

# Unstructured grid modelling to create 3-D Earth models that unify geological and geophysical information

**Peter Lelièvre**, Angela Carter-McAuslan, Cassandra Tycholiz,  
Colin Farquharson and Charles Hurich

`plelievre@mun.ca`



Memorial University,  
Department of Earth Sciences,  
St. John's, Newfoundland, Canada

GAC-MAC Annual Meeting, SS14, May 28, 2012

# Motivation: The common Earth model

## Motivation: The common Earth model

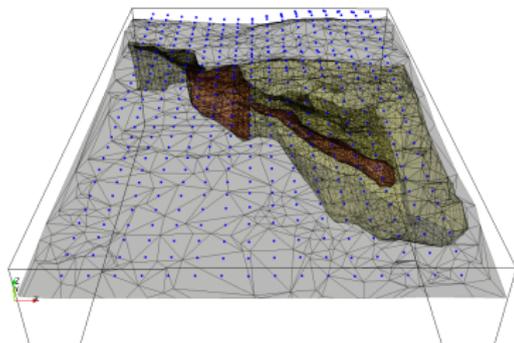
- Earth models used for mineral exploration or other subsurface investigations should be consistent with all available geological and geophysical information

## Motivation: The common Earth model

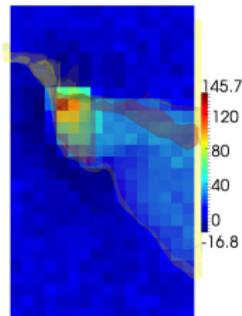
- Earth models used for mineral exploration or other subsurface investigations should be consistent with all available geological and geophysical information
- Geophysical inversion provides the means to unify geological and geophysical data towards the development of a common Earth model

# Geophysical inversion

Forward problem



Earth model (e.g. density)



Survey data (e.g. gravity)

Inverse problem

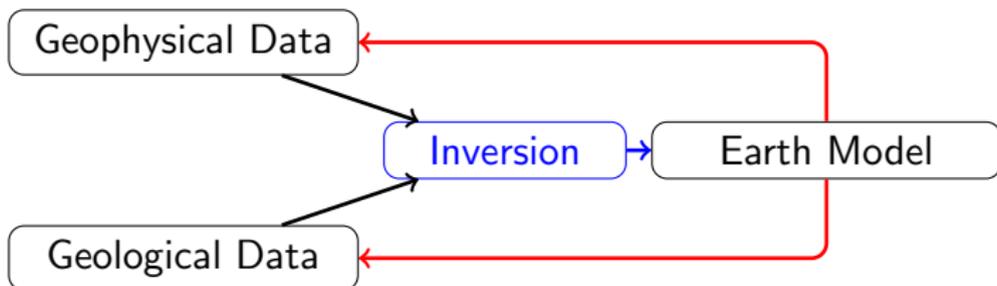


# Geophysical inversion

- Incorporation of geological and geophysical data into inversions is always an iterative process

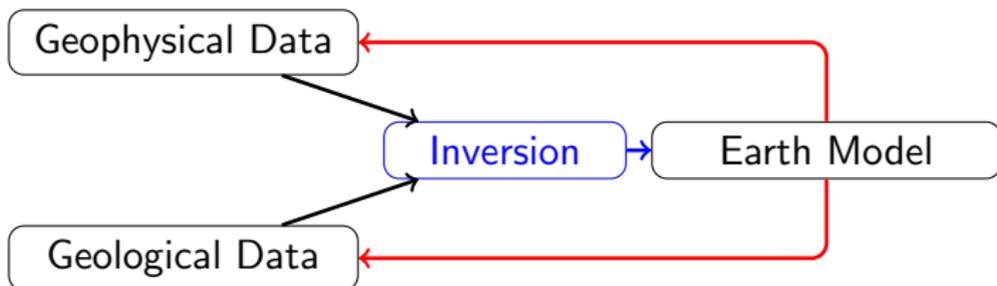
# Geophysical inversion

- Incorporation of geological and geophysical data into inversions is always an iterative process



# Geophysical inversion

- Incorporation of geological and geophysical data into inversions is always an iterative process



