

## Modelling and Inversion for Mineral Exploration Geophysics: A Review of Recent Progress, the Current State-of-the-Art, and Future Directions

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### ABSTRACT

*Many developments have occurred in computer modelling for mineral exploration geophysics over the past decade, both in terms of new techniques being devised and previously underutilized techniques being adopted by industry. As in the past, the continuing improvement in computer performance has facilitated most of the developments. Three dimensional forward and inverse modelling are now possible for essentially all geophysical data-types used in mineral exploration. Inversions can be carried out for meshes with many more cells than was ever before possible, and more sophisticated, more capable, more complex modelling and inversion approaches contemplated. There have also been significant developments in computer modelling for structural geology, particularly in automated ways of constructing interface-based geological models. Some of the more sophisticated geophysical modelling and inversion approaches being developed are specifically targeted at constructing geophysical models that are consistent with these geological models. The minimum-structure, Occam's technique is still the workhorse of inversion for mineral exploration geophysics. This is in large part due to its reliability and robustness. Variations on the minimum-structure approach enhanced with new capabilities are being developed, and some recently developed variations are now seeing uptake by industry. Strategies for integrated imaging, that is, constructing a multiple-physical-property Earth model consistent with multiple geophysical data-types, have been devised: joint inversion, using either a coupling between physical properties based on subsurface spatial variation or coupling based on physical property information; cooperative inversion; and post-inversion lithologic segmentation. Constrained inversion and inverting magnetic data for the magnetization vector are experiencing increased use by industry. Development of inversion methods that produce geophysical models that can be more easily integrated with geological models will continue. Possibilities include geophysical inversion done directly on the interface-based geology models, meaning both geologists and geophysicists would be working with the same, truly integrated Earth model; joint inversion of structural geological data and geophysical data to give a model consistent with both these geological and geophysical data-types; and level-set, clustering or litho-type techniques to construct geophysical models more like a 3D geology map of the subsurface. Geological and geophysical modelling capabilities will also continue to be improved, in particular, the speed with which geophysical data can be synthesized for a given Earth model, and the ease with which an Earth model can be built, manipulated and refined in a graphical environment. Minimum-structure inversion, for a single physical property and using a fine rectilinear mesh, will remain a very practical option for many typical, real-life situations. The relative benefit of using more sophisticated and complex approaches will depend on the range and quality of data available, the complexity of the subsurface under investigation, and the level of detail and sophistication required in the constructed model to answer the exploration question. The developments over the last decade have greatly increased the range of capabilities available to us in our geophysical forward and inverse modelling toolkit. The next decade will see further such developments, with the developments that are truly useful finding their place in industry alongside the now-familiar minimum-structure inversion.*

### INTRODUCTION

Computer modelling in sciences and engineering involves numerical calculation of the state, response or behaviour of a system under certain conditions. In the particular case of mineral exploration geophysics, it is the physical fields—gravitational, magnetic, electric, electromagnetic (EM)—or their derivatives or integrals that are the measured quantities in any survey, and hence it is these that correspond to the “response” that we geophysicists are interested in. The “system” for which we want to synthesize responses (i.e., perform forward modelling) includes the mechanism, whether naturally occurring or human-made, that generates the fields, the means by which the fields (or their derivatives or integrals) are measured, and the properties of the subsurface of Earth. It is the values of the physical properties—density, susceptibility, magnetization, conductivity, chargeability—throughout Earth's subsurface that affect the measured quantities and hence these physical properties and

their spatial variations throughout the subsurface are a fundamental, core component of the system to which we want to be applying our computer modelling.

To carry out our computer modelling, we need to describe the variation of the relevant physical property or properties throughout Earth's subsurface in a form understandable to a computer program and amenable to numerical methods for synthesizing the physical fields. Such a description—“discretization” or “parameterization”—of the spatial variation of a physical property (or properties) is what we call our “model” of Earth, i.e., our geophysical Earth model.

Inverse modelling, or inversion, corresponds to the reconstruction of an unknown component of a system from measurements of a response of the system. In exploration geophysics, the unknown component about which we would like much more knowledge is the subsurface of Earth. In the narrower perspective of the computational problem that is

invariably solved, inversion corresponds to determining values of the parameters in our geophysical Earth model such that the synthesized values of the measured quantities adequately match the values that were actually measured. The assumption, then, is that the Earth model that we have constructed represents, or at least shares some characteristics with, the actual subsurface of Earth.

Forward and inverse modelling in geophysics have advanced substantially over the past decade. However, one major, fundamental issue that remains a challenge is how to translate knowledge about the spatial variation of physical properties in the subsurface to rock type, mineralization and alteration information; that is, translating the output of our inversion process into geological quantities more directly relevant to the exploration questions being asked. This is the “weak link” between geology and geophysics referred to by Palacky (1988), which, at the time of his writing, he considered to be “the most neglected area of the exploration sequence”. Much work on significantly strengthening this link has been undertaken over the last thirty years, including the last decade. This is not the place for a thorough review of physical property studies, nor are we best qualified to present one. However, we think it is important to keep sight of the fundamental importance of this link, and we feel it is useful and appropriate to indicate here some of the recent and ongoing work, particularly on building up collections of physical property values catalogued according to rock type and mineralization, that is enhancing and expanding our understanding of this link.

For example, Mira Geoscience has continued to expand their Rock Property Database System, which was initially described by Parsons and McGauchey (2007). Smith et al. (2012) summarize the measurement techniques for, and uses of, physical properties that were discussed at an Society of Exploration Geophysicists workshop on this topic in 2010. Duff et al. (2012) and Malehmir et al. (2013) determined seismic velocities and densities for ores, mineralized units and host rocks at Voisey’s Bay, Canada, and at three locations in Sweden, respectively. Mitchinson et al. (2013) compiled physical property information from the porphyry deposits studied in Geoscience BC’s *QUEST* projects. Enkin (2014) describes the Geological Survey of Canada’s physical property database (which includes densities, electrical resistivities, susceptibilities, remanent magnetizations and Koenigsberger ratios) built from measurements on numerous samples from throughout British Columbia. Tschirhart and Morris (2014), for the Bathurst Mining Camp, New Brunswick, and Enkin et al. (2016) for the Great Bear magmatic zone, Northwest Territories, investigated relationships between physical properties in attempts to identify combinations that were characteristic of particular rock types or mineralization in these two areas.

Compilations and studies such as those mentioned above involve all possible means of determining physical properties of rocks, including direct measurement on samples in a lab, in-situ measurement via down-hole logging, and estimation of physical property values from geochemical data. A major complicating factor is that there are vastly different scales between geophysical surveys and the measurements performed to collect physical property information, and hence there are vastly

different scales of the volumes of material to which those two types of data are each sensitive. Because of the complexity of how a rock actually affects a physical field or flow of current (e.g., fracturing and mineralization providing conductive pathways through otherwise resistive rocks), the physical property value obtained for a particular rock type in a lab, downhole or by derivation might not be the same as that which a geophysical survey “sees” for that rock type in the subsurface. Furthermore, a non-uniqueness exists in that many different rock types have the same physical properties. These factors make it difficult to design useful recipes, formulae or look-up tables, except in the most restricted situations, to convert a spatial distribution of physical properties in the subsurface (a geophysical Earth model) into a spatial arrangement of rock units in the subsurface (a geological Earth model or 3D geology map of the subsurface). Nevertheless, physical property compilations and catalogues such as those indicated above are moving us closer to being able to devise look-up tables or keys, at least ones that can be used for specific, local scenarios, that enable us to convert our geophysical Earth models into geological Earth models, and ultimately aid in the construction of a common Earth model.

Just as for physical properties, we make no attempt here to thoroughly review the geophysical methods themselves that are used for mineral exploration. Instead we direct an interested reader to a number of excellent recent reviews, some that are new compilations or distillations of existing knowledge, others that cover new developments made over the last decade or so: Vallée et al. (2011) for mineral exploration geophysics in general; Thomas et al. (2016) for rare metal exploration; Nabighian et al. (2005a, b) for gravity and magnetic methods respectively; Loke et al. (2013) for DC resistivity; Zhdanov (2010), and the subsequent Comment by Nabighian (2012), and Smith (2014) for EM methods; and Yin et al. (2015) specifically for airborne EM methods. We also want to concentrate here on a relatively high-level review of modelling and inversion, and so do not consider details of any of the numerical mathematical methods for synthesizing data. Avdeev (2005), Börner (2010) and Newman (2014) provide excellent, thorough, comprehensive recent reviews of modelling techniques for EM methods. Textbooks such as Blakely (1996), as well as individual papers presenting recent novel developments (e.g., Ren et al., 2017, Martin et al., 2017), are the best sources of details for gravity and magnetic modelling techniques. There is also now quite a selection of textbooks on inversion methodology in geophysics: Parker (1994), Tarantola (2005), Ulrych and Sacchi (2005), Menke (2012), Aster et al. (2013), Sen and Stoffa (2013) and Zhdanov (2015). Oldenburg and Pratt (2007) give a comprehensive review of the state-of-the-art of geophysical inversion in the context of mineral exploration ten years ago.

