Forward modelling of geophysical electromagnetic data on unstructured grids using a finite-volume approach

Hormoz Jahandari and Colin G. Farquharson

Memorial University
Department of Earth Sciences
St. John’s, Newfoundland, Canada

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1. Unstructured grids
2. Finite-volume discretization of Maxwell’s equations (direct EM-field and potential formulation)
3. Example for magnetic dipole sources
4. Example for a long grounded wire source
5. Example for a helicopter EM survey
6. Conclusions
Unstructured grids

- Model irregular structures
Unstructured grids

- Topographical features
- Geological interfaces
- Local refinement (at observation points, sources, interfaces)
Dual tetrahedral-Voronoi grids

Grid generator: TetGen (Si, 2004)

tetrahedral grid

Voronoi grid
Staggered finite-volume schemes

- Magnetic field divergence free
- Easy for implementing boundary conditions
- Satisfies the continuity of tangential $E$
- Physically meaningful

Rectilinear dual grid

Rectilinear dual contours
Staggered finite-volume schemes

Dual tetrahedral-Voronoï grid

Delaunay-Voronoï contours

E-field

H-field
Staggered finite-volume schemes

**Direct EM-field method**
- Unknowns are E and/or H
- Simpler
- Smaller system of equations
- Ill-conditioned

**EM Potential \((A - \phi)\) method**
- Unknowns are \(A\) and \(\phi\)
- Larger system of equations
- Well-conditioned
- Allows studying the galvanic and inductive parts
Direct EM-field formulation of Maxwell’s equations

Maxwell’s equations:

\[ \nabla \times E = -i\omega \mu_0 H - i\omega \mu_0 M_p \]
\[ \nabla \times H = \sigma E + J_p \]

Helmholtz equation for electric field

\[ \nabla \times \nabla \times E + i\omega \mu_0 \sigma E = -i\omega \mu_0 J_p - i\omega \mu_0 (\nabla \times M_p) \]

Homogeneous Dirichlet boundary condition:

\[ E = 0 \quad at \infty \]

or

\[ E \cdot \tau = 0 \quad on \ \Gamma \]
Integral form of Maxwell’s equations:

\[
\begin{align*}
\oint_{\partial S^D} \mathbf{E} \cdot d\mathbf{l}^D &= -i\mu_0\omega \iiint_{S^D} \mathbf{H} \cdot d\mathbf{S}^D - i\mu_0\omega \iiint_{S^D} \mathbf{M}_p \cdot d\mathbf{S}^D \\
\oint_{\partial S^V} \mathbf{H} \cdot d\mathbf{l}^V &= \sigma \iiint_{S^V} \mathbf{E} \cdot d\mathbf{S}^V + \iiint_{S^V} \mathbf{J}_p \cdot d\mathbf{S}^V
\end{align*}
\]
Discretized form of Maxwell's equations:

\[
\sum_{q=1}^{W_j^D} E_{i(j,q)} l_{i(j,q)}^D = -i\mu_0\omega H_j S_j^D - i\mu_0\omega M_{pj} S_j^D
\]

\[
\sum_{k=1}^{W_i^V} H_{j(i,k)} l_{j(i,k)}^V = \sigma E_i S_i^V + J_{pi} S_i^V.
\]
Discretized form of Helmholtz equation:

\[
\sum_{k=1}^{W_i^V} \left( \left( \sum_{q=1}^{W_j^D} E_{i(j,q)} l_{i(j,q)}^D \right) \frac{l_{j(i,k)}^V}{S_{j(i,k)}^D} \right) + i \omega \mu_0 \sigma E_i S_i^V \\
= -i \omega \mu_0 \sum_{k=1}^{W_i^V} M_{p j(i,k)} \frac{l_{j(i,k)}^V}{S_{j(i,k)}^D} - i \omega \mu_0 J_{pi}
\]
**EM potential (A-\(\phi\)) formulation of Maxwell’s equations**

- Magnetic vector and electric scalar potentials:
  
  \[
  E = -i\omega A - \nabla \phi \\
  \mu_0 H = \nabla \times A
  \]