- More information  $\Rightarrow$  reduce non-uniqueness  $\Rightarrow$  higher potential to resolve deeper features

## Geological models

3D geological ore deposit models are commonly created during delineation drilling:

## Geological models

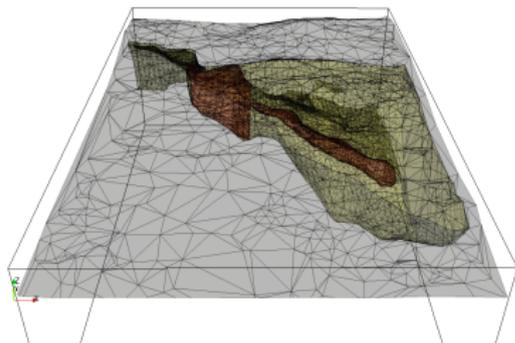
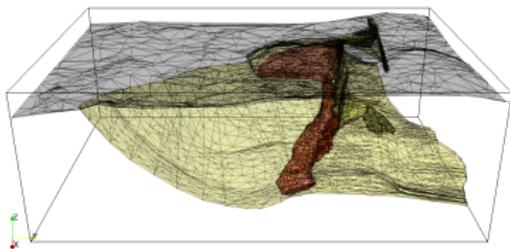
3D geological ore deposit models are commonly created during delineation drilling:

- visualization
- calculate volumes of ore reserves
- accuracy is crucial to determine if deposit is economical

## Geological models

3D geological ore deposit models are commonly created during delineation drilling:

- visualization
- calculate volumes of ore reserves
- accuracy is crucial to determine if deposit is economical
- typically comprise wireframe surfaces of connected triangles that represent geological contacts



## Geophysical models

In contrast, most current 3D geophysical modelling is performed on rectilinear meshes:

## Geophysical models

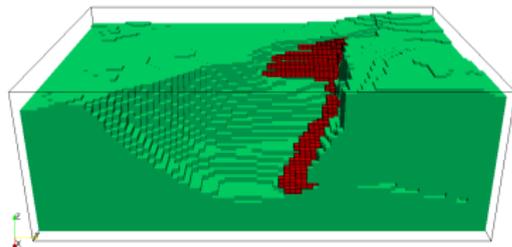
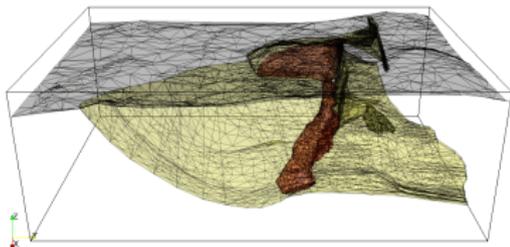
In contrast, most current 3D geophysical modelling is performed on rectilinear meshes:

- simplifies the development of numerical methods

## Geophysical models

In contrast, most current 3D geophysical modelling is performed on rectilinear meshes:

- simplifies the development of numerical methods
- **incompatible with wireframe geological models:**
  - can be impossible to adequately model complicated geology
  - produce pixellated representations

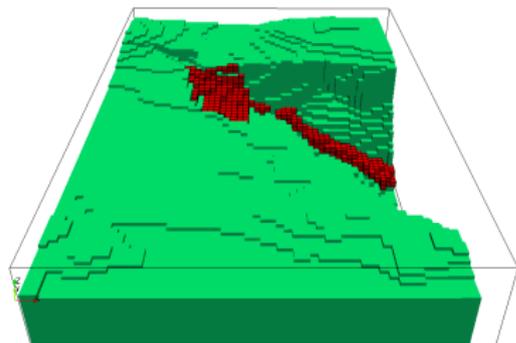
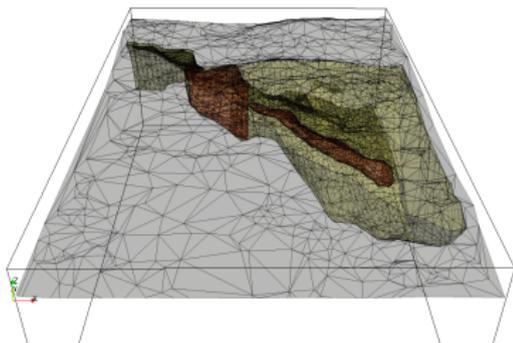


