Joint inversion of seismic travel times and gravity data on 3D unstructured grids with application to mineral exploration

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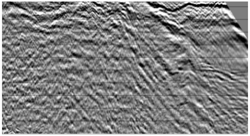
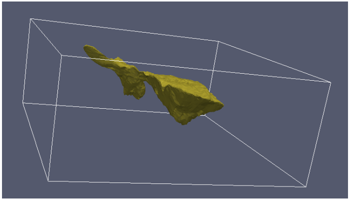
EGU Vienna, SM5.6, May 4, 2010

Problem statement

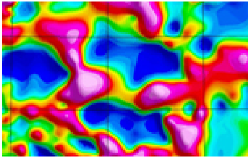
Forward problem



Seismic data



Earth model (density, velocity)



Gravity data

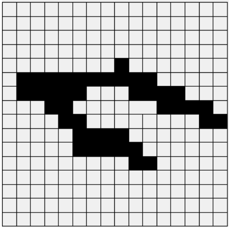
Inverse problem

Motivation: joint inversion

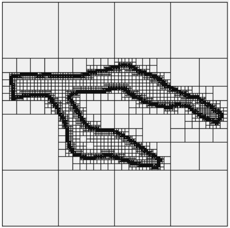
- An Earth model consistent with multiple datasets is more likely to represent the true subsurface than a model consistent with only a single type of data.
- Complicated hard-rock geology can cause difficulties with seismic data processing and interpretation.

Motivation: unstructured grids

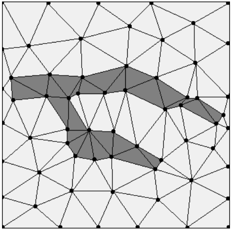
- Efficient generation of complicated subsurface geometries when known *a priori*
- Significant reduction in problem size



Rectilinear



Quadtree, Octree



Unstructured

Two types of data

Gravity data

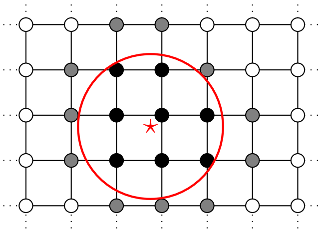
- Analytic response of a triangle, tetrahedron (Okabe, 1979, Geophys.)
- Finite element solution to Poisson's equation

Seismic data

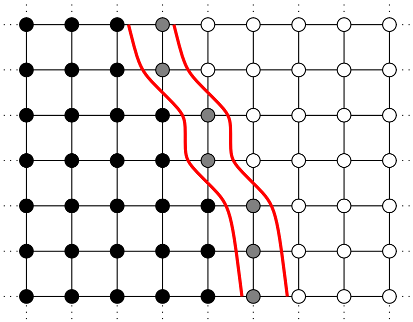
- First-arrival travel times
- Fast Marching Method (Sethian, 1996, P.N.A.S.)

Seismic first-arrivals: fast marching solution

1) Initialization near-source

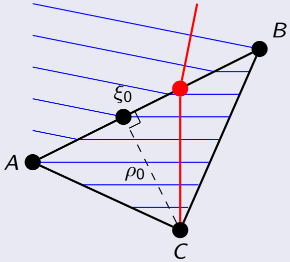


2) Solution-front marching



Seismic first-arrivals: local update

2D triangular grid

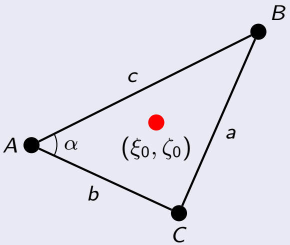


$$t_C = t_A + (t_B - t_A)\,\xi_0 + w c^{-1} \rho_0$$

$$w = \sqrt{s^2 c^2 - (t_B - t_A)^2}$$

(Fomel, 2000, S.E.P.)

