Finite volume modelling of electromagnetic data using unstructured staggered grids

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Unstructured grids

- model irregular structures
Unstructured grids

- topographical features
- geological interfaces
Unstructured grids

- local refinement (at observation points, sources, interfaces)
Maxwell’s equations: describe the behavior of electromagnetic fields.

\[ \nabla \times \mathbf{E} = -i \omega \mu_0 \mathbf{H} - i \omega \mu_0 \mathbf{M}_p \\
\nabla \times \mathbf{H} = \sigma \mathbf{E} + \mathbf{J}_p \]

Helmholtz equation for electric field:

\[ \nabla \times \nabla \times \mathbf{E} + i \omega \mu_0 \sigma \mathbf{E} = -i \omega \mu_0 \mathbf{J}_p - i \omega \mu_0 (\nabla \times \mathbf{M}_p) \]

Boundary condition:

\[ \mathbf{E} \cdot \tau = 0 \quad \text{on } \Gamma \]
Staggered tetrahedral-Voronoi grids

- Tetrahedral grid
- Voronoi grid
Finite-volume method

Integral form of Maxwell’s equations:

\[
\oint_{\partial A^D} \textbf{E} \cdot d\textbf{l}^D = -i\mu_0\omega \iint_{A^D} \textbf{H} \cdot d\textbf{A}^D - i\mu_0\omega \iint_{A^D} \textbf{M}_p \cdot d\textbf{A}^D \\
\oint_{\partial A^V} \textbf{H} \cdot d\textbf{l}^V = \sigma \iint_{A^V} \textbf{E} \cdot d\textbf{A}^V + \iint_{A^V} \textbf{J}_p \cdot d\textbf{A}^V
\]
Discretized form of Maxwell’s equations:

\[
\sum_{q=1}^{W_i^D} E_{i(j,q)} l_{i(j,q)}^D = -i \mu_0 \omega H_j A_j^D - i \mu_0 \omega M_{pj}
\]

\[
\sum_{k=1}^{W_i^V} H_{j(i,k)} l_{j(i,k)}^V = \sigma E_i A_i^V + J_{pi}
\]

Discretized form of Helmholtz equation:

\[
\sum_{k=1}^{W_i^V} \left( \left( \sum_{q=1}^{W_j^D} E_{i(j,q)} l_{i(j,q)}^D \right) \frac{l_{j(i,k)}^V}{A_j^D} \right) + i \omega \mu_0 \sigma E_i A_i^V
\]

\[
= -i \omega \mu_0 \sum_{k=1}^{W_i^V} M_{pj(i,k)} \frac{l_{j(i,k)}^V}{A_j^D} - i \omega \mu_0 J_{pi}
\]

Sparse direct solver: MUMPS (Amestoy et al., 2006)
Interpolation inside tetrahedra: edge vector basis functions
Grid generator: TetGen (Si, 2004)
Inclusion of EM sources

- grounded wire:

- point vertical magnetic dipole:
Example 1: long grounded wire

- 100 m wire along the $x$ axis operating at 3 Hz
- Dimensions of the prism: $120 \times 200 \times 400$ m
- $\sigma_{\text{ground}} = 0.02 \text{ S/m}$; $\sigma_{\text{prism}} = 0.2 \text{ S/m}$
- Observation points along the $x$ axis

![Diagram of a long grounded wire along the x-axis with dimensions and observation points indicated.]
Example 1: long grounded wire

- dimensions of the domain: $40 \times 40 \times 40 \ km$
- number of tetrahedra: 162,689 ; number of unknowns: 189,105
Example 1: long grounded wire

- grid refined at the source, observation points and the prism
- computation time: 40 s; memory: 4 Gbytes (on Apple Mac Pro; 2.26 GHz Quad-Core Intel Xeon processor)
Example 1: long grounded wire

- without prism (homogeneous halfspace)
- total field
- FV vs IE (Farquharson and Oldenburg, 2002)

- with and without prism
- total field
- FV only

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Circles: IE; in phase; positive
Black line: FV; in phase; positive
Crosses: IE; quadrature; negative
Gray line: FV; quadrature; negative

Black solid line: in phase; with anomaly
Gray dashed line: in phase; no anomaly
Gray solid line: quadrature; with anomaly
Black dashed line: quadrature; no anomaly
Example 1: long grounded wire

- Scattered field
- FV vs IE

In phase

Quadrature

Secondary $E_x$ (V/m)

$x$ (m)

Full line: finite volume
Circles: integral equation

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Example 1: long grounded wire

- horizontal section \((z = -150 \text{ m})\)
- total electric field (in phase and quadrature)

- vertical section \((y = 0 \text{ m})\)
- total electric field (in phase and quadrature)
Example 2: magnetic dipole transmitter-receiver pairs

- graphite cube in brine (physical scale modelling measurements)
- transmitter-receiver pairs along the $x$ axis at $z = 2$ cm
- dimensions of the cubic graphite: $14 \times 14 \times 14$ cm
- $\sigma_{\text{brine}} = 7.3 \ S/m$; $\sigma_{\text{prism}} = 63,000 \ S/m$
- frequencies: 1, 10, 100, 200, 400 kHz
Example 2: magnetic dipole transmitter-receiver pairs

- grid refined at the sources, observation points and the prism
Example 2: magnetic dipole transmitter-receiver pairs

- scattered H-field: (total−free-space)/free-space
- FV vs PSM (Farquharson et al., 2006)

![Graphs showing in phase and quadrature responses at 1 kHz](image)
Example 2: magnetic dipole transmitter-receiver pairs

- scattered H-field: \( \frac{\text{total} - \text{free-space}}{\text{free-space}} \)
- FV vs PSM

In phase

Quadrature

\[ H_z \text{ (\%)} \]

\[ x \text{ (m)} \]

\[ 10 \text{ kHz} \]

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Example 2: magnetic dipole transmitter-receiver pairs

- scattered H-field: \((\text{total} - \text{free-space})/\text{free-space}\)
- FV vs PSM

In phase

100 kHz

Quadrature

100 kHz
Example 2: magnetic dipole transmitter-receiver pairs

- scattered H-field: \( \text{total−free-space}/\text{free-space} \)
- FV vs PSM

![Graphs showing in phase and quadrature measurements for 200 kHz](image-url)
Example 2: magnetic dipole transmitter-receiver pairs

- Scattered H-field: \((\text{total} - \text{free-space})/\text{free-space}\)
- FV vs PSM

**In phase**

**Quadrature**

400 kHz

\[ H_z (\%) \]

\[ x (m) \]
Example 2: magnetic dipole transmitter-receiver pairs

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature

In phase

Quadrature
Example 2: magnetic dipole transmitter-receiver pairs

- total H-field (1 kHz)
- horizontal and vertical sections
Example 2: magnetic dipole transmitter-receiver pairs

- total H-field (10 kHz)
- horizontal and vertical sections

![Graphs showing magnetic field distributions](image-url)
Example 2: magnetic dipole transmitter-receiver pairs

- total H-field (100 kHz)
- horizontal and vertical sections

![Diagram showing horizontal and vertical sections of in-phase and quadrature total H-field](image-url)
Conclusions

- A finite-volume technique is used for modelling the electromagnetic data. This technique uses the staggered tetrahedral-Voronoï grid.
- The Helmholtz equation is discretized and solved using a sparse direct solver (MUMPS).
- The scheme has been tested for two models: one with a long grounded wire source; another one for magnetic source-receiver pairs with large conductivity contrasts.
- For the both examples, the results from the FV scheme are in good agreement with those from the IE method (example 1) and the physical scale modelling measurements (example 2).
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