

Introduction

Joint inversion has the potential to significantly improve inversion results by reducing the non-uniqueness of the inverse problem. One of the challenges of joint inversion is coupling the multiple physical property models. If a coupling approach is used that is inconsistent with the physical truth then inversion artifacts can occur and may lead to incorrect interpretations. **In this work, we investigated a fuzzy c-means clustering approach to lithologically couple seismic velocity and density in joint inversions of first-arrival traveltimes and gravity data.**

Geological scenario

We conducted a suite of joint inversion tests on synthetic data generated from a geologically realistic model. The synthetic model, Figure 1, is inspired by magmatic massive sulphide deposits such as the Voisey's Bay Ni-Cu-Co deposit, the Sudbury Ni-Cu-PGE deposits and the Thompson Ni-Cu deposits. These deposits comprise sulphide lenses hosted in a mafic or ultramafic magmatic or volcanic rock. Our model contains a pyrrhotite and pentlandite dominated sulphide lens at the base of a gabbroic intrusion within a quartzite footwall.

The physical properties assigned to the three rock units are based on the work of Salisbury et al. (2000) who demonstrated that the relationship between the density and velocity of silicate rocks can be approximated using the Nafe-Drake curve, Figure 2. The model consists of a very dense, slow sulphide lens; a moderately dense, fast gabbro intrusion; and a low density, slow quartzite background.

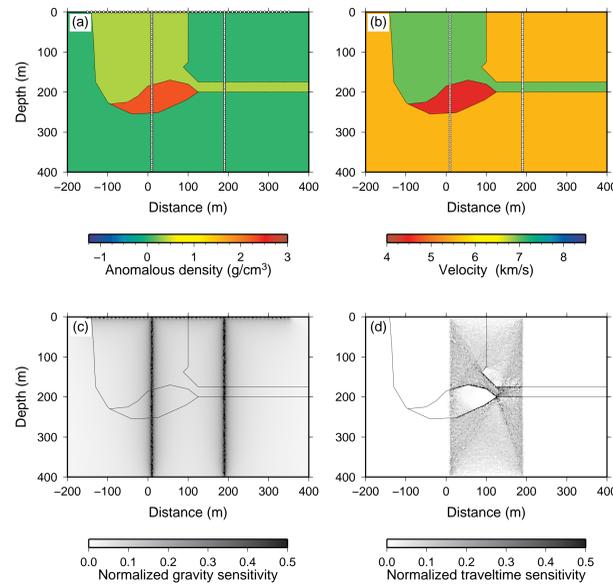


Figure 1 : Panels (a) and (b) show the density and velocity, respectively, of the true synthetic model. Anomalous densities are with respect to a background of 2.5 g/cm³. White dots in (a) indicate the locations of the surface and downhole gravity data, and in (b) they indicate the locations of the downhole transmitters (right-most borehole) and receivers (left-most borehole). Panels (c) and (d) show RMS volume-adjusted sensitivities for each cell of the modelling mesh for the gravity and traveltimes data respectively. The sensitivities are normalized by the largest value in the mesh (the grey scale maximums have been reduced to improve contrast in the images).

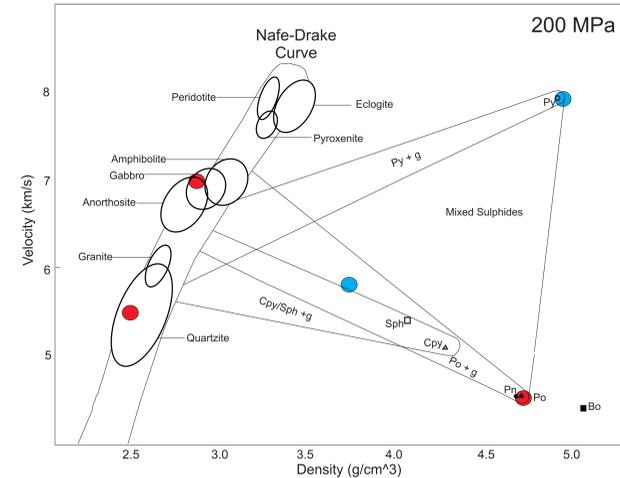


Figure 2 : Physical properties for various rocks and minerals, including those of interest in this work, modified from Salisbury et al. (2000). Red dots indicate the values used for our true synthetic model and blue dots indicate values used for inaccurate a priori information.