3D tetrahedral grid



$$t_D = t_A + (t_B - t_A)\,\xi_0 + \dots$$
$$+ (t_C - t_A)\,\zeta_0 + \tilde{w} \varphi^{-1} \rho_0$$

Fermat's Principle

Joint optimization problem

Single dataset

- Objective function

$$\Phi = \beta \Phi_d + \Phi_m$$

- Data misfit

$$\Phi_d = \sum_i \left(\frac{d_i^{pred}(m) - d_i^{obs}}{\sigma_i} \right)^2$$

- Model structure

$$\Phi_m = (\text{smallness term}) + (\text{smoothness term})$$

Joint optimization problem

Single dataset

$$\Phi = \beta \Phi_d + \Phi_m$$

Two datasets

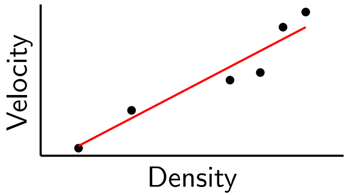
$$\Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \Phi_{joint}$$

$$\Phi_{joint} = \sum_j \gamma_j \Psi_j \left(m_1, m_2 \right)$$

Measures of model similarity: compositional

Measured analytic relationship

- linear-linear
- log-linear
- log-log
- etc.

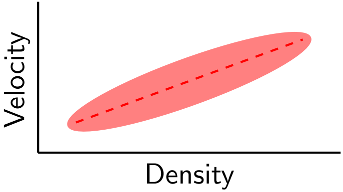


$$\Psi\left(m_1, m_2\right) = \sum_{i=1}^{M} \left(a m_{1,i} + b m_{2,i} + c\right)^2$$

Measures of model similarity: compositional

Assumed analytic relationship

- "Some" (linear) relationship
- Correlation from statistics
- Independent of scale of physical properties

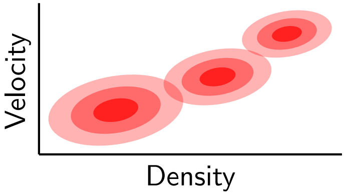


$$\Psi\left(m_1, m_2\right) = \left(\frac{\sum_{i=1}^{M} \left(m_{1,i} - \mu_1\right)\left(m_{2,i} - \mu_2\right)}{M\sigma_1\sigma_2} \pm 1 \right)^2$$

Measures of model similarity: compositional

Measured statistical relationship

- Probability density function
  e.g. combination of Gaussians
- Fuzzy C-means clustering
  (Paasche & Tronicke, 2007, Geophys.)

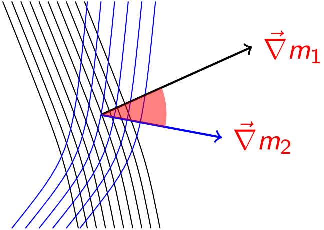


$$\Psi\left(m_1, m_2\right) = \sum_{k=1}^{C} \sum_{i=1}^{M} w_{ik}^2 \left(\left(m_{1,i} - u_{1,k}\right)^2 + \left(m_{2,i} - u_{2,k}\right)^2 \right)$$

Measures of model similarity: structural

Assumed spatial correlation
(changes occur in same place)

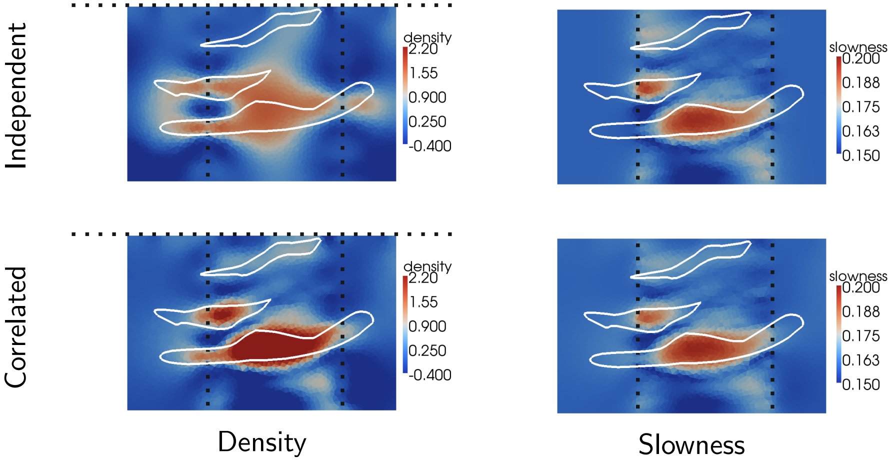
- "Structural" similarity (versus "compositional")
- Cross-gradients (Gallardo & Meju, 2004, J.G.R.)
- Independent of scale of physical properties



$$\Psi\left(m_1, m_2\right) = \left\| \vec{\nabla} m_1 \times \vec{\nabla} m_2 \right\|^2$$