$55 \times 82 \times 31 = 139,810$  cells and stair-casing still evident

## Geophysical models

In contrast, most current 3D geophysical modelling is performed on rectilinear meshes:

- simplifies the development of numerical methods
- **incompatible with wireframe geological models:**
  - can be impossible to adequately model complicated geology
  - produce pixellated representations



$55 \times 82 \times 31 = 139,810$  cells and stair-casing still evident

# Why unstructured meshes?

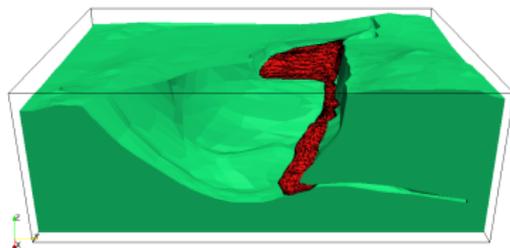
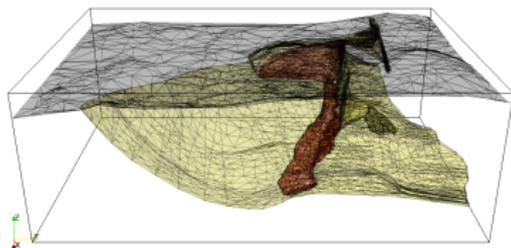
## Incompatibility:

- most current 3D geological Earth models comprise wireframe surfaces (tessellated triangles)
- most current 3D geophysical modelling is performed on rectilinear meshes (rectangular prisms)

# Unstructured meshes

Unstructured meshes provide:

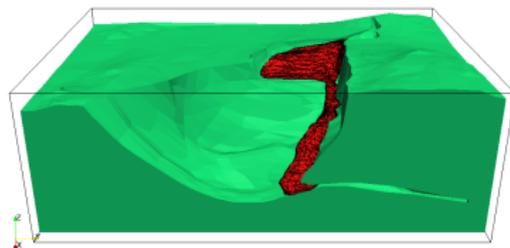
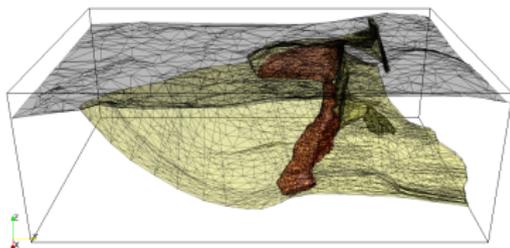
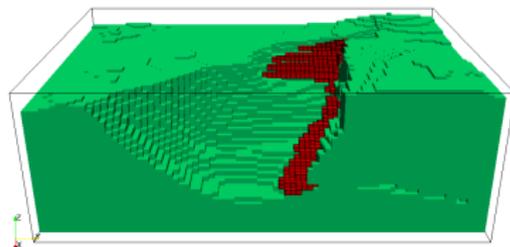
- efficient generation of complicated geometries
- significant reduction in problem size (57,132 vs. 139,810)



# Unstructured meshes

Unstructured meshes provide:

