Parameterization of the Earth's Subsurface to Flexibly Emphasize Distinct Rock Units

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Motivation

- Typical geophysical inversions discretize the Earth into many cells and seek smoothly varying models.
- In contrast, geologists' interpretations about the Earth typically involve contacts between distinct rock units.
- **There are benefits to performing fundamentally different inversions that seek the interfaces between proposed rock units.**

Sharp Interface Volumetric Inversion

- Standard approach: minimization of objective function including data misfit and smoothness regularization:

$$\Phi(\mathbf{m}) = \|\mathbf{W}_d(F[\mathbf{m}] - \mathbf{d})\|^2 + \beta\|\mathbf{Dm}\|^2$$

- Typical "minimum structure" inversions use ℓ_2-norms (sum-of-squares) to obtain smooth models.
- **Piecewise-constant models, with regions of uniform values between sharper features, can be recovered using different smoothness measures**:

$$\psi(\mathbf{x}) = \sum_i \rho(x_i) \quad , \quad \rho(x_i) = (x_i^2 + \epsilon^2)^{p/2}$$

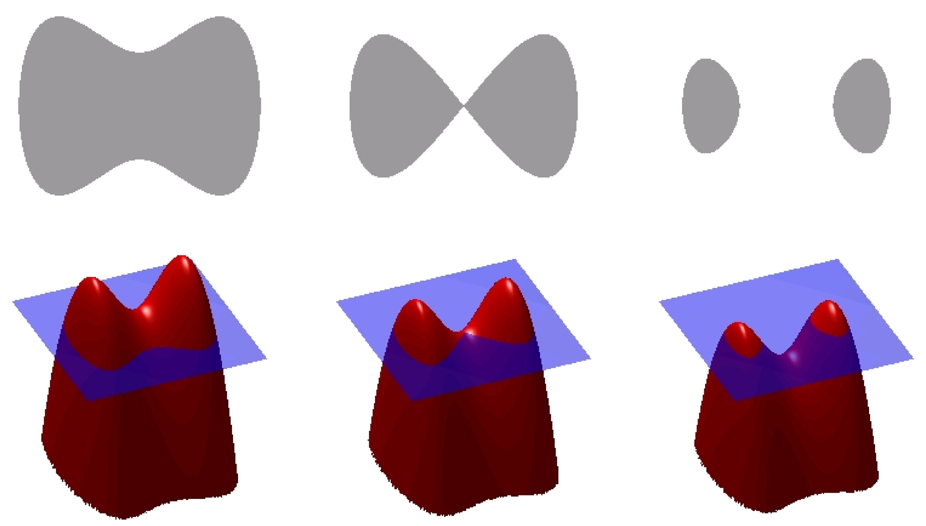
- Clustering methods can provide further improvements.

Level Set Parameterization

- An interface (a contact) is parameterized as the 0-level set of a higher dimensional "level set" function.
- The model values on an underlying mesh are determined by the level set function φ as follows:

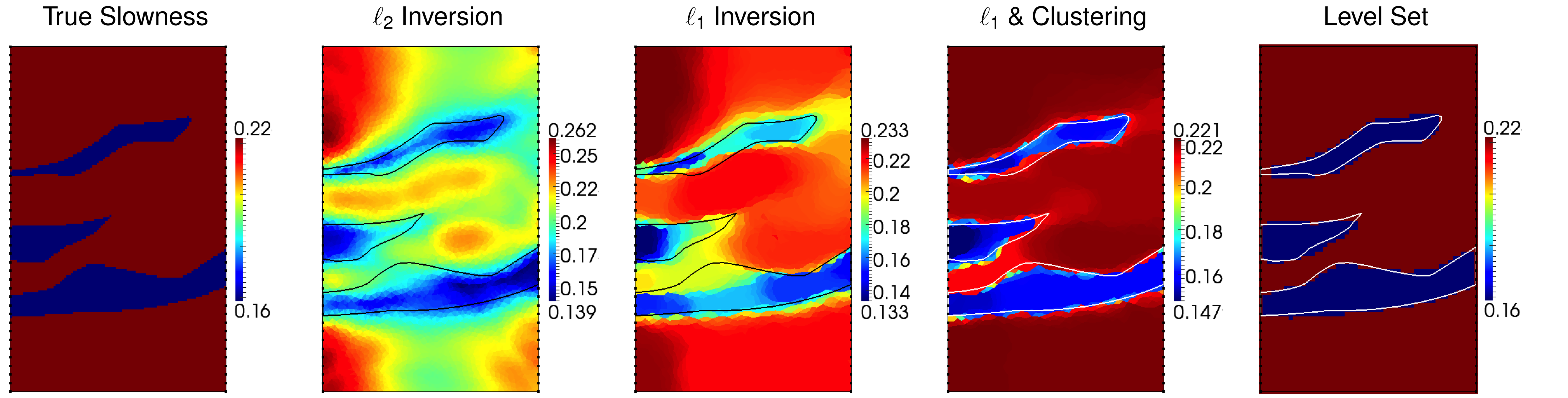
$\varphi \geq 0$, in the inclusion,
$\varphi < 0$, in the background,
$\varphi = 0$, on the interface.