Over the last decade, the increase in the capabilities of computers has continued to facilitate and drive the main trends and developments in modelling and inversion in geophysics. At the time of writing, a basic PC available from any of the main consumer-electronics or home-entertainment stores for well under a thousand dollars comes with, for example, a 2GHz quad-core processor and 8GB of RAM. A high-end PC costing five thousand dollars might come with a 3GHz 8-core processor,

32GB of RAM and one or two GPUs (graphics processors with hundreds or thousands of compute cores). Modest computer clusters comprising dozens of such high-end PCs connected via specialized high-speed networking (which amount to “computers” with 100s of cores and 1000s GB RAM), as well as the remote, nebulous “cloud”, are being used by research groups and modelling and inversion service providers. With the increase in the amount of memory that is typically available, larger and larger problems (in terms of the number of unknowns to be determined) can be fit into a computer and hence solved. With the increase in the speed of computers, and the increase in availability of multi-processor machines that enable parallel computing, the size of problem that can be run in a convenient or acceptable length of time has increased dramatically. We are now at the stage where 3D modelling and inversion for general Earth models is possible for all geophysical methods used for mineral exploration, with no major numerical approximations or model simplifications required.

The increases in computer capabilities have also effectively eased the restrictions on how we operate. The simplest, most basic 3D computations no longer max out our computational resources, as was once the case. This has opened up more freedom and possibilities in virtually all aspects of modelling and inversion for exploration geophysics. We are seeing Earth models that are both larger in spatial extent and with finer discretizations (and hence many more cells in total) than could ever have been contemplated ten years ago. We are also seeing an expanding range of modelling and inversion approaches, especially ever more sophisticated techniques that themselves offer more, or different, possibilities: for example, joint inversion, different model parameterizations, inversion for magnetization vector. It is these trends—existing methods applied to larger and larger models, and development of more sophisticated approaches—that we feel have been the most prevalent and significant over the last decade or so. We also feel these represent a subtle change in the direction of advances in modelling and inversion compared to previous decades when simply managing to perform any kind of 3D inversion was a goal in itself.

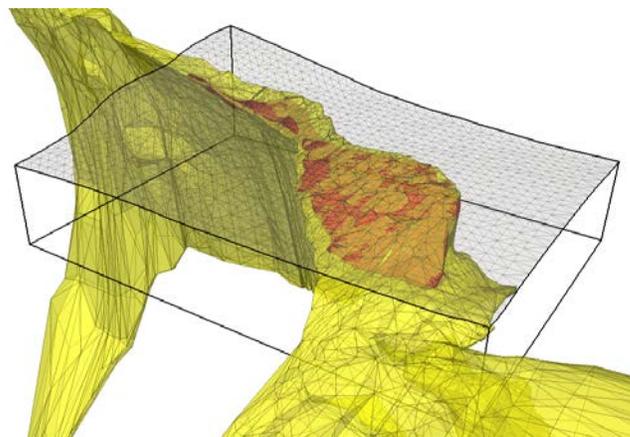
The remainder of this document is organized as follows. We discuss parameterizations of Earth models next, both because 1) the nature of the parameterization of our geophysical Earth model is fundamental to how we do our modelling and inversion (and how we *can* do our modelling and inversion), and 2) because of the expansion in types of parameterizations being used by new software as a consequence of no longer being restricted to the simplest possible 3D parameterizations. We then consider the minimum-structure, or Occam’s, style of inversion, which still dominates geophysical inversion, for very good reason. There are a number of developments that are built upon the general strategy of minimum-structure inversion and which have recently become available or seen an increase in usage. Most notable amongst these are constrained inversion and joint inversion, which we consider next. After these we touch on magnetic inversion methods for determining not susceptibility but the full magnetization vector from magnetic data, which is another inversion technology that is now receiving increased interest from industry. We end with a look at some emerging technologies, including inversion methods that are definitely not

minimum-structure approaches, and some thoughts on what the future might hold for forward and inverse modelling methods for mineral exploration geophysics.

## TYPES OF EARTH MODELS

### Geological Models: Tessellated Wireframe Surfaces

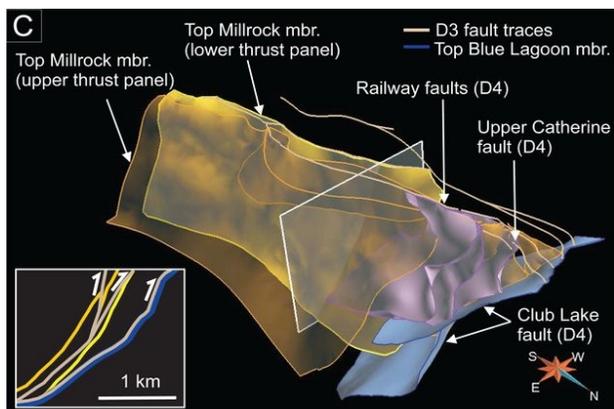
One trend over the last decade that we geophysicists have perhaps been slow to recognize, certainly as far as the authors are concerned, is the ever increasing use of 3D computer Earth models by geologists (see, for example: Caumon et al., 2009; Caumon, 2010; Jessell et al., 2014; Caumon et al., 2016; and even Wikipedia, 2017). These geological Earth models are primarily concerned with describing contacts between different rock units and faults, i.e., interfaces, in the subsurface. It is the locations and orientations of these interfaces, both in absolute terms and relative to one another, that is important to geologists; having a description of how, for example, a physical property varies everywhere throughout the volume of the subsurface is not so useful to them. Geological Earth models therefore involve parameterizations of surfaces, either explicit or implicit. Explicit parameterizations are typically in terms of wireframe surfaces made up of tessellations of triangular, planar facets (see Figure 1). Such wireframe surfaces, for which the sizes of the triangular facets can be as small or as large as required by the complexity of the interface being modelled, allow essentially any kind of arbitrary, general surface to be represented in 3D space.



**Figure 1:** An example of a wireframe geological model made up of surfaces tessellated with triangles. This is a simplified model of the Ovoid ore-body (red) and troctolite feeder zone (yellow) of the Voisey’s Bay deposit, Labrador, Canada. The faint grey surface is the topography. (From Lelièvre et al., 2012a.)

Geological Earth models are typically constructed using point locations of the interfaces observed from intercepts in boreholes and surface outcrop, measured strikes and dips of the interfaces, and observed locations and orientation information for units known to be on one side of an interface or the other. Geological models are used to visualize the subsurface geological features and their geometrical relationships (Figure 2), perhaps allowing for improved understanding of the processes that led to their formation, to compute volumes of possible resources, and to provide a starting point from which detailed models for mine

planning can be built. Some recent examples of the use of 3D geological models include Perrouy et al. (2014), Milicich et al. (2014), Martin-Izard et al. (2015), Vollgger et al. (2015), Schetselaar et al. (2016) and Bell et al. (2017). Building geological models used to involve the painstaking process of interpolating the surfaces between their known locations by hand, essentially drawing these surfaces in 3D space using what amounted to computer-aided design (CAD) software. However, within the last decade or so, from the field of computational structural geology, there has been considerable development of computer algorithms and software that can automatically construct surfaces through 3D space from sparse, discrete knowledge of locations and orientations of the surfaces taking into account structural geological rules and understanding. For recent developments, see, for example, Hillier et al. (2014), de la Varga and Wellmann (2016) and Gonçalves et al. (2017). For a compendium of 3D geology model building software, see [www.3d-geology.de](http://www.3d-geology.de) (Wycisk, 2017).

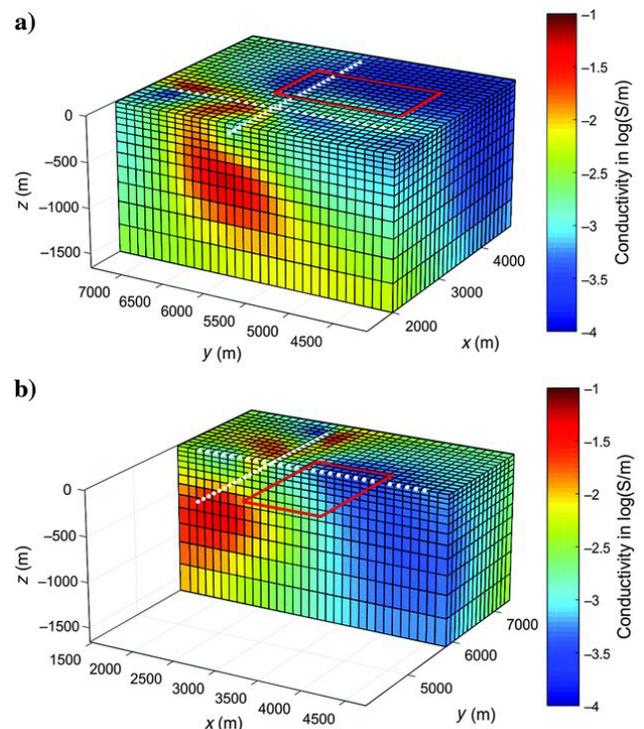


**Figure 2:** Selected features of the geological model of the Flin Flon-Callinan-777 volcanogenic massive sulphide (VMS) ore system in the Flin Flon belt of Manitoba and Saskatchewan, Canada, constructed by Schetselaar et al. (2016). (Figure adapted from Schetselaar et al., 2016.)

