Joint inversion of seismic travel times and gravity data on 3D unstructured grids with application to mineral exploration

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Joint inversion of seismic travel times and gravity data
Motivation: joint inversion

- An Earth model consistent with multiple datasets is more likely to represent the true subsurface than a model consistent with only a single type of data.

- Complicated hard-rock geology can cause difficulties with seismic data processing and interpretation.
Motivation: unstructured grids

- Efficient generation of complicated subsurface geometries when known \textit{a priori}
- Significant reduction in problem size

Rectilinear  
Quadtree, Octree  
Unstructured
Two types of data

Gravity data
- Analytic response of a triangle, tetrahedron (Okabe, 1979, Geophys.)
- Finite element solution to Poisson’s equation

Seismic data
- First-arrival travel times
- Fast Marching Method (Sethian, 1996, P.N.A.S.)
Seismic first-arrivals: fast marching solution

1) Initialization near-source

2) Solution-front marching
Seismic first-arrivals: local update

Two types of data
Seismic first-arrivals

2D triangular grid

\[ t_C = t_A + (t_B - t_A) \xi_0 + wc^{-1} \rho_0 \]
\[ w = \sqrt{s^2 c^2 - (t_B - t_A)^2} \]

(Fomel, 2000, S.E.P.)

3D tetrahedral grid

\[ t_D = t_A + (t_B - t_A) \xi_0 + \ldots + (t_C - t_A) \zeta_0 + \tilde{w} \varphi^{-1} \rho_0 \]

Fermat’s Principle
Joint optimization problem

Single dataset

- Objective function
  \[ \Phi = \beta \Phi_d + \Phi_m \]

- Data misfit
  \[ \Phi_d = \sum_i \left( \frac{d_i^{\text{pred}}(m) - d_i^{\text{obs}}}{\sigma_i} \right)^2 \]

- Model structure
  \[ \Phi_m = (\text{smallness term}) + (\text{smoothness term}) \]
Joint optimization problem

Single dataset

$$\Phi = \beta \Phi_d + \Phi_m$$

Two datasets

$$\Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \Phi_{joint}$$

$$\Phi_{joint} = \sum_j \gamma_j \psi_j (m_1, m_2)$$
Measures of model similarity: compositional

Measured analytic relationship

- linear-linear
- log-linear
- log-log
- etc.

\[ \Psi (m_1, m_2) = \sum_{i=1}^{M} (a m_{1,i} + b m_{2,i} + c)^2 \]
Assumed analytic relationship

- “Some” (linear) relationship
- Correlation from statistics
- Independent of scale of physical properties

\[
\Psi(m_1, m_2) = \left( \frac{\sum_{i=1}^{M} (m_{1,i} - \mu_1)(m_{2,i} - \mu_2)}{M\sigma_1\sigma_2} \pm 1 \right)^2
\]
Measures of model similarity: compositional

Measured statistical relationship

- Probability density function
  e.g. combination of Gaussians

- Fuzzy C-means clustering
  (Paasche & Tronicke, 2007, Geophys.)

\[
\Psi(m_1, m_2) = \sum_{k=1}^{C} \sum_{i=1}^{M} w_{ik}^2 \left( (m_{1,i} - u_{1,k})^2 + (m_{2,i} - u_{2,k})^2 \right)
\]
Measures of model similarity: structural

Assumed spatial correlation (changes occur in same place)

- “Structural” similarity (versus “compositional”)
- Cross-gradients (Gallardo & Meju, 2004, J.G.R.)
- Independent of scale of physical properties

\[ \Psi (m_1, m_2) = \| \vec{\nabla} m_1 \times \vec{\nabla} m_2 \|^2 \]
2D example: recovered models
(true = 0.0, 2.0 g/cc ; 0.16, 0.22 s/km)
2D example: recovered models
(true = 0.0, 2.0 g/cc ; 0.16, 0.22 s/km)
2D example: density versus slowness
3D example: true model and data coverage

Voisey’s Bay sulfide deposit, Labrador, Canada

Gravity data

Sources (red) & receivers (blue)
3D example: recovered models

Independent

Joint (correlated)
Joint inversion of seismic travel times and gravity data

3D example: recovered models

- Density
- Slowness

Independent
Joint
Joint inversion of seismic travel times and gravity data
We use unstructured 2D and 3D grids, allowing for efficient generation of complicated subsurface geometries.
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We have developed a local update for the Fast Marching Method on 3D tetrahedral grids.
Introduction
Forward modelling
Joint inversion
Examples
Summary

Summary

1. We use unstructured 2D and 3D grids, allowing for efficient
generation of complicated subsurface geometries.

2. We have developed a local update for the Fast Marching
Method on 3D tetrahedral grids.

3. We employ many joint similarity measures; those applied
should depend on one’s existing knowledge of the subsurface.
(additional slides follow)
Algorithm: single beta, pareto search

\[ \Phi = \beta_0 (\Phi_{d1} + \Phi_{d2}) + \Phi_{m1} + \Phi_{m2} \]

Initialize:
\[ \beta_0 = \text{low}, \quad \gamma_j = 0 \]

Minimize \( \Phi \)

\( \beta_1 = \beta_0 \)
\( \beta_2 = \beta_0 \)

Adjust \( \beta_0 \) towards pareto-optimality

Pareto-optimal misfits?

Nelder-Mead simplex adjustment of \( \beta_1 \) and \( \beta_2 \)

Achieved target misfits?

Increase \( \gamma_j \) slowly (STOP when limits are hit)

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Joint inversion of seismic travel times and gravity data
Algorithm: two betas, simplex search

\[ \Phi = \beta_1 \Phi_d + \beta_2 \Phi_d + \Phi_m + \Phi_m \]

Initialize:
\[ \beta_0 = \text{low}, \quad \gamma_j = 0 \]

\[ \beta_1 = \beta_0 \]
\[ \beta_2 = \beta_0 \]

Minimize \( \Phi \)

Adjust \( \beta_0 \) towards pareto-optimality

Nelder-Mead simplex adjustment of \( \beta_1 \) and \( \beta_2 \)

Minimize \( \Phi \)

Increase \( \gamma_j \) slowly (STOP when limits are hit)

Pareto-optimal misfits?

Achieved target misfits?

YES

NO
Algorithm: heating of joint measures

\[ \Phi = \beta_1 \Phi_{d1} + \beta_2 \Phi_{d2} + \Phi_{m1} + \Phi_{m2} + \sum \gamma_j \Psi_i \]

Initialize:
\[ \beta_0 = \text{low}, \quad \gamma_j = 0 \]

Minimize \( \Phi \)

Adjust \( \beta_0 \) towards pareto-optimality

Pareto-optimal misfits?

Nelder-Mead simplex adjustment of \( \beta_1 \) and \( \beta_2 \)

Achieved target misfits?

Increase \( \gamma_j \) slowly (STOP when limits are hit)

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The joint similarity measure(s) applied should depend on one’s existing knowledge of the subsurface.

- *nonlinearity, multiple minima*

**Algorithm**

**Measures of model similarity**

**Examples**

**Measures of model similarity: strength and behavior**

The joint similarity measure(s) applied should depend on one’s existing knowledge of the subsurface.

- **Strength of constraint**
  - Poor numerical behavior
    - **cross-gradient**
    - **correlation**
    - **analytic relationship**
  - **clustering**
    - **PDF**
    - **FCM**

*nonlinearity, multiple minima*
2D example: recovered models
(true = 0.0, 2.0 g/cc ; 0.16, 0.22 s/km)

Clustered (slow)

Clustered (fast)

Density

Slowness

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