- The interface changes as the level set function evolves to minimize the objective function.
- **The level set method naturally handles topology changes (merges, separations) without adding algorithmic complexity.**



An illustration of the concept of the level set method: the intersection of the 0-level (blue) with the level set function (red) generates the lower dimensional bodies (grey). Used with permission from Oleg Alexandrov at the Wikipedia project.

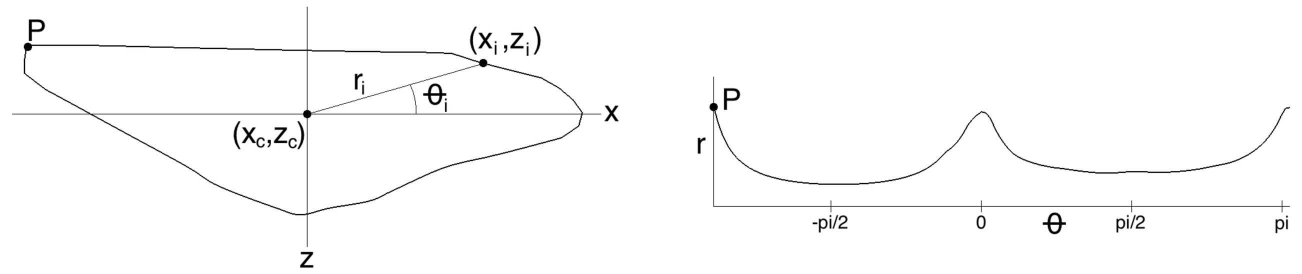
Cross-Well Tomography Example



True and recovered models for a cross-well tomography example. The outlines of the true bodies are superimposed in black or white on top of the recovered models. Lines of down-hole sources and receivers are displayed as black dots. The colour-bars show slowness in s/km. The meshes are 60 m by 100 m.

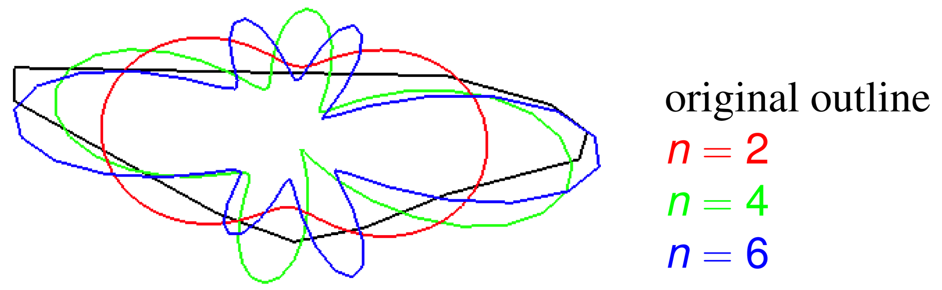
Spherical Harmonic Parameterization

- In 2D, a polygon representing the outline of a body can be represented by a control point (x_c, z_c) and the distance r of the vertices from that point:

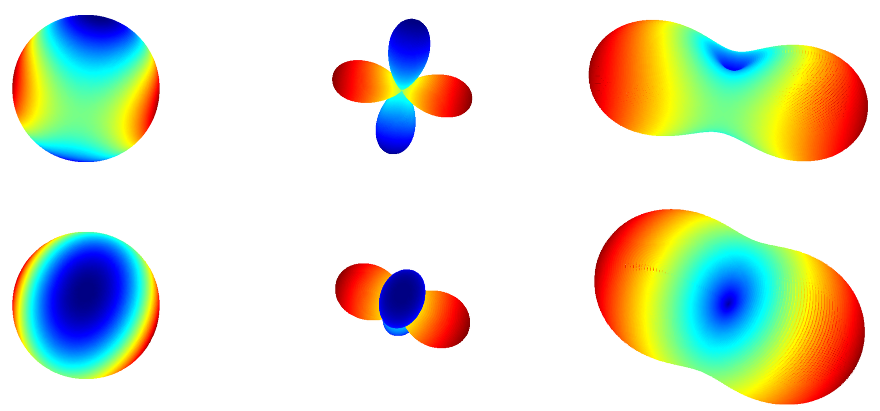


- The information can be expressed as a Fourier series:

$$r(\theta) = r_0 + \sum_{k=1}^{n}\left(a_k\cos(k\theta) + b_k\sin(k\theta)\right)$$



- Spherical harmonics can be used to extend to 3D.