Methods

In our deterministic minimum-structure inversions, we minimize the objective function of Lelièvre et al. (2012):

$$\Phi(m_1, m_2) = \lambda_1 \Phi_d(m_1) + \alpha_1 \Phi_{m1}(m_1) \dots + \lambda_2 \Phi_d(m_2) + \alpha_2 \Phi_{m2}(m_2) + \rho \Psi(m_1, m_2) \quad (1)$$

The two Φ_d terms measure the data misfit for each of the two datasets. The two Φ_m terms measure the amount of structure in each of the two physical property models, m_1 and m_2 . The Ψ joint coupling term measures the similarity between the two models. In this study, guided by physical property information, e.g. Figure 2, we consider a coupling measure based on the fuzzy c-mean (FCM) clustering approach of Paasche and Tronicke (2007):

$$\Psi(m_1, m_2) = \sum_{i=1}^C \sum_{k=1}^M w_{ik}^2 \left((m_{1,k} - u_{1,i})^2 + (m_{2,k} - u_{2,i})^2 \right) \quad (2)$$

where M is the number of model cells, C is the number of clusters, u_1 and u_2 define the a priori cluster centres, and the membership weights w_{ik} relate the physical property values for the k^{th} cell to the i^{th} cluster.

Given that the cluster centres ($u_{1,i}, u_{2,i}$) are specified a priori in our approach, we performed a post-inversion cluster analysis to assess whether or not the physical properties in the jointly recovered models clustered as prescribed. If they don't, this may indicate that something is wrong with the a priori knowledge: a cluster centre may be specified incorrectly, or one or more additional cluster centres may be missing or unnecessary. We used the Xie-Beni validity index (XBI) of Xie & Beni (1991) to determine the optimal number of clusters, C :

$$XBI = \frac{\Psi(m_1, m_2)}{Ms^2} \quad (3)$$

where s is the minimum distance between any pair of cluster centres.

Independent inversions

Surface and downhole gravity data and cross-well tomography traveltimes were calculated from the true model and contaminated with random noise. Figure 1 shows the gravity and seismic surveys and a sensitivity analysis. Figure 3 shows the results of inverting these two datasets independently.

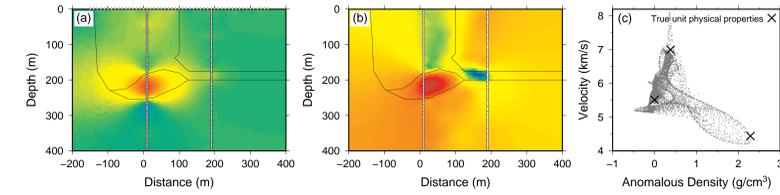


Figure 3 : Results from independent inversions: a) anomalous density model, b) velocity model, c) cross-plot of the recovered densities and velocities in every cell of the inversion mesh. Colour scales are as in Figure 1.

Joint inversion with accurate a priori information

In this test, accurate a priori information was provided to the joint inversion, providing clearly improved results over the independent inversions.

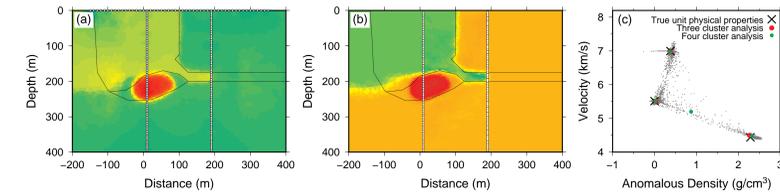


Figure 4 : Results from joint inversion with the three cluster centres for the true model prescribed.