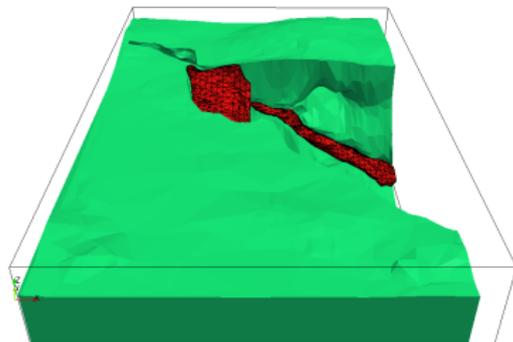
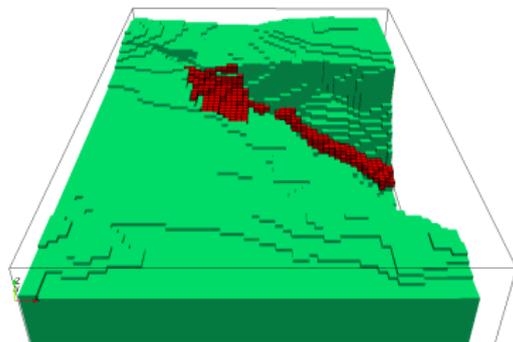
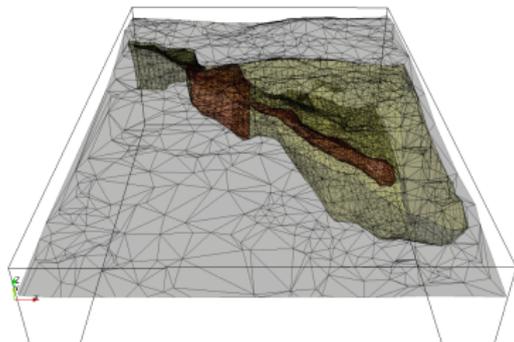
- efficient generation of complicated geometries
- significant reduction in problem size (57,132 vs. 139,810)



# Unstructured meshes

Unstructured meshes provide:

- efficient generation of complicated geometries
- significant reduction in problem size (57,132 vs. 139,810)



# Unstructured meshes

Advantages:

- efficient generation of complicated geometries
- significant reduction in problem size

# Unstructured meshes

## Advantages:

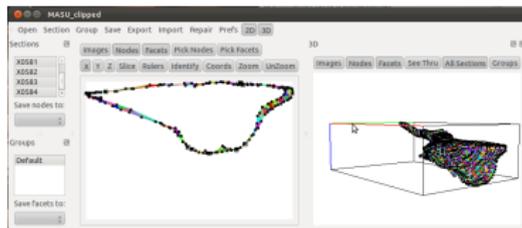
- efficient generation of complicated geometries
- significant reduction in problem size

## Challenges:

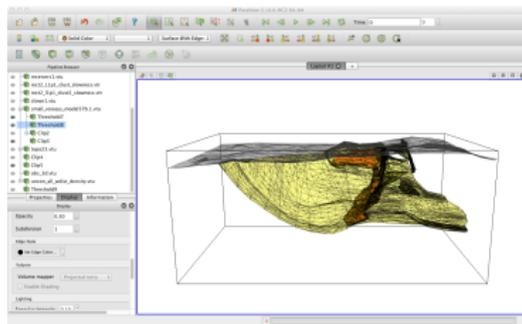
- create, manipulate and visualize Earth models
- mathematics of numerical modelling

# Wireframe creation, manipulation and visualization

- Gocad
- FacetModeller
- Blender
- ParaView



(FacetModeller)



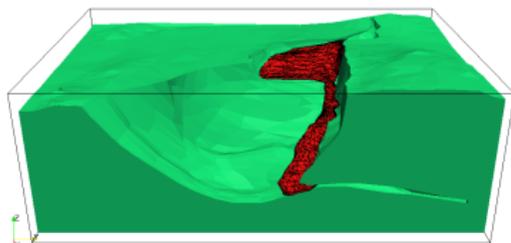
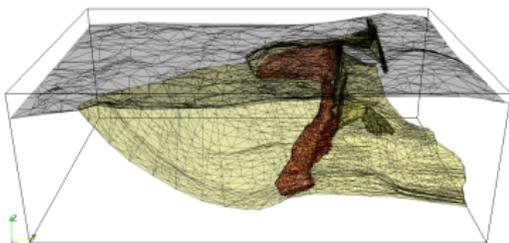
(ParaView)

## Volumetric discretization of wireframes

- 2D: Triangle (J. R. Shewchuck)
- 3D: TetGen (H. Si)

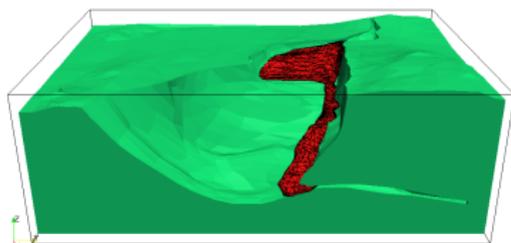
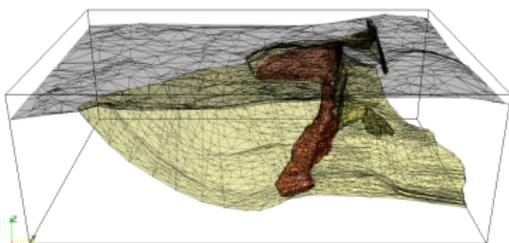
# Volumetric discretization of wireframes