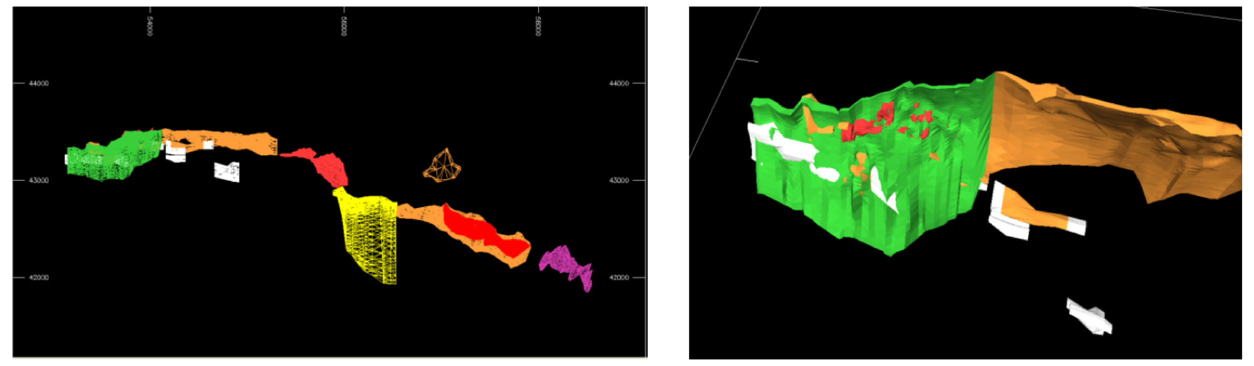


Two views (top and bottom) of a sphere (a zero-degree spherical harmonic, left), a higher degree spherical harmonic (middle), and the addition of both (right). The rainbow colour-scale indicates radial deviation (blue negative, red positive) for the body at right.

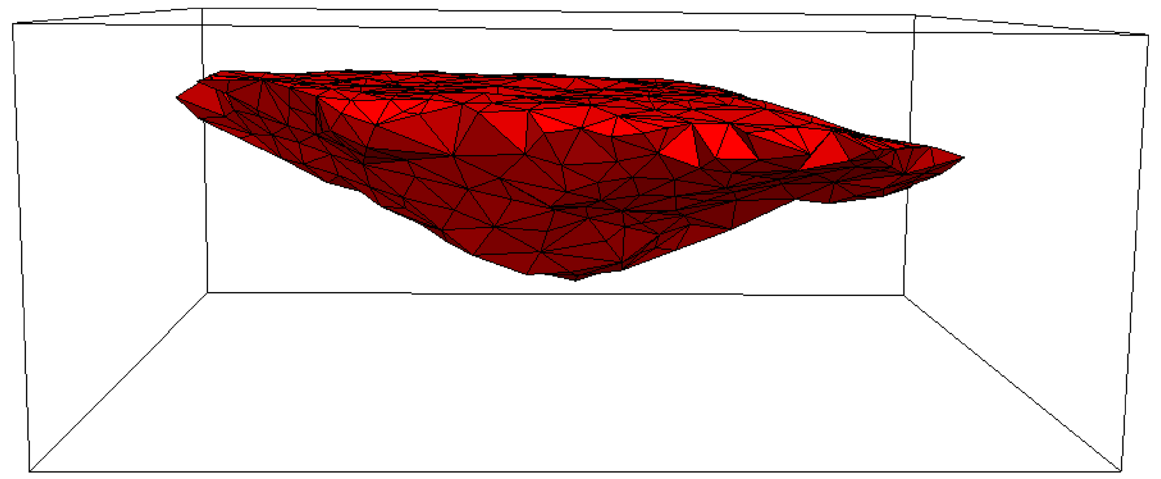
- Regularization involves keeping fewer or more high coefficients.

Wireframe Parameterization

- Geological models typically comprise wireframe surfaces representing geological contacts between rock units.



- A wireframe surface, or section thereof, can be parameterized by its node coordinates in a Cartesian or spherical system.