### The Geophysics Standard: Rectilinear Meshes

Rectilinear meshes have been the standard means of discretizing a general, arbitrary geophysical Earth model for many years. The model volume is subdivided into rectangular cuboid cells by planes normal to the three Cartesian coordinate axes and it is typical to consider the physical property to be constant within each cell (see Figure 3). These meshes are the easiest to generate: one simply specifies the numbers and dimensions of cells in each of the  $x$ -,  $y$ - and  $z$ -directions. More importantly, however, these are the easiest type of mesh for which to formulate numerical mathematical methods such as finite-difference solutions to partial differential equations. Also, numerical methods for these meshes typically have the nicest numerical properties (e.g., number and pattern of non-zero elements in the matrices for these numerical methods; convergence rates of iterative solvers for systems of equations involving these matrices) when compared with those for other types of meshes, particularly if the cells all have the same size (i.e., a uniform mesh). Rectilinear meshes are limited, however, in how well they can represent topography and the shapes of real

subsurface structures and interfaces unless a very fine mesh is used, which leads to longer computation times. Also, rectilinear meshes do not lend themselves well to local refinement at, for example, source and observation locations, nor padding zones with coarse discretizations in the outer parts of a mesh, without introducing cells with large aspect ratios that can affect numerical calculations. Nevertheless, as Figure 3 suggests, if topography is minimal, if the sources and receiver locations for a survey align with a Cartesian coordinate system, and if the exploration questions can be answered via inversion using a fairly coarse, mostly uniformly discretized mesh, then rectilinear meshes can be a perfectly good option.

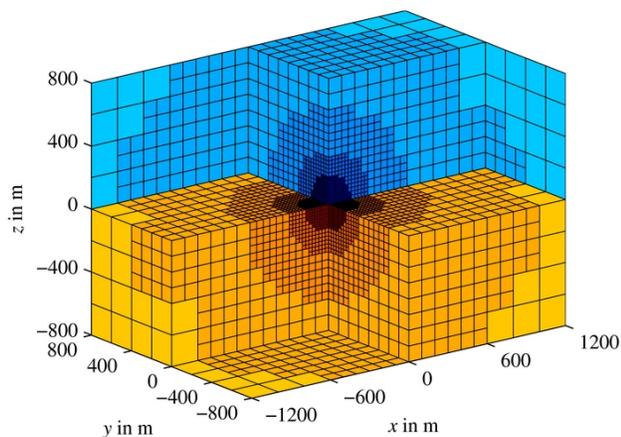


**Figure 3:** An example of a classic geophysical Earth model parameterized in terms of a rectilinear mesh. The black lines indicate the cell boundaries. These effectively constitute planes normal to the three Cartesian directions. The physical property, in this case conductivity, is constant within each cell. Two different views are shown: panel (a) shows a slice normal to the  $x$ -direction, panel (b) shows a slice normal to the  $y$ -direction. This model was constructed from the inversion of time-domain EM data from the Lalor deposit, Manitoba, Canada, by Yang and Oldenburg (2016). (Adapted from Yang and Oldenburg, 2016.)

### Geophysical Models: Non-conforming, OcTree Meshes

OcTree meshes are a semi-structured option that overcomes some of the disadvantages of rectilinear meshes. With OcTree meshes the subsurface is still subdivided using planar surfaces normal to the three Cartesian directions, but these planes are no longer infinite, meaning cells can be successively subdivided or amalgamated as one moves through the mesh (see Figure 4). These meshes share to some extent the advantages of rectilinear

meshes: they are reasonably easy to generate and adapt, numerical methods for synthesizing data using these meshes are natural extensions of those for rectilinear meshes, and these numerical methods retain some of the numerical “niceness” that the corresponding methods for rectilinear meshes exhibit. Also, OcTree meshes can be locally refined or coarsened. However, fundamentally the cell boundaries in an OcTree mesh are still all aligned with the Cartesian axes, and, even although the ability for local refinement can lessen this issue, it is impossible to avoid some amount of stair-casing when building arbitrarily oriented surfaces. The use of non-conforming, OcTree meshes for EM modelling and inversion has been presented by, for example, Haber and Heldmann (2007), Haber and Schwarzbach (2014) and Grayver and Bürg (2014), and for magnetic inversion by Davis and Li (2013).



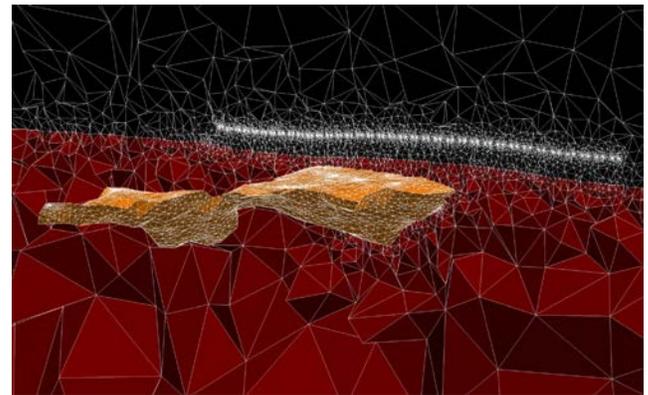
**Figure 4:** An example of an OcTree mesh. This mesh shows local refinement around a source-receiver location for an airborne EM survey (blue: air; yellow: ground; black lines: cell boundaries). (From Haber and Schwarzbach, 2014.)

### Geophysical Models: Unstructured Tetrahedral Meshes

Rectilinear and OcTree meshes are not directly compatible with geological models parameterized in terms of surfaces: the surfaces will invariably cut through the brick-like cells in the mesh, with the cut cells having to be assigned an average of the physical properties from the two sides of the surface. Performing such averaging so that the data synthesized using the rectilinear mesh are faithful to the original, interface-based geological model is not necessarily difficult (although not trivial for some physical properties and geophysical methods, most notably electrical conductivity and electrical and EM methods), and the synthesized data can be accurate enough if the rectilinear or OcTree mesh is of a sufficiently fine discretization. However, an alternative means of parameterizing the geophysical Earth model is to discretize the volumes between the surfaces in the geological Earth model using an unstructured tetrahedral mesh (Figure 5). In principle, one can take each triangular facet in a tessellated surface, construct a tetrahedral cell that has this triangular facet as one of its faces, and build outwards from the surface filling up the volumes with tetrahedral cells. The volumetric discretization is then entirely consistent with the tessellated surfaces. These meshes are unstructured in the sense

that there is not a simple recipe for locating a particular cell, as is the case for rectilinear meshes for which one can prescribe the number of cells one has to go in the  $x$ -,  $y$ - and  $z$ -directions to reach the cell of interest. Hence, the numerical methods for synthesizing data using rectilinear meshes do not naturally extend to unstructured tetrahedral meshes, and numerical methods using tetrahedral meshes generally require wholesale derivation from scratch.

As with OcTree meshes, unstructured tetrahedral meshes can be locally refined or coarsened. In a similar manner to the triangles of a tessellated surface, the cells in an unstructured tetrahedral mesh can grow or shrink as one moves from one part of the mesh to another. Tetrahedral meshes are therefore particularly well suited to refinement around transmitters and observation locations, which can be important for accuracy when synthesizing data, and to coarsening away from regions of interest and in the outer parts of a model, thus being economical in terms of the total numbers of cells in a mesh.



**Figure 5:** An example of a geophysical Earth model parameterized in terms of an unstructured tetrahedral mesh. This view shows (in orange) the Ovoid ore-body at Voisey’s Bay, Labrador, Canada, extruding from a section through a homogeneous background (red), the true topography (red-black interface), and refinement around the source and receiver locations for a DIGHEM flight line. The white lines indicate the edges of the tetrahedral cells. (From Ansari et al., 2017.)

There has been a surge over the past decade in the development of modelling and inversion techniques that use unstructured tetrahedral meshes. Rucker et al. (2006) and Günther et al. (2006) describe modelling and inversion of DC resistivity data, and consider, in particular, the ability of unstructured tetrahedral meshes to accurately represent real topography. Ren and Tang (2010) and Weiss et al. (2016) also consider DC resistivity, with Weiss et al. using unstructured tetrahedral meshes to model highly conducting fractures and borehole casing. Um et al. (2010), Schwarzbach and Haber (2013), Puzryev et al. (2013), Ansari and Farquharson (2014), Jahandari and Farquharson (2014) and Usui (2015) are examples for EM methods. Lelièvre et al. (2012b) consider gravity and seismic travel-time data in a mineral exploration context, and Jahandari and Farquharson (2013) synthesize gravity data via the solution of Poisson’s equation.

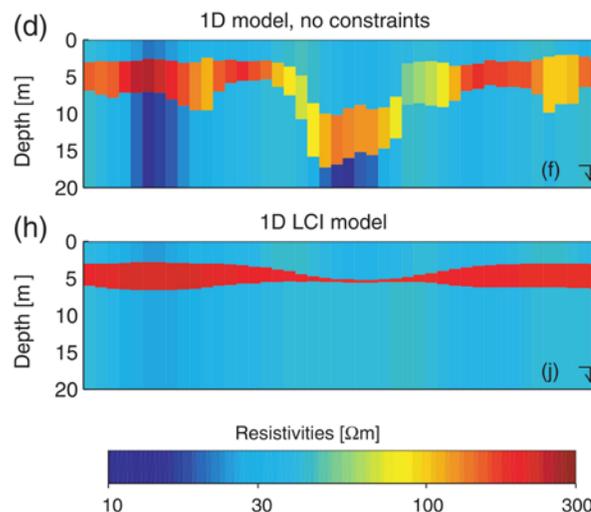
Unstructured tetrahedral meshes can conform exactly with surfaces tessellated in terms of triangles in geological Earth

models, effectively meaning that the one single model is parameterized in a way that allows for both geological and geophysical modelling. However, unstructured tetrahedral meshes are much more complicated to work with than rectilinear meshes, in particular, requiring specialized software for their generation and manipulation. Also, many numerical methods for synthesizing data require a “quality” tetrahedral discretization, i.e., one for which there is a minimal amount of narrow, pointy tetrahedra, with the performance of the numerical method being significantly degraded when applied to poor quality meshes. This issue is also relevant to rectilinear meshes when cells contain large aspect ratios; OcTree meshes largely dispense with the issue. For tetrahedral meshes, this issue can be exacerbated when starting construction of a model from a collection of tessellated surfaces, e.g., Figure 1. These surfaces need to be welded together into a “watertight” wireframe arrangement that unequivocally divides the Earth model into distinct regions. This welding process inevitably results in many small, elongated triangular facets on the composite tessellated surfaces, which then result in narrow, pointy tetrahedra and a poor quality mesh. However, specialized software for generating good quality tetrahedral meshes that honour surfaces tessellated in terms of triangles is available (e.g., *TetGen*, Si, 2015), and there are research groups actively working on methods for meshing surfaces and volumes specifically in the context of building and working with geological and geophysical Earth models (see, e.g., Pellerin et al., 2014, Zehner et al., 2015).