- 2D: Triangle (J. R. Shewchuck)
- 3D: TetGen (H. Si)
- the triangular wireframe surface facets become the faces of tetrahedra in the volumetric model



# Volumetric discretization of wireframes

- 2D: Triangle (J. R. Shewchuck)
- 3D: TetGen (H. Si)
- the triangular wireframe surface facets become the faces of tetrahedra in the volumetric model



- geological and geophysical models can share the same modelling mesh; they can be the same model

## Forward modelling on unstructured meshes

We have developed modelling methods for various data types:

- gravity (Hormoz Jahandari)
- gravity gradiometry (Cassandra Tycholiz)
- magnetics (Cassandra Tycholiz)
- seismic first-arrivals (Peter Lelièvre)
- geoelectric (Amir Javaheri)
- electromagnetic (Hormoz Jahandari, Masoud Ansari)

## Standard deterministic inversion approach

- Objective function

$$\Phi = \Phi_d + \beta\Phi_m$$

- Data misfit

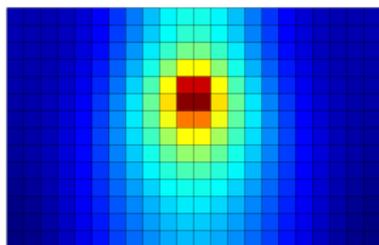
$$\Phi_d = \sum_i \left( \frac{d_i^{pred}(m) - d_i^{obs}}{\sigma_i} \right)^2$$

- Model structure (regularization)

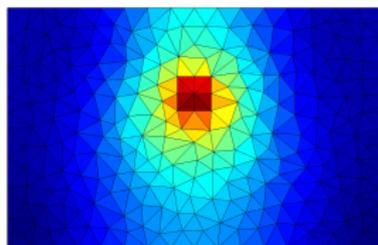
$$\Phi_m = [\text{smallness term}] + [\text{smoothness term}]$$

# Regularization

The same regularization is possible on unstructured or rectilinear meshes provided that appropriate matrix operators can be created



regular rectilinear

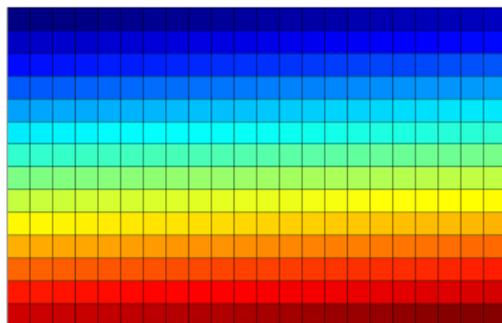


triangular unstructured

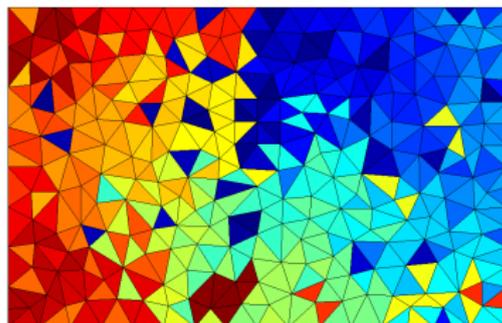
## Computational challenges

Algorithms can be designed that exploit mesh structure:

- sparsity structure of spatial matrix operators
- compression of full sensitivity matrices