**Helmholtz equation in terms of potentials with Coulomb gauge**

\[
\nabla \times \nabla \times A - \nabla (\nabla \cdot A) + i\omega \mu_0 \sigma A + \sigma \mu_0 \nabla \phi = \mu_0 J_p + \mu_0 \nabla \times M_p
\]

**Conservation of charge**

\[
-\nabla \cdot J = \nabla \cdot J_p \\
i\omega \nabla \cdot \sigma A + \nabla \cdot \sigma \nabla \phi = \nabla \cdot J_p
\]

- Homogeneous Dirichlet boundary condition:

\[
(A, \phi) = 0 \quad \text{at } \infty
\]

or

\[
(A \cdot \tau, \phi) = 0 \quad \text{on } \Gamma
\]
EM potential \((A-\phi)\) formulation of Maxwell’s equations

Relations used for the finite-volume discretization

\[
\nabla \times H - \mu_0^{-1} \nabla \psi + i\omega \sigma A + \sigma \nabla \phi = J_p + \nabla \times M_p
\]

(1)

\[
\mu_0 H = \nabla \times A
\]

(2)

\[
\psi = \nabla \cdot A
\]

(3)

\[
i\omega \nabla \cdot \sigma A + \nabla \cdot \sigma \nabla \phi = \nabla \cdot J_p
\]

(4)
The system of equations for the direct method

- Decompose $E$ into real and imaginary parts:

$$E = E_{re} + iE_{im}$$

- Resulting block matrix equation:

$$
\begin{pmatrix}
A & -B \\
B & A
\end{pmatrix}

\begin{pmatrix}
E_{re} \\
E_{im}
\end{pmatrix}

= 
\begin{pmatrix}
0 \\
S_{re}
\end{pmatrix}
$$
The system of equations for the $A$-$\phi$ method

- Decompose $A$ and $\phi$ into real and imaginary parts:
  \[
  A = A_{re} + iA_{im} \quad ; \quad \phi = \phi_{re} + i\phi_{im}
  \]

- Resulting block matrix equation:
  \[
  \begin{pmatrix}
  A & -\omega B & C & 0 \\
  \omega B & A & 0 & -C \\
  0 & -\omega D & E & 0 \\
  \omega D & 0 & 0 & E
  \end{pmatrix}
  \begin{pmatrix}
  A_{re} \\
  A_{im} \\
  \phi_{re} \\
  \phi_{im}
  \end{pmatrix}
  =
  \begin{pmatrix}
  S_1 \\
  0 \\
  S_2 \\
  0
  \end{pmatrix}
  \]
Direct EM-field method: MUMPS sparse direct solver (Amestoy et al, 2006)

EM Potential method: BCGSTAB iterative solver from SPARSKIT (Saad, 1990)
Example 1: magnetic dipole transmitter-receiver pairs

- Graphite cube in brine (physical scale modelling measurements)
- Transmitter-receiver pairs along the $x$ axis at $z = 2 \, cm$
- Dimensions of the cubic graphite: $14 \times 14 \times 14 \, cm$
- $\sigma_{brine} = 7.3 \, S/m$; $\sigma_{prism} = 63,000 \, S/m$
- Frequencies: 1, 10, 100, 200, 400 \, kHz
Example 1: magnetic dipole transmitter-receiver pairs

- Grid refined at the sources, observation points and the prism
Example 1: magnetic dipole transmitter-receiver pairs

- Grid refined at the sources, observation points and the prism
Example 1: magnetic dipole transmitter-receiver pairs

- Scattered H-field: \((\text{total } - \text{free-space})/\text{free-space}\)
Example 1: magnetic dipole transmitter-receiver pairs

- Scattered H-field: \((\text{total−free-space})/\text{free-space}\)

In phase

- 10 kHz

Quadrature

- 10 kHz
Example 1: magnetic dipole transmitter-receiver pairs

Scattered H-field: (total−free-space)/free-space
Example 1: magnetic dipole transmitter-receiver pairs

- Scattered H-field: (total−free-space)/free-space

![In phase and Quadrature plots for scattered H-field at 200 kHz](image)

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Example 1: magnetic dipole transmitter-receiver pairs

- Scattered H-field: \((\text{total} - \text{free-space})/\text{free-space}\)

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**In phase**

- **400 kHz**

**Quadrature**

- **400 kHz**
Example 1: magnetic dipole transmitter-receiver pairs

- Galvanic part ($-\nabla \phi$)

