A finite-volume solution to the geophysical electromagnetic forward problem using unstructured grids

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| 1 | Unstructured grids                        |
| 2 | A finite-volume discretization of Maxwell’s equations |
| 3 | Example for magnetic dipole sources       |
| 4 | Example for a long grounded wire source   |
| 5 | Some accuracy studies                     |
| 6 | Conclusions                               |
Model irregular structures
Unstructured grids

- Topographical features
- Geological interfaces
Unstructured grids

- Local refinement (at observation points, sources, interfaces)
Dual tetrahedral-Voronoï grids

- **Grid generator:** TetGen (Si, 2004)

![Tetrahedral grid](image1)

![Voronoï grid](image2)
Staggered finite-volume schemes

- Magnetic field divergence free
- Easy for implementing boundary conditions
- Satisfies the continuity of tangential $E$
- Physically meaningful
Staggered finite-volume schemes

Dual tetrahedral-Voronoï grid

Delaunay-Voronoï contours
Maxwell’s equations

- Maxwell’s equations:
  \[ \nabla \times E = -i\omega \mu_0 H - i\omega \mu_0 M_p \]
  \[ \nabla \times H = \sigma E + J_p \]

Helmholtz equation for electric field

\[ \nabla \times \nabla \times E + i\omega \mu_0 \sigma E = -i\omega \mu_0 J_p - i\omega \mu_0 (\nabla \times M_p) \]

- Homogeneous Dirichlet boundary condition:
  \[ E = 0 \quad at \ \infty \]
  or
  \[ E \cdot \tau = 0 \quad on \ \Gamma \]
General features of the finite volume method

- Naturally supports unstructured grids
- Simple in idea
- Uses the integral form of equations
- Uses the average values of quantities
Integral form of Maxwell’s equations:

\[ \oint_{\partial A^D} \mathbf{E} \cdot d\mathbf{l}^D = -i\mu_0\omega \iint_{A^D} \mathbf{H} \cdot d\mathbf{A}^D - i\mu_0\omega \iint_{A^D} \mathbf{M}_p \cdot d\mathbf{A}^D \]

\[ \oint_{\partial A^V} \mathbf{H} \cdot d\mathbf{l}^V = \sigma \iint_{A^V} \mathbf{E} \cdot d\mathbf{A}^V + \iint_{A^V} \mathbf{J}_p \cdot d\mathbf{A}^V \]
Discretized form of Maxwell’s equations:

\[
\begin{align*}
W_j^D \sum_{q=1}^{w} E_{i(j,q)} I_{i(j,q)}^D &= -i \mu_0 \omega H_j A_j^D - i \mu_0 \omega M_{pj} A_j^D \\
W_i^V \sum_{k=1}^{w} H_{j(i,k)} I_{j(i,k)}^V &= \sigma E_i A_i^V + J_{pi} A_i^V.
\end{align*}
\]
Discretized Helmholtz equation

Discretized form of Helmholtz equation:

\[
\sum_{k=1}^{W_i^V} \left( \left( \sum_{q=1}^{W_j^D} E_{i(j,q)} I_{i(j,q)}^D \right) \frac{I_{j(i,k)}^V}{A_{j(i,k)}^D} \right) + i \omega \mu_0 \sigma E_i A_i^V
\]

\[
= -i \omega \mu_0 \sum_{k=1}^{W_i^V} M_{p(j,i,k)} \frac{I_{j(i,k)}^V}{A_{j(i,k)}^D} - i \omega \mu_0 J_{p_i}
\]
Decompose $E$ to real and imaginary parts:

$$E = E_{re} + iE_{im}$$

Resulting block matrix equation:

$$
\begin{pmatrix}
A & -B \\
B & A
\end{pmatrix}
\begin{pmatrix}
E_{re} \\
E_{im}
\end{pmatrix}
=
\begin{pmatrix}
S_{im} \\
S_{re}
\end{pmatrix},
$$
Finite-volume discretization

- Sparse direct solver: MUMPS (Amestoy et. al, 2006)
- Interpolation inside tetrahedra: vector basis functions

\[ E(x, y, z) = \sum_{i=1}^{6} N_i(x, y, z) E_i, \]

\begin{center}
\includegraphics[width=0.5\textwidth]{tetrahedron.png}
\end{center}
Inclusion of EM sources

Grounded wire:
Inclusion of EM sources

- Point vertical magnetic dipole:
Example 1: magnetic dipole transmitter-receiver pairs