### Geophysical Models: Hybrid 1D/3D

The meshes considered above for geophysical models are all used for what we will refer to as “fixed-mesh” inversions, that is, inversions in which the physical property in each cell is sought but the locations of the cell boundaries remain unchanged. As will be elaborated on below, inversion techniques based on such meshes have many appealing characteristics. However, because the cell boundaries cannot move and the variation of the physical property in the subsurface is effectively being represented in pixellated, or “voxellated”, form, these meshes require fine discretization in order to reconstruct features and boundaries at arbitrary locations in the model. An alternative inversion strategy is to let the cell boundaries move in an inversion as well as letting the physical property in the cells vary. This strategy has the potential for requiring many fewer cells in the model. Also, this strategy offers the possibility of getting sharp interfaces in the constructed model, which is not easily accomplished for fixed-mesh inversion methods. However, letting all cell boundaries vary in an inversion as well as physical properties leads to a very unstable, non-unique, challenging problem. A number of groups have therefore adopted a hybrid mesh that resembles a rectilinear mesh but in which the horizontal boundaries between the cells in any column can move up and down during an inversion (Figure 6). Smith et al. (1999) use such a model for 2D inversion of magnetotelluric data in which they use 1D approximations based on the columns of cells for sensitivity calculations. de Groot-Hedlin and Constable (2004) use such a mesh (with full 2D calculation of the sensitivities on a finer rectilinear mesh) to obtain sharp sub-horizontal interfaces in their constructed models. Auken and Christiansen (2004) for DC resistivity data, and Auken et al. (2005) and Brodie and

Sambridge (2006) for EM data for example, use this type of mesh to construct essentially layered models with variable-depth interfaces that are particularly appropriate for predominantly stratified geology. Fullagar et al. (2008) use this kind of mesh for gravity and magnetic inversion both to construct models that contain sharp interfaces, albeit sub-horizontal, and to limit the total number of cells in a model.



**Figure 6:** Examples of a hybrid mesh comprising vertical columns each with a small number of cells. The vertical boundaries between columns do not move during an inversion, but the depths to the horizontal interfaces between cells, as well as the physical property within each cell, are allowed to vary. The model in the upper panel was constructed with no constraints on the lateral variation from column to column whereas, for the one in the lower panel, lateral smoothness constraints were applied to both the depths of the horizontal interfaces and the resistivities. (Adapted from Auken et al., 2005).

### Miscellaneous Model Thoughts: Meshes, Large Models/Airborne Surveys

The aforementioned meshes are not the only means of parameterizing an Earth model, but are currently the most common. Another type that has seen some limited use is the hexahedral curvilinear mesh. These meshes are like rectilinear meshes in that they are structured, they have the same connectivity relationships, and the surfaces defining cell boundaries extend to infinity. However, these infinite surfaces can now be non-orthogonal, and warped and bowed rather than planar. These meshes are somewhat flexible in terms of being able to represent curved surfaces (e.g., topography), but only as long as these surfaces are mostly horizontal or vertical. Also, these meshes do not really offer the possibility of substantial local refinement or coarsening. Some recent examples of the use of hexahedral curvilinear meshes are Cai et al. (2014) and Grayver and Kolev (2015; non-conforming hexahedral) for EM, and Kordy et al. (2016a, b) for magnetotellurics.

It has also been suggested that the mesh, or rather the cloud of nodes, that is used for the numerical mathematics of

synthesizing the data be decoupled from the mesh used to parameterize the Earth model. This would circumvent the need for the mesh describing the model to also satisfy quality constraints required by the numerical modelling method. The model mesh can then be chosen based on what is most convenient for describing the model, and the modelling mesh can be designed based on what the modelling method needs. An example of such a “mesh-free” approach for synthesizing data in a geophysical context is the magnetotelluric modelling by Wittke and Tezkan (2014).

The Earth models considered so far are aimed at representing a general, arbitrarily complex subsurface. However, modelling methods that are based on analytic or mostly analytic formulae for simple geometrical shapes and polygons (e.g., *Potent*, *GM-SYS*, *IGMAS+*) still have a significant role to play. Such forward-modelling methods are essentially instantaneous on modern computers, which facilitates interactive trial-and-error modelling sessions in which the model can be modified by point-and-click interactions in a GUI and the data is continually re-generated in “real time” as the model is altered. Although not an inversion method in the sense of automatically generating an Earth model from a set of observations, and despite it being labour intensive and time consuming, such a trial-and-error “inversion” does allow the operator to bring all their knowledge and experience (geological, geophysical, local) to bear on the modelling and interpretation of a particular data-set. However, having an expert in the driver's seat may bias the results and lead to an incomplete investigation of “model space”, that is, the wide array of model possibilities offered by the non-unique problem of honouring the geological and geophysical data.

Last but definitely not least for this Earth Models section, there are still scenarios that are computationally challenging even with the advances in computer capabilities over the last decade, and for which compromises have to be made in order to carry out an inversion. This is the case for airborne surveys, in general, and airborne EM surveys, in particular. Data-sets from airborne surveys can involve huge numbers of observations, cover large, regional-scale areas, and have observation locations that are only metres apart. A model used for inverting such a data-set needs, in principle, to span the whole survey area and to do so with a fine enough discretization so that local-scale features such as ore-bodies can be reproduced to the desired resolution. The computational expense is compounded for airborne EM surveys because each different transmitter location requires its own modelling, and for most airborne EM survey types each different location of the bird corresponds to a new transmitter location as well as a new observation location. A number of inversion strategies have been devised for constructing a single model from a whole airborne data-set. These strategies typically consider two tiers or levels of mesh: one for the whole model, and another that is adaptable and movable which is used for the modelling and sensitivity computations in the neighbourhood of each observation location. Cox and Zhdanov (2007) and Cox et al. (2010) present a “moving footprint” strategy for EM data in which one mesh is used, but only the cells in the vicinity of each transmitter-receiver pair are included in the forward modelling and sensitivity computations relevant to the location of that transmitter-receiver pair. Yang and Oldenburg (2012) present a method for airborne EM that progresses from a coarse mesh and

inversion of a sub-set of the data to more refined meshes and a more complete sub-set of the data. Yang et al. (2014) consider two meshes, one for the whole model and a rectilinear one tailored for forward modelling each transmitter-receiver pair in turn. Haber and Schwarzbach (2014) present a similar method in which the forward modelling mesh is an OcTree mesh. Scheunert et al. (2016) and Ullmann et al. (2016) use a combination of meshes and only use 3D forward modelling and sensitivities for the parts of the data-set for which 1D-based inversion is deemed inadequate.

## MINIMUM-STRUCTURE INVERSION

Minimum-structure, or “Occam's”, style of inversion has been the workhorse for inverse modelling in exploration geophysics for at least the last two decades. This style of inversion uses cell-based, fixed-mesh models (e.g., rectilinear, OcTree, tetrahedral) with the values of the physical property in the cells being the parameters sought in the inversion. The characteristic feature of this style of inversion is that a measure of the roughness of the model is combined with a measure of data misfit in the overall, composite objective/goal/cost/penalty function that is minimized. The very deliberate strategy is to find the simplest possible model that reproduces the measured survey data (to an adequate level given the noise in the measurements). Not only this, but the original, standard measure of model complexity, namely, the measure of model roughness/smoothness or variability, is a quadratic, convex function of the model parameters. These characteristics—inverting for physical property only and not locations of boundaries plus physical properties, finding the simplest possible model, and quantifying “simplest” in terms of a quadratic measure of model variability—mean that minimum-structure inversions are reliable and robust. They typically produce a useful model that fits the data at the first attempt (re-running from a suite of different starting models and starting conditions is not usually necessary), and their successful running tends not to be unduly affected by noisy data. It is because of this reliability and robustness that minimum-structure inversions have proved to be so useful and so successful. It is likely that they will continue to be the main tool in the geophysical inversion and modelling tool-box for many years to come.

The original minimum-structure inversion method has been adapted and extended in many ways. Arguably the two most important of which are constrained inversion and joint inversion, to which we devote the next two sections. But there have been many other adaptations, large and small. One grouping of extensions involves different measures of model complexity (and different measures of data misfit). A defining characteristic of the original minimum-structure approach, which uses a sum-of-squares, i.e., quadratic, measure of model roughness, is that the features in the constructed model are smooth, smeared-out and fuzzy. This is a very different representation of the subsurface from the interfaces and distinct units of geological Earth models. Variants of the minimum-structure method have been proposed that use a sum-of-absolute-values, or similar, measure for model roughness (e.g., Zhdanov et al., 2004, Farquharson, 2008). Such measures tend to result in a model with sharp, distinct interfaces separating mostly uniform blocks. Doing this within the framework of a minimum-structure

inversion and using a fixed-mesh model (preferably one with a fine discretization) allows for an arbitrary, complex model to be constructed, in contrast to the hybrid methods mentioned in the Earth Models section above. However, as we shall comment on again in the context of constrained and joint inversion, every adaptation of and extension to the standard, basic minimum-structure approach takes us further and further from the well-behaved, reliable, robust optimization problem of the original. Despite one's best efforts in designing a reliable and robust and careful optimization algorithm, it is possible that in increasing the sophistication and hoped-for capabilities of an inversion method one ends up with an algorithm that is no longer robust and reliable enough to be useful.