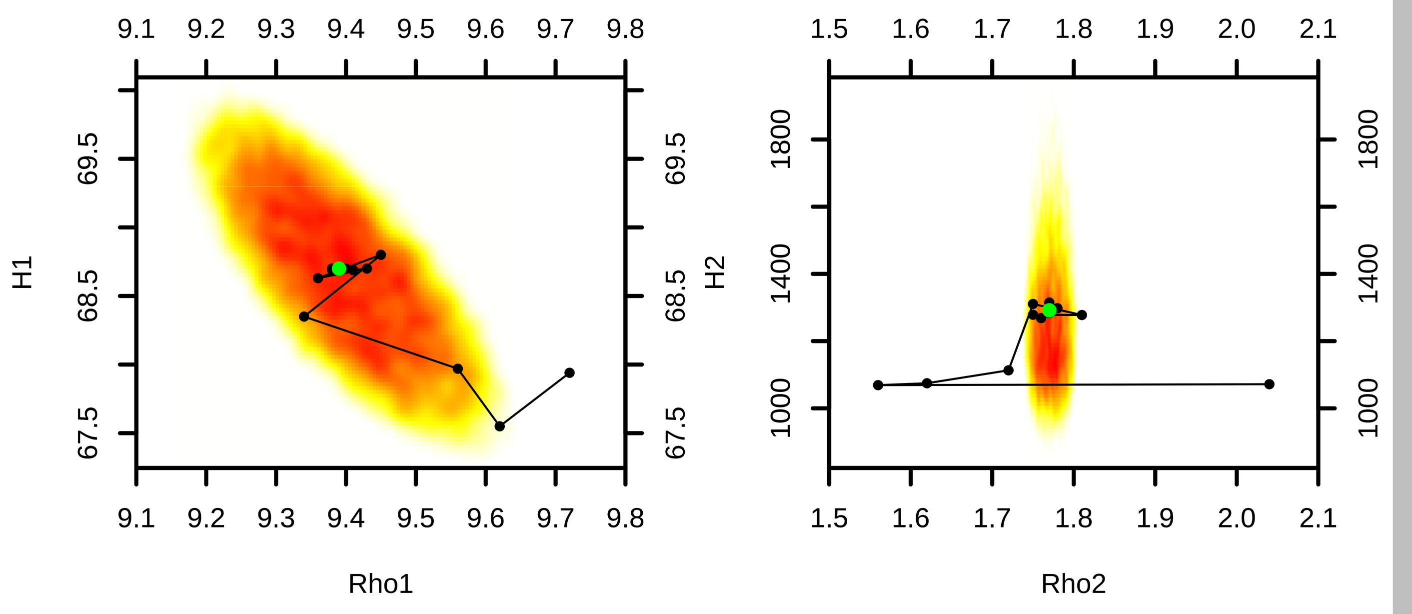


A wireframe of tessellated triangles representing an isolated 3D body.

- **Geological and geophysical models can be specified using this same parameterization: they are, in essence, the same Earth model.**
- Regularizing the inverse problem is somewhat complicated for this parameterization:
  - enclosed volume,
  - surface area,
  - surface curvature.
- Constraints may be required to avoid intersections.
- **The wireframe and spherical harmonic parameterization schemes call for the use of global optimization methods.**

Global Optimization

- Particle Swarm Optimization (PSO) simulates the social behaviour of animals, e.g. a swarm of bees searching for food.
- The particles are aware of their current position, previous best personal position, and global or group best position; the particles are moved according to the objective function values at those positions.
- It is relatively easy to develop hybrid PSO methods for stochastic interpretation.



Evolution of the global best PSO solution (black dots, green dot is final solution) plotted over Bayesian likelihood for a 2-layer MT example.

Further Reading

- T. Danek, M. Wojdyla and C. G. Farquharson, 2012, Bayesian inversion of geophysical data using combined particle swarm optimization and Metropolis sampling, 2012 EGU General Assembly Geophysical Research Abstracts Vol. 14, EGU2012-5024.
- C. G. Farquharson, 2008, Constructing piecewise-constant models in multidimensional minimum-structure inversions, Geophysics, 73, K1–K9.
- P. G. Lelièvre, C. G. Farquharson and C. A. Hurich, 2011, Inversion of first-arrival seismic traveltimes without rays, implemented on unstructured grids, Geophysical Journal International, 185, 749–763.
- P. Zheglova and C. G. Farquharson, *submitted*, Level set method in seismic inversion: 2D reconstruction of boundaries, 2012 SEG Annual Meeting Abstracts.

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