Joint inversion with an inaccurate cluster centre

In these tests, inaccurate a priori information was provided to the joint inversion. Three cluster centres were included but one was prescribed erroneously. The results show clear artifacts resulting from the inaccurate a priori information, and the cross-plots of density and velocity provide an indication that there is indeed something wrong with the a priori information.

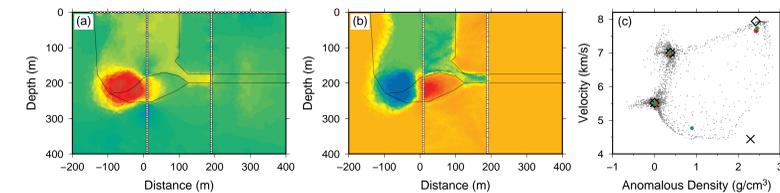


Figure 5 : Results from joint inversion with the cluster centre for the massive sulphides replaced with one consistent with pyrite.

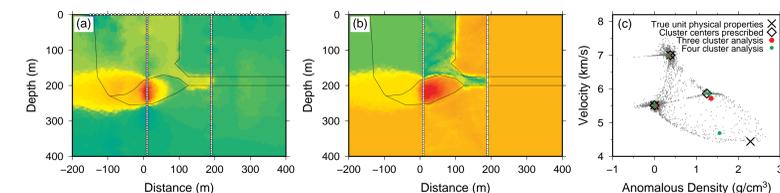


Figure 6 : Results from joint inversion with the cluster centre for the massive sulphides replaced with one consistent with disseminated mixed sulphides.

Joint inversion with inaccurate number of clusters

In these tests, the inaccuracy in the a priori information related to the number of clusters: either too many or too few were prescribed. Some undesirable artifacts are present but, again, the cross-plots provide an indication that there is something wrong with the a priori information.

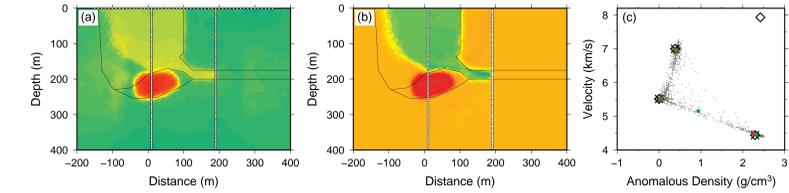


Figure 7 : Results from joint inversion with all three true clusters prescribed plus an additional cluster consistent with pyrite.

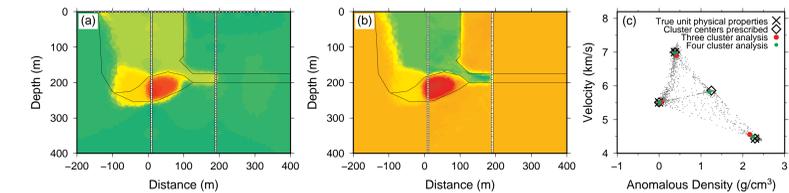


Figure 8 : Results from joint inversion with all three true clusters prescribed plus an additional cluster consistent with disseminated mixed sulphides.

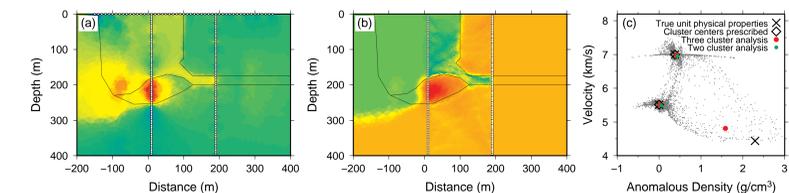


Figure 9 : Results from joint inversion with only two of the clusters for the true model prescribed. The cluster for the massive sulphides is absent.

Conclusion

The tests clearly demonstrate the benefits of joint inversion using FCM coupling, provided the a priori information is accurate. This work also illustrates the effects of including inaccurate a priori physical property information and suggests approaches to assess whether such inaccurate information may have been used.

Acknowledgements

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References

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