2D example: recovered models

(true = 0.0, 2.0 g/cc ; 0.16, 0.22 s/km)



2D example: recovered models

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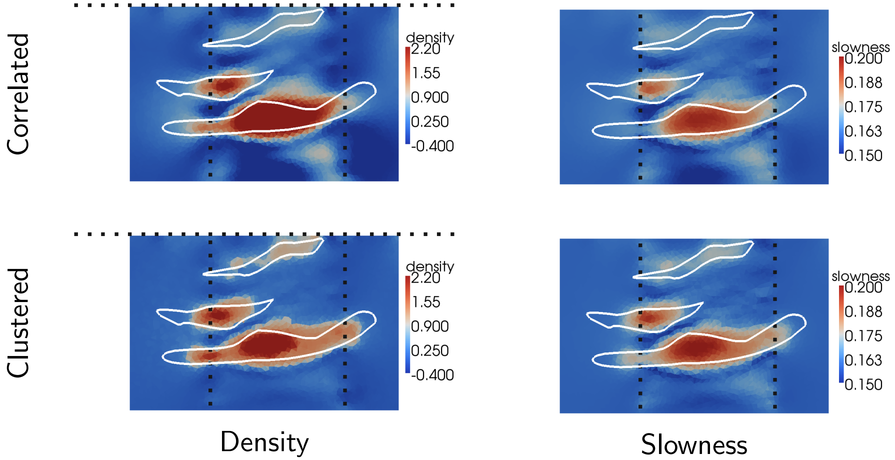
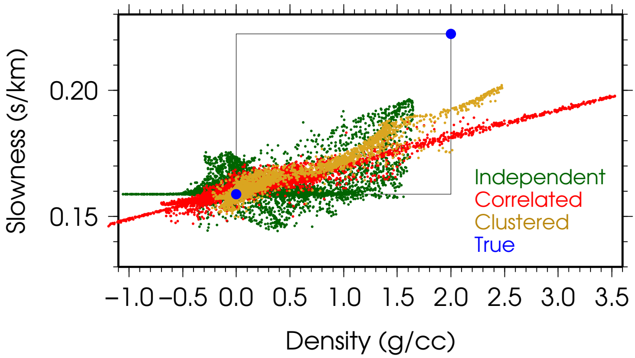
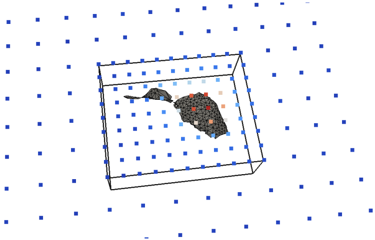