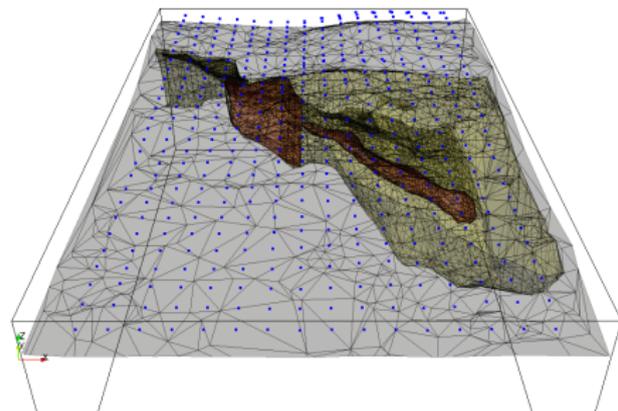
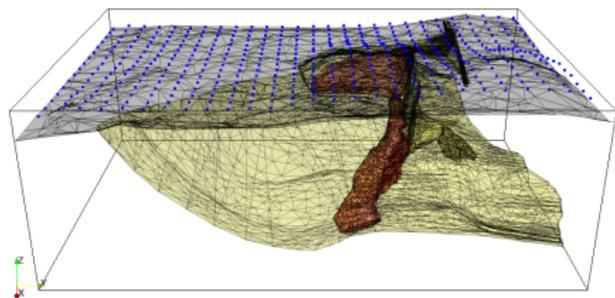
regular rectilinear



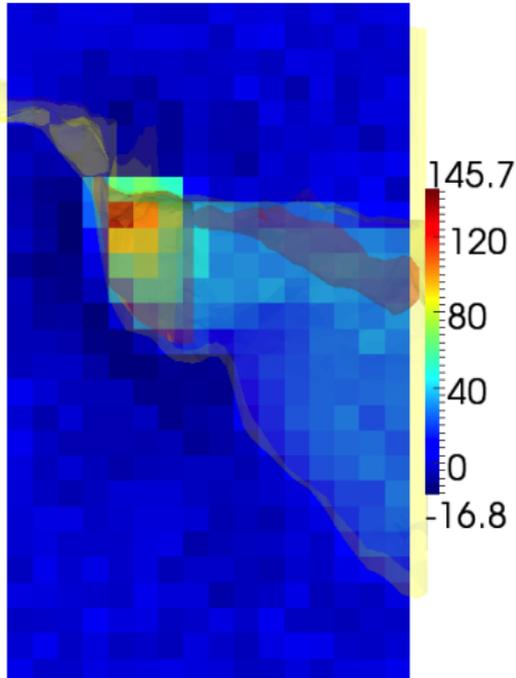
triangular unstructured

# Voisey's Bay example

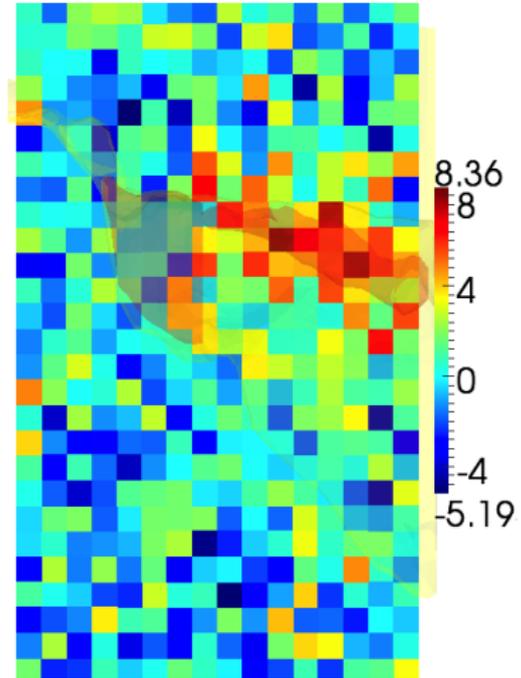
- nickel-copper-cobalt sulfide deposit
- north-east coast of Labrador, Canada