Another, more recent, suite of adaptations of the minimum-structure approach also attempts to generate models comprising distinct, mostly uniform units, but doing so not by minimizing some measure of the variability or roughness of a model, but by encouraging the physical property in each cell to be close to one of two or more prescribed values. Examples include the "multinary" approach of Zhdanov and Cox (2013; multinary as in an extension of binary) and the clustering approaches of Lelièvre et al. (2012), Sun and Li (2015), Carter-McAuslan et al. (2015), Li and Sun (2016) and Maag and Li (2016). Lelièvre et al. and Carter-McAuslan et al. are concerned with joint inversion (more below), and Li and Sun consider inversion for the full magnetization vector, but their respective clustering approaches are equally applicable to (scalar) single-property inversion.

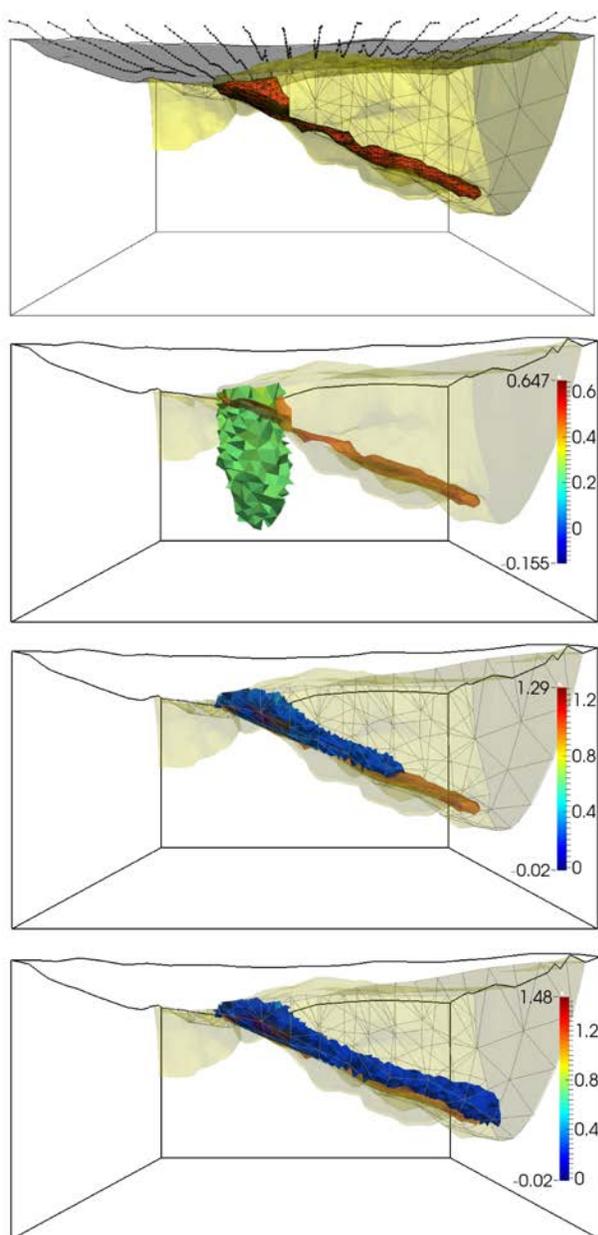
It is also possible to derive minimum-structure-like inversion algorithms from a statistical view of the subsurface (as well as the data). Instead of minimizing a measure of model roughness, the algorithm is designed to find the maximum of a probability density function that quantifies the probability of a certain character of the subsurface such as correlation length. This information about the model is the "a priori" information provided to the inversion. Such an approach is known as Bayesian, Maximum Entropy, or Maximum Likelihood inversion. Because of the explicit consideration of probabilities and statistics, this approach allows, perhaps, for a more complete, rigorous treatment of the noise in the measured data (noise quantified in terms of its probability density function; noise correlated between measurements), and for a more statistics-based, less-certain quantification of what we know about the subsurface. In practice, however, little quantitative information is available about the measurement noise and the character of the subsurface, meaning that only default, basic information can be incorporated into an inversion. In this case, these inversion algorithms tend to give very similar results to those generated by minimum-structure approaches. There have been fewer applications of this style of inversion to mineral exploration geophysics compared to other areas of geophysics. One example is Chasseriau and Chouteau (2003) who consider gravity inversion and use kriging to describe anticipated variation in the subsurface. Shamsipour et al. (2010) and Geng et al. (2014) are other examples of related stochastic, geostatistical approaches.

## CONSTRAINED MINIMUM-STRUCTURE INVERSION

Basic, default minimum-structure inversion is useful and completely appropriate in a greenfield exploration context in which essentially nothing is known about the subsurface beyond what is available from the geophysical data-set being inverted. However, when some knowledge of the subsurface exists, either from geological mapping or drilling or other geophysics, the diffuse, smeared-out, fuzzy-blob character of models constructed by basic minimum-structure inversions can be frustrating, misleading and clearly erroneous. Given this context, the most important and significant development in modelling and inversion for exploration geophysics over the last decade, in terms of relatively new technology getting used and being useful, has arguably been the growth in the use of constrained minimum-structure inversion. In this variant, one or two terms are included in the overall penalty function being minimized and the attached mathematical constraints: one that measures how similar the constructed model is to a user-supplied reference model, and one that is designed to restrict the physical property within each cell to lie between user-specified upper and lower bounds. The reference model and the upper and lower bounds can vary throughout the subsurface, as can a weighting incorporated into the measure of closeness to the reference model so that the influence of this measure can vary throughout the subsurface. It is also possible to incorporate spatially varying weights into the measure of model roughness to encourage sharp jumps in the physical property at specified locations. The ability to incorporate such constraints into a minimum-structure-style inversion was recognized from the early days of this approach. However, it has only really been over the last decade, or less, that constrained minimum-structure inversion has started to see significant use by industry.

Nick Williams in his Ph.D. thesis provides a comprehensive description and assessment of constrained minimum-structure inversion, illustrated by applications to real-life scenarios (Williams, 2008; Figure 7 below). Some other case history-type examples of the use of constrained minimum-structure inversion include: Farquharson et al. (2008) for gravity data over the Ovoid ore-body, Voisey's Bay; Malehmir et al. (2009) for gravity data over the Kristineberg mining area of Sweden; Boszcuk et al. (2011) for gravity inversion at the Matagami mining camp, Québec; Spicer et al. (2011) for gravity and magnetic data over the Rambler Rhyolite, Newfoundland; and Tschirhart et al. (2013) for magnetic data from the Amer Lake area, Nunavut. Lelièvre and Oldenburg (2009b), for rectilinear meshes, and Lelièvre and Farquharson (2013), for unstructured tetrahedral meshes (Figure 8), increase the ways in which a constructed model can be constrained or influenced by introducing the ability to enhance or suppress variation in the model along prescribed directions (throughout all or parts of the subsurface).





**Figure 8:** An example of constrained inversion, using an unstructured tetrahedral mesh, of a synthetic airborne gravity gradiometry data-set generated for a simplified model of the Eastern Deeps ore-body at Voisey’s Bay (Lelièvre and Farquharson, 2013). Top panel: the topographic (grey), gneiss-troctolite (yellow) and troctolite-sulphide (brown) surfaces in the true model, and the approximated gneiss-troctolite surface (wireframe) used in the constrained inversion (observation locations indicated by black dots). Second panel: density isosurface in model constructed from unconstrained inversion. Third panel: isosurface in model constructed from inversion constrained by upper and lower bounds on density inside and outside estimated troctolite-gneiss surface. Bottom panel: same as third panel, but with extension down-dip encouraged. Colour scales show density in  $\text{g}/\text{cm}^3$ .

“Joint inversion” is generally taken to mean an inverse problem that involves two (or more) physical property Earth models, two (or more) types of geophysical data, a single, combined inversion procedure, and some form of coupling between the physical property models such that, at the conclusion of the inversion, the multiple physical property models can be thought of as the physical property distributions for a single, unified proxy geological model. The inversion procedure is usually designed to be the natural extension of a minimum-structure approach: instead of the penalty function consisting of one measure of data misfit and one measure of model complexity, it now comprises a data-misfit measure for each data-type, a roughness measure for each physical property model, and a measure of coupling (or rather, a measure of lack of coupling) between the physical property models. Joint inversion methods can be divided into two main types, differentiated based on the type of coupling between the physical property models: structural and compositional.