2D example: density versus slowness

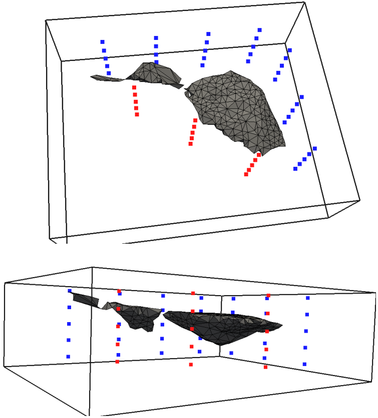


3D example: true model and data coverage

Voisey's Bay sulfide deposit, Labrador, Canada

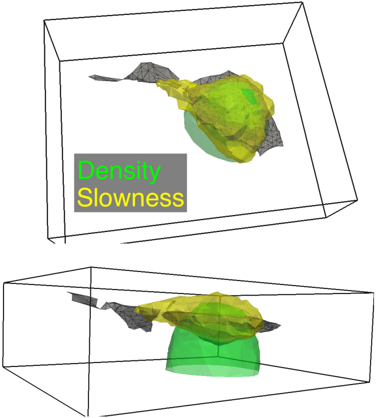


Gravity data

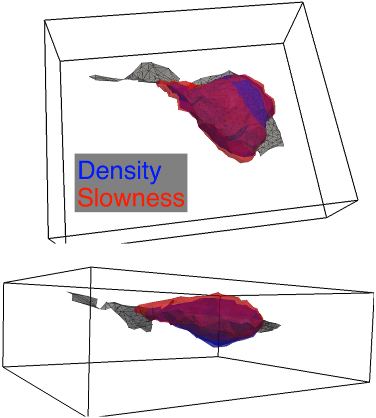


Sources (red) & receivers (blue)

3D example: recovered models

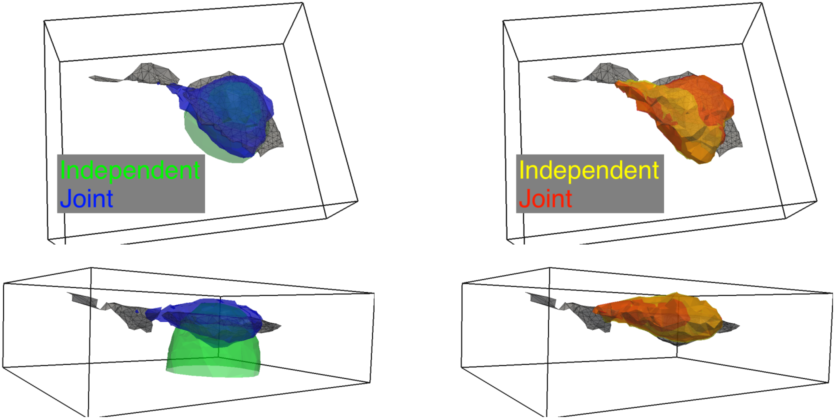


Independent



Joint (correlated)

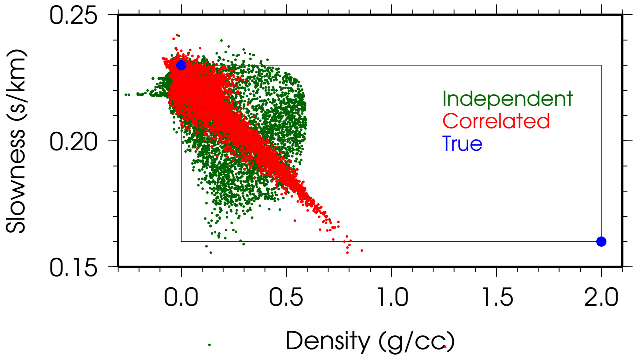
3D example: recovered models



Density

Slowness

3D example: density versus slowness



Summary

1. We use unstructured 2D and 3D grids, allowing for efficient generation of complicated subsurface geometries.

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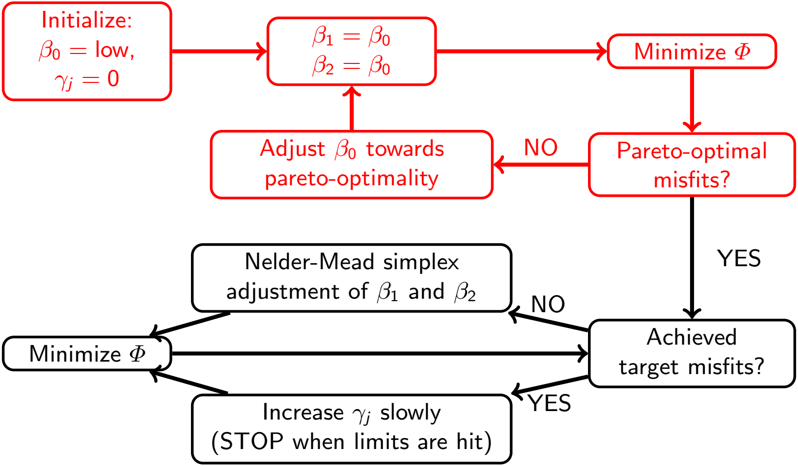
2. We have developed a local update for the Fast Marching Method on 3D tetrahedral grids.