# Gravity gradiometry (tensor) data

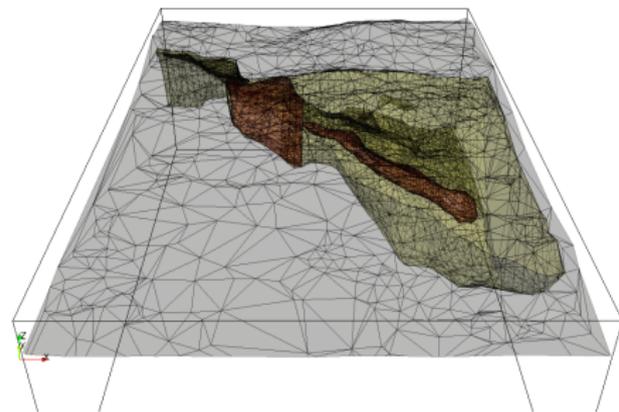
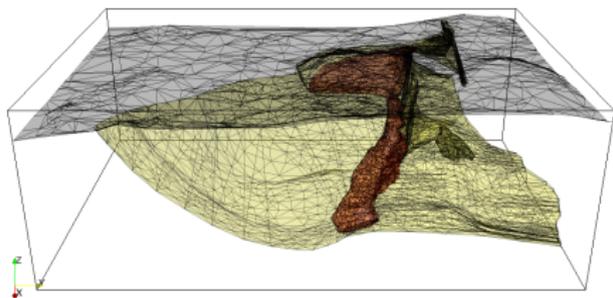


noisy zz-component



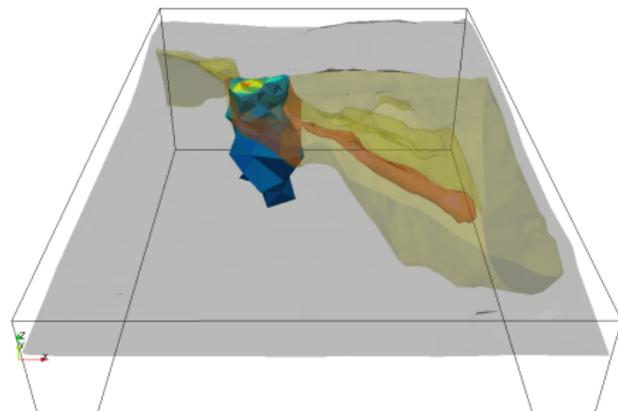
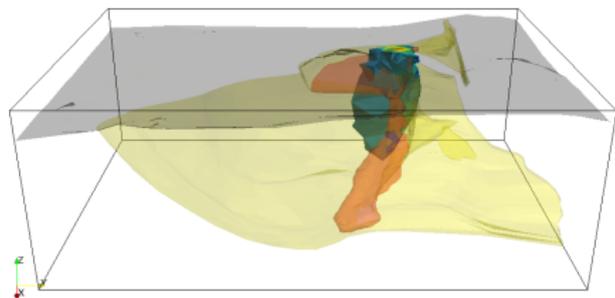
noise plus signal from extension

# True model



# Unconstrained inversion

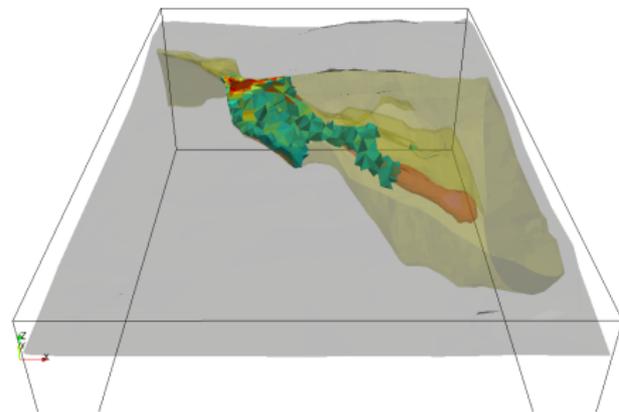
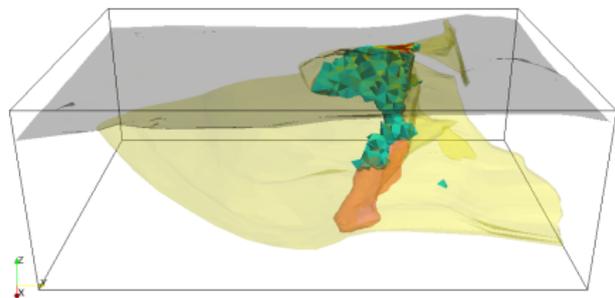
(gravity gradiometry data only)