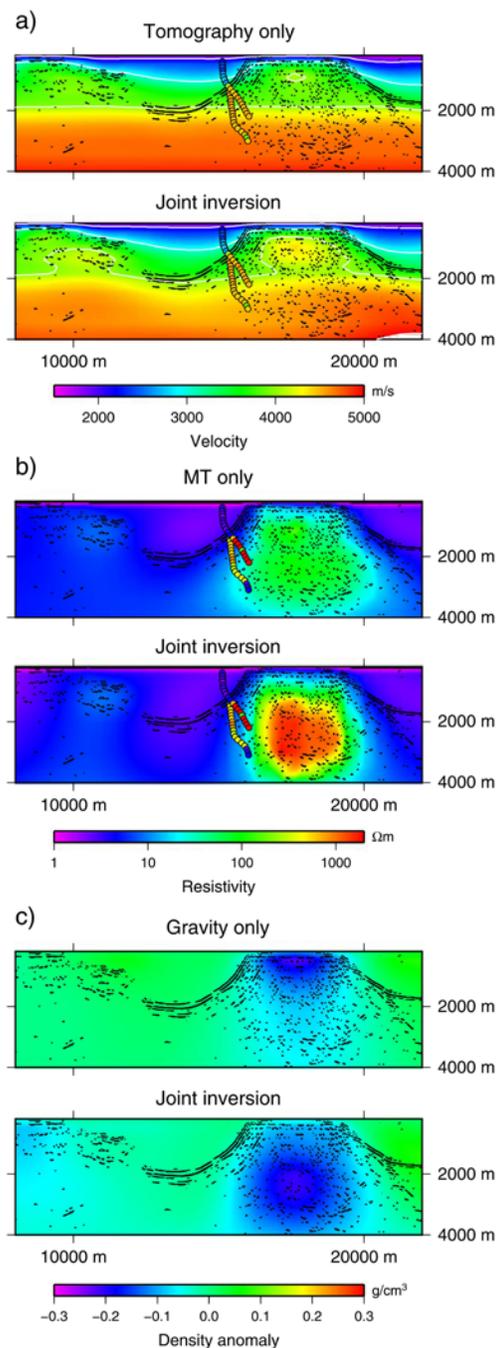
### Structural Joint Inversion

In structural joint inversion, the coupling term in the penalty function is designed to encourage similar spatial variations of the physical properties in the same parts of the model. Haber and Oldenburg (1997) use a coupling term that quantifies the cell-by-cell difference between normalized spatial derivatives of the two respective physical property models. This term is small when both physical property models are doing the same thing in the same place, for example, increasing or decreasing in a particular direction, or having an abrupt jump in value associated perhaps with a geological boundary. Gallardo and Meju (2003) introduce the “cross-gradient” measure that quantifies the vector cross product of the spatial gradients of the physical property models. This again is small when both physical property models are changing in the same way in the same place. However, this measure is also small when one or other of the physical properties is uniform, or when both physical properties are changing along the same direction but in opposite senses (i.e., one increasing and one decreasing). These additional characteristics of the cross-gradient coupling term mean that it is happy to construct a model containing, for example, an ore-body that is all denser than the background but only the bottom part of which is more susceptible than the background: the existence of the susceptible–not susceptible transition within the ore-body is not penalized by the cross-gradient measure. Also, the cross-gradient approach has the benefit that it does not require any knowledge about the relationships between the physical properties in the area under investigation, in contrast to composition-based coupling terms. Because of the generality of the approach, cross-gradient joint inversion has received a lot of interest; see, for example, Fregoso and Gallardo (2009), Moorkamp et al. (2011), Moorkamp et al. (2013; Figure 9) and Meju and Gallardo (2016). However, as we indicated above, the cross-gradient term is particularly non-unique. This non-uniqueness makes the optimization problem considerably more difficult to solve than a single-property minimum-structure inversion, and provides less potential to produce dramatic improvements than the composition-based joint inversion approach, which we consider next.

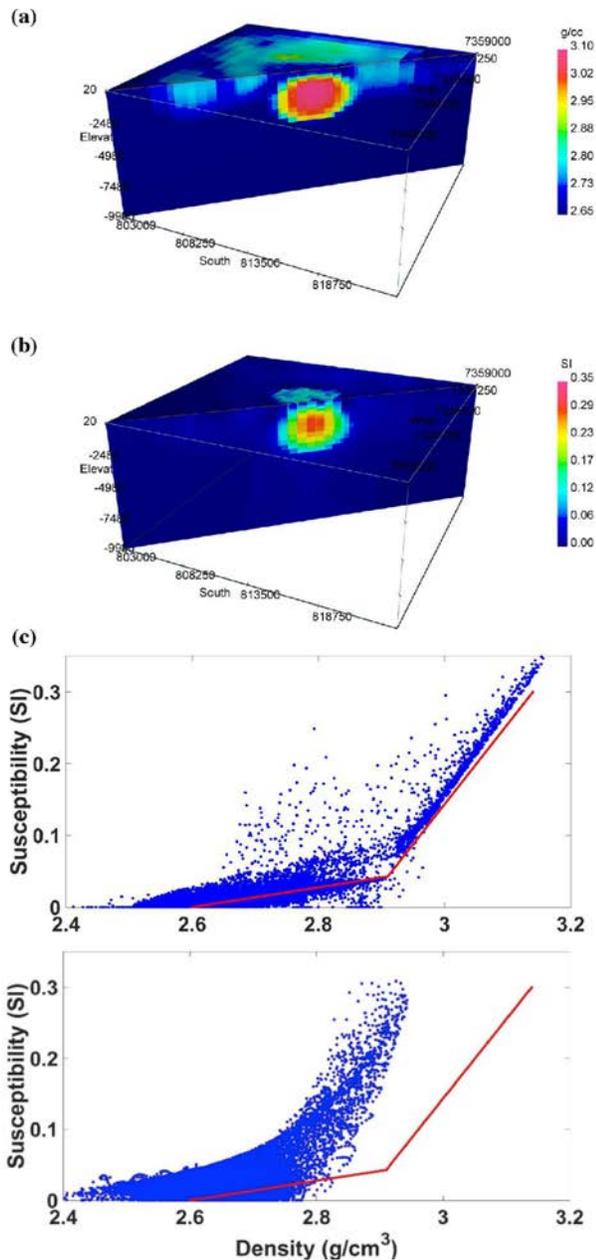
### Compositional Joint Inversion

The coupling between the physical property models in a composition-based joint inversion is achieved through user-provided specification of the relationships between the physical properties. A number of types of relationships have been considered including strict formula-based relationships; a looser, correlation-based relationship (a single relationship or subdivided into multiple segments); and clustering around a small number of prescribed points, or line segments, in the physical property cross-plot (see bottom panels of Figure 10). The coupling term is then designed to quantify how well these relationships are being satisfied by the models constructed in the inversion, and hence to encourage the physical property values in the same cell in the two (or more) physical property models to satisfy the prescribed relationships. An early example of compositional joint inversion was presented by Bosch and McLaughy (2001) for 2D inversion of gravity and magnetic data. There has been significant recent work, particularly in terms of incorporating and assessing different types of physical property relationships. Examples include: Lelièvre et al. (2012a), Carter-McAuslan et al. (2015) and Sun and Li (2016) for the joint inversion of gravity and seismic travel-time data; Zhang and Revil (2015) for gravity and DC resistivity data; and Sun and Li (2017; Figure 10) for gravity and magnetic data. This compositional approach to joint inversion is dependent, by design, on knowing the physical properties of the various rock units in the volume of subsurface under investigation, or knowing the relationship between those different physical properties. This knowledge can either come from actual physical property measurements, or can be based on estimated or anticipated values of the rocks that are thought to be present. Having such real or even anticipated information about physical properties is perhaps unrealistic for large-scale crustal studies. However, in the mineral exploration context, it is entirely conceivable that sufficient physical property information is available for the target region, or that the range of likely rock types and hoped-for mineralization is sufficiently narrow that physical property values and relationships can be estimated with reasonable confidence.

The structural and compositional joint inversion approaches indicated above are typically implemented by designing an optimization problem such that the combination of physical property models that gives the minimum value of the particular penalty function corresponds to the desired type of “answer” to the inverse problem. Measures of model complexity are included in this optimization problem, so such implementations of joint inversion do retain (hopefully) some of the robustness and reliability of the classic, single-property minimum-structure inversion approach. However, just as for the other, less extensive adaptations of the minimum-structure method mentioned earlier, the more involved and complicated the optimization problem, even with model simplicity still an integral part of it, the less robust and reliable the optimization process becomes. It is very attractive to think that a single, automated, self-contained joint inversion procedure might allow us to construct geophysical Earth models that are consistent with each other, with physical property information, and hence maybe with geology. However, the extra complexity of the inverse problem and corresponding reduction in robustness and



**Figure 9:** Sections through 3D models constructed via independent (top sections in panels a, b & c) and joint (bottom sections) inversions of seismic travel-time, magnetotelluric and gravity data from over a salt dome in the Barents Sea (Moorkamp et al., 2013). The joint inversion was performed using a cross-gradient measure. It can be seen that, in particular, the density low now coincides with the resistivity high in the joint inversion result, and the resistivity high is enhanced. (The coloured circles indicate boreholes used to assess the accuracy of the models. Back lines indicate reflections on a seismic section coincident with the displayed sections. Figure adapted from Moorkamp et al., 2013.)



**Figure 10:** Density (a) and susceptibility (b) models constructed from the joint inversion of gravity and magnetic data from Boden, Sweden, by Sun and Li (2017). The joint inversion was performed using a compositional coupling term based on the relationship defined by the red line in the physical property cross-plots shown in panel (c). The red line in panel (c) was derived from physical property measurements from the area. The blue dots in panel (c) indicate the densities and susceptibilities in the cells in the constructed models. These densities and susceptibilities mostly follow the provided (and expected) physical property relationship. The lower graph in panel (c) shows the cell densities and susceptibilities resulting from independent inversions. (Adapted from Sun and Li, 2017.)

reliability of the inversion procedure could very well restrict the true usefulness of joint inversion in a practical, industrial, commercial setting.