- Graphite cube in brine (physical scale modelling measurements)
- Transmitter-receiver pairs along the $x$ axis at $z = 2 \text{ cm}$
- Dimensions of the cubic graphite: $14 \times 14 \times 14 \text{ cm}$
- $\sigma_{\text{brine}} = 7.3 \text{ S/m} ; \sigma_{\text{prism}} = 63,000 \text{ S/m}$
- Frequencies: $1, 10, 100, 200, 400 \text{ kHz}$
Example 1: magnetic dipole transmitter-receiver pairs

- Grid refined at the sources, observation points and the prism
Example 1: magnetic dipole transmitter-receiver pairs

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Example 1: magnetic dipole transmitter-receiver pairs

- Scattered H-field: (total−free-space)/free-space
- FV (circles) vs PSM (red), IE (orange), and FE (black)
Example 1: magnetic dipole transmitter-receiver pairs

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![Graphs showing In phase and Quadrature responses at 100 kHz](image-url)
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- Scattered H-field: (total−free-space)/free-space
- FV (circles) vs PSM (red), IE (orange), and FE (black)
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Example 1: magnetic dipole transmitter-receiver pairs

In phase

Quadrature

In phase

Quadrature

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Example 2: long grounded wire

- 100 m wire along the x axis operating at 3 Hz
- Dimensions of the prism: 120 × 200 × 400 m
- $\sigma_{\text{ground}} = 0.02 \text{ S/m} ; \sigma_{\text{prism}} = 0.2 \text{ S/m}$
- Observation points along the x axis
Example 2: long grounded wire

- Dimensions of the domain: $40 \times 40 \times 40$ km
- Number of tetrahedra: 162,689; number of unknowns: 189,105
Example 2: long grounded wire

- Grid refined at the source, observation points and the prism
- Computation time: 40 s; memory: 4 Gbytes (on Apple Mac Pro; 2.26 GHz Quad-Core Intel Xeon processor)
Example 2: long grounded wire

- Grid refined at the source, observation points and the prism
- Computation time: 40 s; memory: 4 Gbytes (on Apple Mac Pro; 2.26 GHz Quad-Core Intel Xeon processor)
Example 2: long grounded wire

- Without prism (homogeneous halfspace)
- Total field
- FV vs IE (Farquharson and Oldenburg, 2002)

**Without prism (homogeneous halfspace)**

**Total field**

**FV vs IE (Farquharson and Oldenburg, 2002)**

- Circles: IE; in phase; positive
- Black line: FV; in phase; positive
- Crosses: IE; quadrature; negative
- Gray line: FV; quadrature; negative

**With and without prism**

**Total field**

**FV only**

- Black solid line: in phase; with anomaly
- Gray dashed line: in phase; no anomaly
- Gray solid line: quadrature; with anomaly
- Black dashed line: quadrature; no anomaly
Example 1: long grounded wire

- Scattered field
- FV vs IE

In phase

Quadrature

Secondary $E_x$ (V/m)

$x$ (m)
Example 2: long grounded wire

- Horizontal section \((z = -150 \text{ m})\)
- Total electric field (in phase and quadrature)

- Vertical section \((y = 0 \text{ m})\)
- Total electric field (in phase and quadrature)
Example 2: accuracy studies

- Refinement at the observation points
- Exact solutions: solution due to a fine grid
Example 2: accuracy studies

- **Improvement in grid quality**
- **Quality criteria: maximum radius-edge ratio**

![Graph showing grid quality improvement](image-url)

- **In phase**
- **Quadrature**
- **Full line: 8th order**
- **Dashed line: 4th order**
- **Dotted line: 2nd order**

Cumulative error (V/m)

Grid quality

1e−08
1e−07
1e−06
1e−05

1 2
maximum radius-edge ratio
Example 2: accuracy studies

- **Refinement at the line source**
- **Line source divided into equal segments**

![Graph showing cumulative error (V/m) vs. source segments size (m) for wire sources in phase, quadrature, full line: 2nd order, dashed line: 1st order, dotted line: 0.25th order.](image-url)
Conclusions

- A finite-volume technique is used for modelling the total field EM data. This technique uses the staggered tetrahedral-Voronoï grid.
- The aim is making use of the features of unstructured grids for efficient modeling of the subsurface and for local refinements in the grid.
- The Helmholtz equation is discretized and solved using a sparse direct solver (MUMPS).
- The scheme has been tested for two models: one with a long grounded wire source; another one for magnetic source-receiver pairs with large conductivity contrasts.
- For the both examples, the results from the FV scheme are in good agreement with those from the literature.
- Accuracy studies show the relatively higher importance of refinement at the observation points and improvement in grid quality, and relatively lower importance of refinement at the sources.
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References