3. We employ many joint similarity measures; those applied should depend on one's existing knowledge of the subsurface.

(additional slides follow)

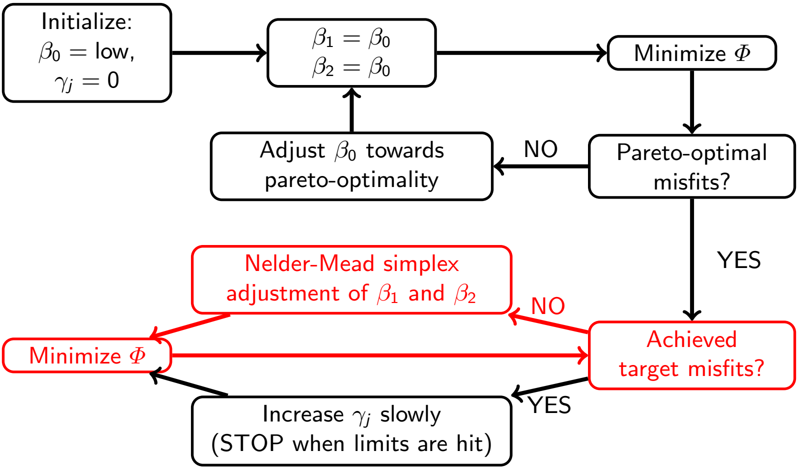
Algorithm: single beta, pareto search

$$\Phi = \beta_0 \left(\Phi_{d1} + \Phi_{d2}\right) + \Phi_{m1} + \Phi_{m2}$$



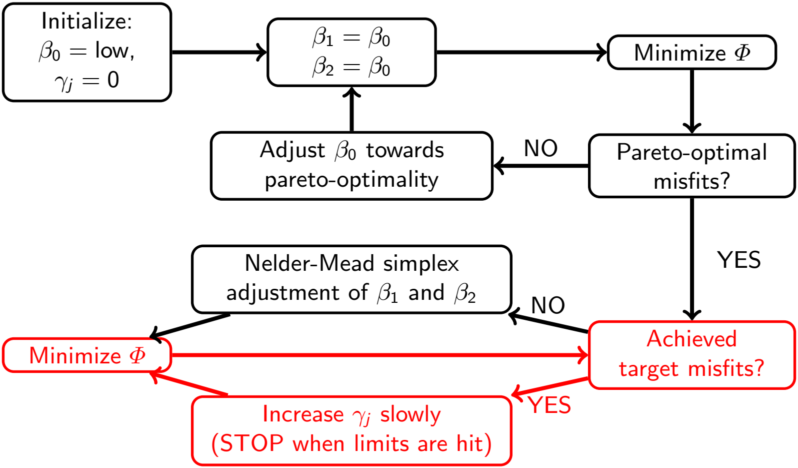
Algorithm: two betas, simplex search

$$\Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2}$$



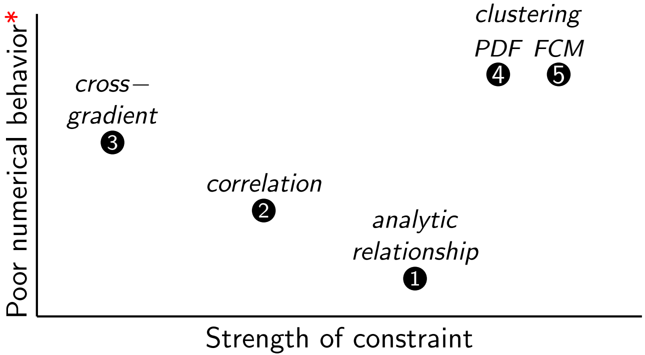
Algorithm: heating of joint measures

$$\Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \sum \gamma_j \Psi_i$$



Measures of model similarity: strength and behavior

The joint similarity measure(s) applied should depend on one's existing knowledge of the subsurface.



* nonlinearity, multiple minima

2D example: recovered models

(true = 0.0, 2.0 g/cc ; 0.16, 0.22 s/km)