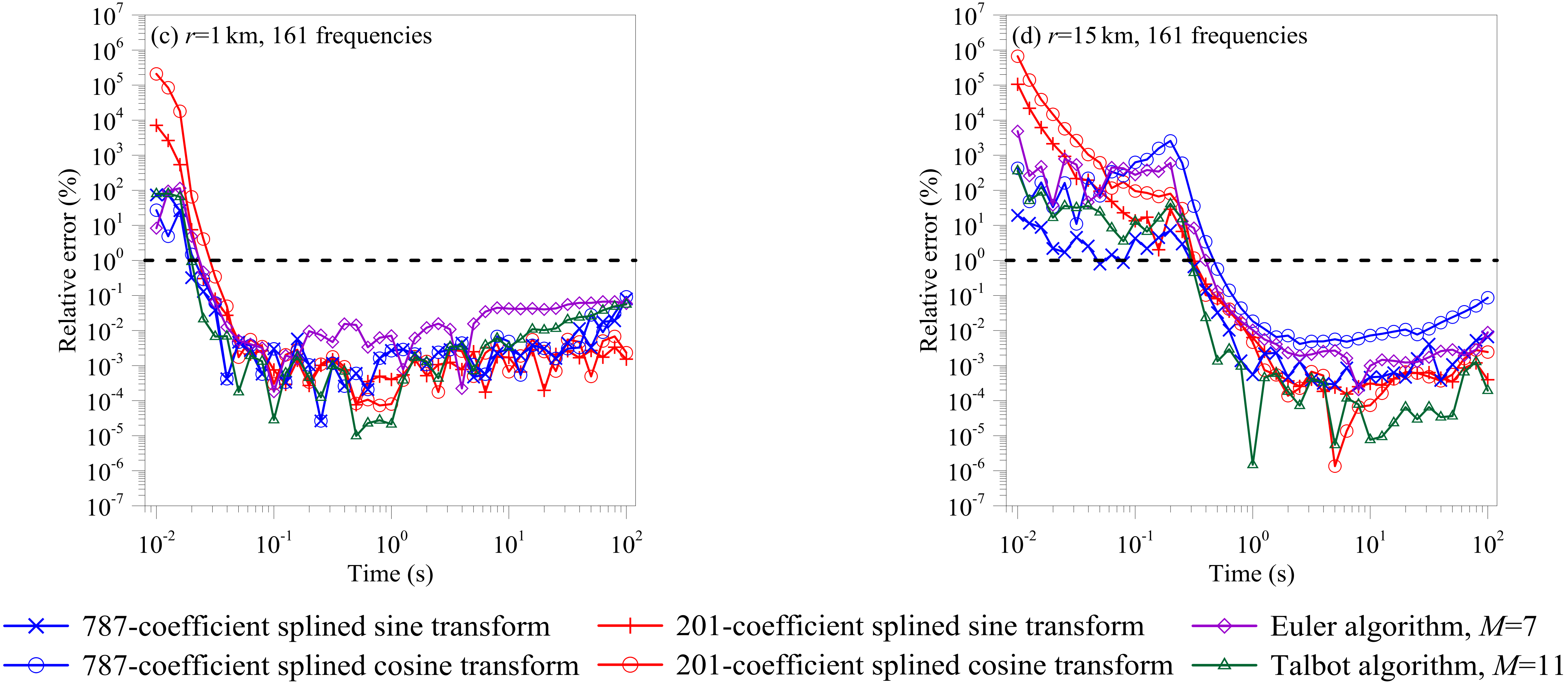

Conclusions

- The Gaver-Stehfest algorithm with variable-precision arithmetic could provide accurate solutions, but it is lowly efficient and seriously problem-dependent.
- The Euler and Talbot algorithms with double-precision arithmetic are less problem-dependent, and have the capacity for yielding more accurate solutions than the splined cosine and sine transforms.

References

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Thanks for your attention!