- Inductive part ($-i\omega A$)

- Total electric field:
  $$E = -\nabla \phi - i\omega A$$
Example 1: magnetic dipole transmitter-receiver pairs

- **Galvanic part** $(-\sigma \nabla \phi)$

- **Inductive part** $(-i\omega \sigma A)$

- **Total current density:**
  
  $$J = -\sigma \nabla \phi - i\omega \sigma A$$
Example 2: long grounded wire

- 100 m wire along the x axis operating at 3 Hz
- Dimensions of the prism: 120 × 200 × 400 m
- $\sigma_{\text{ground}} = 0.02 \, S/m$; $\sigma_{\text{prism}} = 0.2 \, S/m$
- Observation points along the x axis
Example 2: long grounded wire

- Dimensions of the domain: $40 \times 40 \times 40 \ km$
- Number of tetrahedra: 162,689
Example 2: long grounded wire

- EM-field method: MUMPS (40 s; memory 4 Gbytes)
- (on Apple Mac Pro; 2.26 GHz Quad-Core Intel Xeon processor)
Example 2: long grounded wire

- Potential method: BCGSTAB with \( lfil=3 \) and ILUT preconditioner (345 s; memory 0.8 Gbytes; 2000 iterations; residual norm \( 10^{-12} \))
- (on Apple Mac Pro; 2.26 GHz Quad-Core Intel Xeon processor)
Example 1: long grounded wire

- Scattered electric field

In phase
- \( E_x \) vs. \( x \) (m)

Quadrature
- \( E_x \) vs. \( x \) (m)

Secondary \( E_x \) (V/m)

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Example 2: long grounded wire

- Galvanic part \((-\nabla \phi)\)

- Inductive part \((-i\omega A)\)

- Total electric field:
  \[ E = -\nabla \phi - i\omega A \]
Example 2: long grounded wire

- **Galvanic part** \((-\sigma \nabla \phi)\)

- **Inductive part** \((-i\omega\sigma A)\)

- **Total current density**:
  \[ J = -\sigma \nabla \phi - i\omega\sigma A \]
Example 3: Ovoid, HEM survey (direct method)

- Ovoid: massive sulfide ore body, Voisey’s Bay, Labrador, Canada
- HEM survey of the region has been simulated
- Transmitter and receiver towed below the helicopter 30 m above ground
- Transmitter-receiver separation was 8 m and the frequency was 900 Hz
- \( \sigma_{\text{ground}} = 0.001 \) and \( \sigma_{\text{ovoid}} = 100 \) S/m were chosen by try-and-error
- Number of tetrahedra: 190, 121; Number of unknowns: 223, 650
Example 3: Ovoid, HEM survey (direct method)

Topography of the region
Example 3: Ovoid, HEM survey (direct method)

White dots show the observation points.
Example 3: Ovoid, HEM survey (direct method)

The plan view
Example 3: Ovoid, HEM survey (direct method)

- Grid refined at the sources and observation points

![Diagram showing grid refinement at sources and observation points]
Example 3: Ovoid, HEM survey (direct method)

- FV results (circles) vs real HEM data (lines)
Conclusions

- A finite-volume approach is used for modelling the total field EM data. This method uses the staggered tetrahedral-Voronoi grids.
- The aim is to make use of the features of unstructured grids for efficient modeling of the subsurface and for local refinements in the grid.
- Both the direct EM-field formulation and the potential formulation of Maxwell’s equation are discretized and solved using a sparse direct solver (MUMPS) and an iterative solver (BCGSTAB).
- The schemes have been tested for two models with simple geometries: one with a long grounded wire source and a small conductivity contrast; another one for magnetic source-receiver pairs with large conductivity contrast.
- For the both examples, the results from the two FV schemes are in good agreement with those from the literature.
The direct EM-field scheme is ill-conditioned and it can not be easily solved using the iterative solvers. The only option is using a direct solver. The $A - \phi$ scheme, in the other hand, is better conditioned and it can be solved using iterative solvers.

An example is also included in which helicopter-borne EM data is simulated for a model with irregular geometry and with topography. The results from the direct EM-field method show good agreement with the real data.
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