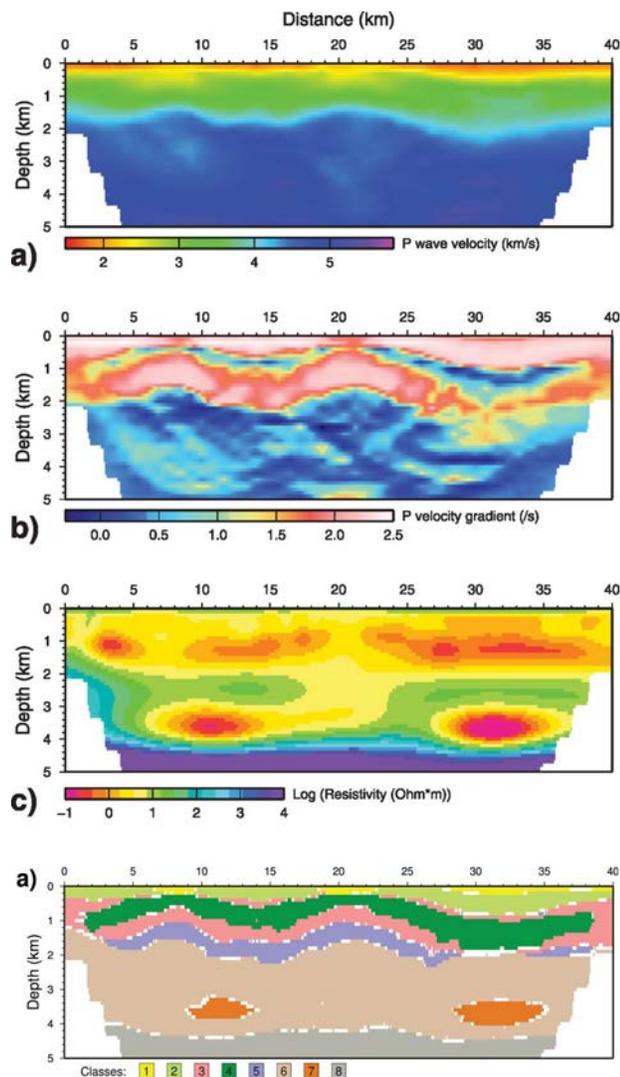
### Cooperative Inversion

Cooperative inversion is a slightly different approach to constructing two physical property models that each reproduce their respective geophysical data-sets but which are also consistent with each other and perhaps with physical property information. This approach involves an iterative scheme in which individual single-property inversions are alternated, with the updated physical property model from one of the single-property inversions used to constrain the next inversion to get an updated version of the other physical property model. Some variants also further adapt, at each iteration, the models going into the single-property inversions or the reference models used to constrain the single-property inversions based on known or estimated physical property information. Cooperative inversion has the particular advantage that each individual physical-property inversion happens on its own, and so two completely separate, pre-existing single-property inversion programs can be used: there is not the need for a single, specialized, monolithic program. Paasche and Tronicke (2007, for GPR and seismic travel-time data) modify the single-property models at each iteration so that the values in each cell fall within clusters on the physical property cross-plot, with the cluster centres varying during the iterative procedure based on the inversion results. McMillan and Oldenburg (2014) cooperatively invert airborne TEM, CSAMT and DC resistivity data-sets from the Antonio gold deposit, Peru, in order to construct a single conductivity model that is consistent with all the data-sets. Kamm et al. (2015) invert gravity and magnetic data from a gabbro intrusion close to Boden, Sweden, revising the individual single-property reference models at each iteration to lie between the constructed models existing at that iteration and physical property information known from measurements on samples from the region. Kamm et al. (2015) manage to honour the same physical property relationship as do Sun and Li (2017) — see our Figure 10 — albeit using cooperative inversion instead of joint inversion.

### Lithologic Segmentation

Further along on the spectrum of methods for integrated imaging are approaches that analyze multiple different single-property models constructed by independent, single-property inversions in order to identify patterns and variations common to the respective models that can perhaps be identified with specific, distinct geological units (Figure 11). Paasche et al. (2006) use the “fuzzy *c*-means” technique to identify clustering on the 3D cross-plot of seismic velocities, GPR velocities and GPR attenuation in the cells in the independently constructed models of these three properties. Each cluster that is identified in the 3D cross-plot is then associated with a particular rock unit, and all the cells whose combination of physical property values lies within the cluster are assigned this rock unit. In this way, what is effectively a geology map of the subsurface volume is generated. Also, Bedrosian et al. (2007) look for correlations between resistivities and seismic velocities produced by independent magnetotelluric and seismic travel-time inversions; Bauer et al. (2012; our Figure 11) and Fraser et al. (2012) use

the “self-organizing maps” (SOM) technique to identify correspondences between single-property models; and Martinez and Li (2015) analyze density and susceptibility models constructed by individual minimum-structure inversions to discriminate lithology for the Quadrilátero Ferrífero, Brazil. An up-to-date overview of post-inversion segmentation and classification methods such as those above is given by Paasche (2016).



**Figure 11:** An example of joint interpretation via post-inversion lithologic segmentation. The data are for the Groß Schönebeck geothermal site in Germany (Bauer et al., 2012). (a) Seismic velocity from refraction travel-time inversion, (b) vertical gradient of the seismic velocity model in (a), and (c) resistivity from magnetotelluric inversion. An SOM procedure was applied to these three sections, the result of which is the identification of and segmentation into the zones shown in the lowest section above (second panel a). (Adapted from Bauer et al., 2012.)

### Integrated Imaging: Concluding Thoughts

All of the integrated interpretation approaches mentioned above aim to exploit the different, often complementary sensitivities and resolutions of multiple different geophysical survey types in order to construct a single, unified, integrated geophysical Earth model that represents the spatial distribution of multiple physical properties in the subsurface. It is assumed that such a multiple-physical-parameter Earth model, with different zones of the model characterized by different combinations of physical properties that can possibly be identified with particular rock types, is a significantly better, more likely representation of the subsurface than any single-property model constructed from the inversion of a single data-set.

### INVERSION FOR FULL MAGNETIZATION VECTOR

The developments in inverse modelling technology considered so far have been general in the sense that they can, in principle, be applied to, and are relevant to, any kind of geophysical data-type. However, there is one important technological advance that has been made during the last decade that is relevant to just one data-type, namely, inversion of magnetic data for magnetization. Minimum-structure inversions of magnetic data have historically constructed models of magnetic susceptibility, and have assumed that the magnetization in any one cell is purely an induced magnetization with the inducing field equal to the background Earth’s field. This assumption is valid for many real-life cases, but not all. If significant remanent magnetization is present, the resultant magnetization of a cell, which is the vector sum of the simple induced magnetization and the remanent magnetization, can be in a very different direction from the background field. The direction of the background Earth’s field, and hence direction of the induced magnetization, is known, but not the direction of any remanent magnetization.

There have been a number of attempts to construct the spatial variation of the full magnetization vector throughout the subsurface, with an increased pulse of activity occurring over the last couple of years. Lelièvre and Oldenburg (2009a) parameterize the model in terms of the three components of the magnetization vector in each cell of the mesh, either Cartesian components parallel and perpendicular to Earth’s field or magnitude, dip and azimuth, and carry out a minimum-structure style of inversion on this parameterization. Using a minimum-structure style of inversion results in a simple spatial variation of the components of magnetization throughout the model. However, inverting for the three vector components, rather than for one single scalar quantity, significantly increases the non-uniqueness of the inverse problem. To circumvent this, Li et al. (2010) investigate a two-step process in which the direction of magnetization is first estimated using one of a number of previously existing techniques and then incorporated into a minimum-structure style of inversion for the strength of magnetization. In the same paper, Li et al. also investigate inverting directly for only the strength of the magnetization, which can be more robustly and readily determined than the full magnetization vector. Ellis et al. (2012), MacLeod and Ellis (2015) and Li and Sun (2016) all invert for the full magnetization vector. MacLeod and Ellis use a non-sum-of-squares measure of spatial variability and a subsequent SOM-

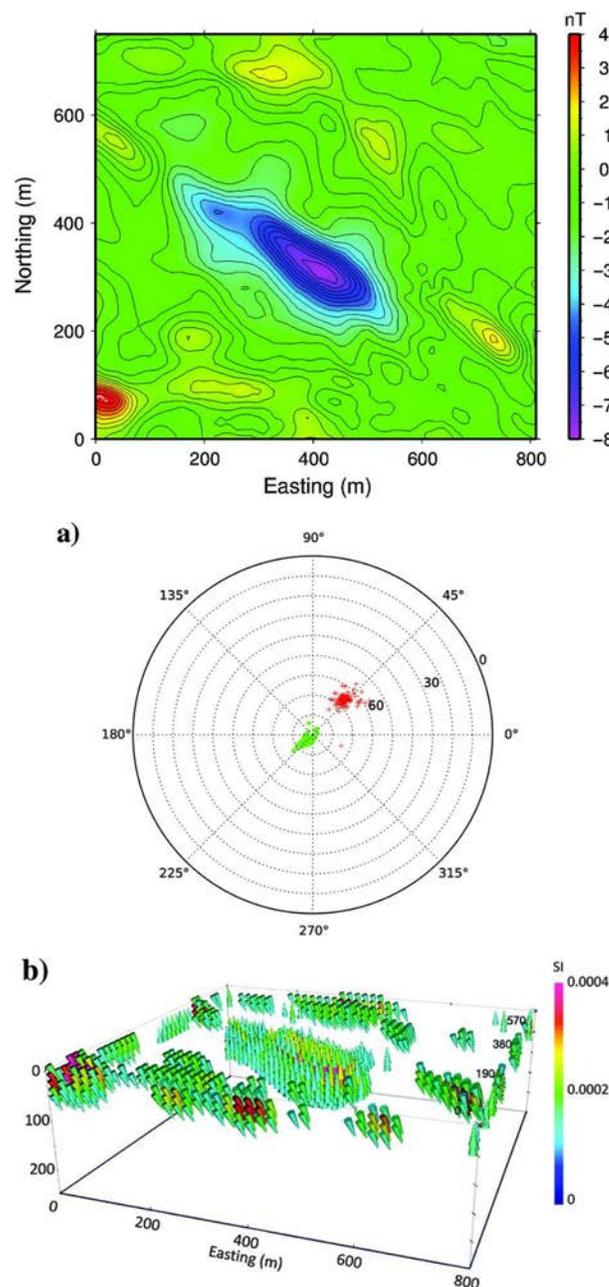
based analysis to consolidate the orientations in the constructed magnetization model into a small number of distinct, discernible directions. Li and Sun use a clustering technique to achieve the same goal directly in the inversion process (Figure 12). This also helps to mitigate the additional non-uniqueness resulting from inverting for a vector quantity.