Recovery of shallow sulphide body only

## Geologically-constrained inversion

(gneiss-troctolite surface and appropriate bounds)



Indication of extension  $\Rightarrow$  refine geological model; collect downhole data

# Conclusion

- most current 3D geological Earth models comprise wireframe surfaces

# Conclusion

- most current 3D geological Earth models comprise wireframe surfaces
- in contrast, most current 3D geophysical modelling is performed on rectilinear meshes

## Conclusion

- most current 3D geological Earth models comprise wireframe surfaces
- in contrast, most current 3D geophysical modelling is performed on rectilinear meshes
- working with unstructured meshes allows for efficient incorporation of complicated *a priori* geometries (forward modelling; constrained inversions)

# Acknowledgements

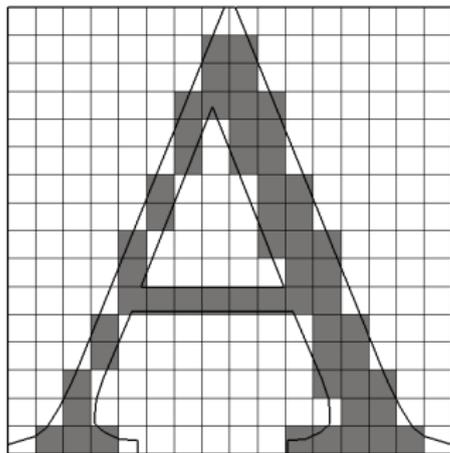
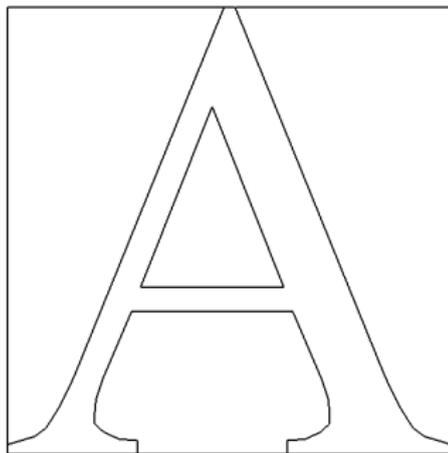
- ACOA  
(Atlantic Canada Opportunities Agency)
- NSERC  
(Natural Sciences and Engineering Research Council of Canada)
- Vale



Additional Slides Follow

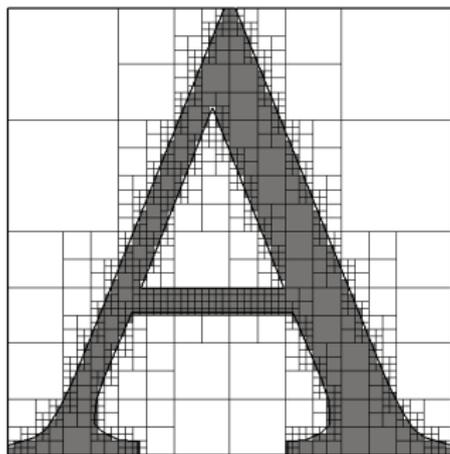
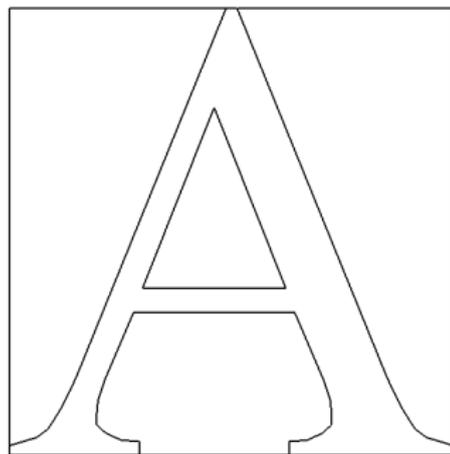
## Rectilinear meshes

- may require infeasibly many cells for adequate representation
- pixellated representation
- 256 cells



## Quadtree & octree meshes

- need fewer cells and are still structured
- pixellated representation
- 946 cells (4096 in underlying regular mesh)



# Unstructured triangular & tetrahedral meshes

- efficient generation of complicated geometries
- significant reduction in problem size
- 183 cells