### SOME THOUGHTS ON THE FUTURE

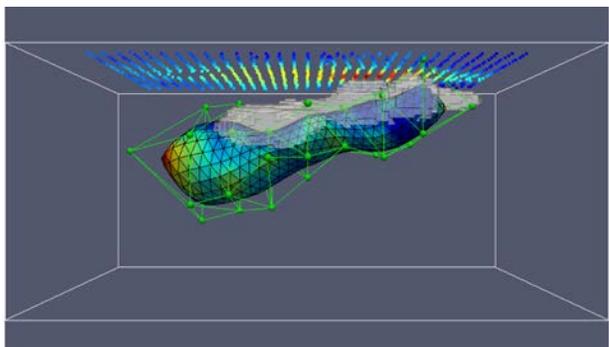
Before concluding this document, we would like to take the liberty of speculating on what kinds of developments in modelling and inversion for mineral exploration geophysics we think will happen over the next decade or so, both new methods that are developed and recently presented methods that we anticipate will be adopted by industry.

As mentioned earlier, geological Earth models are parameterized in terms of the surfaces that separate the various geological units in the subsurface. Such a representation of the subsurface, which essentially amounts to a 3D geology map, is the kind of Earth model that is most consistent with the way geologists think about the subsurface. There can be variations internal to a geological unit that are inconsequential to a geologist as these variations are all still within the range of possibilities categorized as being the same rock type. If there is too dramatic a change, the unit might simply be subdivided by a contact or fault into multiple, appropriately categorized units. This is not the case for geophysical Earth models, of course, for which any spatial variation of a physical property in the subsurface, whether a jump across an interface or a gradual change across some volume, can quite possibly affect the measured geophysical data. Nevertheless, given the increasing use of interface-based 3D computer geological Earth models, why not do our geophysical modelling and especially inversion on these exact same interface-based Earth models parameterized in terms of surfaces? In this way, the geological Earth model and the geophysical Earth model could be the exact same model.

Inverting for the location and even shapes of boundaries is not new in geophysics; this was, after all, the only style of inversion available before it became possible to use general, cell-based models. However, those early inversions were for simple geometrical shapes or tessellated surfaces described by just a few parameters. Any modern incarnation of a moving-interface inversion procedure needs to be applicable to the types of complex, real-life models shown in Figures 1 and 2. Inversion for the locations and shapes of surfaces is a much less stable and poorly behaved style of inverse problem compared to fixed-mesh minimum-structure inversion, and so inverting for the surfaces in the likes of Figures 1 and 2 is a somewhat daunting challenge. However, the goal of doing geophysical inversion in a way that is fully integrated with geological models is very appealing, and perhaps our knowledge and computer capabilities are sufficiently advanced now that we can make moving-interface inversions possible. Furthermore, joint inversion using moving-interface approaches is generally no more difficult than moving-interface inversion of individual data types, provided appropriate physical property information is available. A recent modern attempt at moving-interface inversion by Lelièvre et al. (2015) is illustrated in Figure 13.



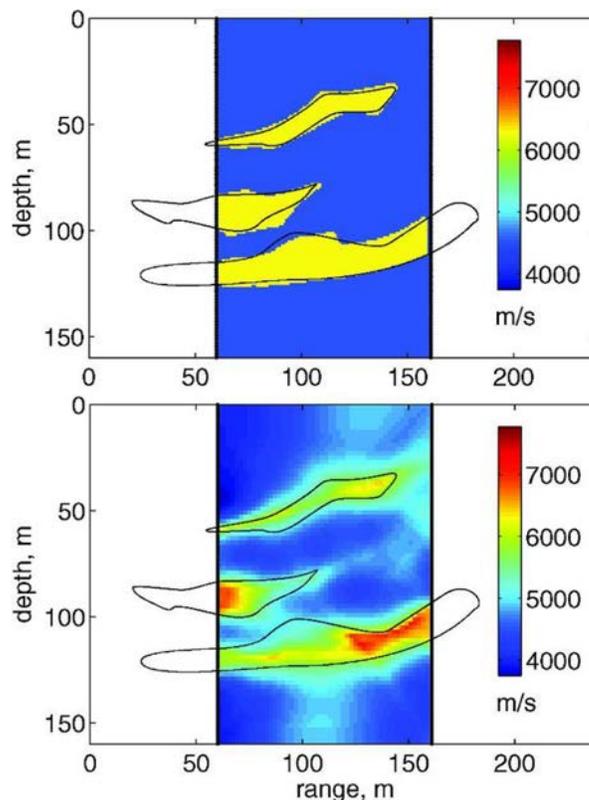
**Figure 12:** An example of inverting for the full magnetization vector (Li and Sun, 2016). Top: the total-field magnetic anomaly data from Victoria Island, Northwest Territories, Canada that were inverted. Middle (panel a): the magnetization directions in the constructed magnetization model (green: vertically up; red: inclination of  $60^{\circ}$ – $70^{\circ}$  down, declination of  $45^{\circ}$ ). Bottom (panel b): directions of magnetization (cones) and effective susceptibilities (colour scale) in the constructed model. (Adapted from Li and Sun, 2016.)



**Figure 13:** An example of a moving-interface inversion of gravity data. The data (coloured dots) are from an IOCG deposit in South Australia. The model comprises the tessellated wireframe surface (indicated by black triangles and coloured surface), with uniform, but different, densities inside and out. The green lines and nodes indicate the “control” frame from which the actual tessellated wireframe surface is constructed by subdivision. Only the parameters defining this control frame are sought in the inversion. The colouring of the wireframe surface indicates uncertainty in its constructed location (hotter colours mean greater uncertainty). The ghost-like surface is an isosurface from a typical minimum-structure inversion. (Adapted from Lelièvre et al., 2015).

Inverting directly for the surface geometries in what is essentially a geological Earth model would give the most complete integration of geophysical and geological models. However, the inverse problem of explicitly moving surfaces around in an interface-based model is an unstable, fragile style of inverse problem riddled with local minima, which requires a global optimization technique for its solution. There are other inversion approaches for constructing models dominated by uniform regions separated by sharp interface-like jumps that use fixed-mesh models and thus can be solved using iterative, descent-based optimization methods like those used for minimum-structure inversions. One approach (mentioned above, and which has been recognized for a couple of decades) is to use specially designed measures of model roughness in an otherwise standard minimum-structure inversion. Another approach, which has seen some recent interest and which explicitly works with uniform regions, is the level-set approach. This approach considers the interface or interfaces in the model to be the “level sets”, i.e., contour lines for a 2D model or isosurfaces for a 3D model, of a surface in a higher-dimensional space (3D if our Earth model is 2D, 4D if our Earth model is 3D). The rationale behind the level-set approach is that a relatively simple and straightforward surface in 4D can result in complex level sets in 3D, including multiple enclosed regions in the 3D model from the same simple surface in 4D. Hence, arbitrarily complicated interfaces in a 3D Earth model, including multiple enclosed bodies, can be constructed by manipulation of a much simpler surface in a higher-dimensional space, with, importantly, the simplicity of this surface resulting in a well-behaved inversion procedure. Figure 14 shows results from 2D inversion of seismic travel-time data from Zheglava et al. (2013). Other recent examples include: Lu and Qian (2015), who present a 3D level-set inversion of gravity gradiometry data; and Li and Qian (2016) and Zheglava and Farquharson (2016), who both

consider joint inversion of gravity and seismic travel-time data. Level-set inversion methods certainly seem capable of constructing geophysical Earth models that have the same kind of piecewise constant character as a 3D geology map of the subsurface. However, examples presented in the literature to date have all been fairly simplistic (compare Figure 14 with Figures 1 and 2) and it is unclear how complex a model a level-set approach might be able to handle. Also, the model constructed by any particular run of a level-set inversion depends somewhat on the starting model, and, as there is typically little or no regularization on the model, artefacts not required by the data can be present in the constructed model. Level-set approaches will never be as robust and reliable as minimum-structure approaches. It is as yet unclear whether they are reliable and robust enough, and able to deal with sufficiently complex models, to be useful. We are also unsure how easily different types of constraining information might be incorporated into level-set inversion approaches.



**Figure 14:** Top panel: Result of level-set inversion of synthetic cross-borehole seismic travel-time tomography data. Thin black lines indicate the outlines of the zones of anomalous velocity in the true model. Vertical thick black lines are the boreholes. Bottom panel: Result from a typical minimum-structure inversion. (Adapted from Zheglava et al., 2013.)

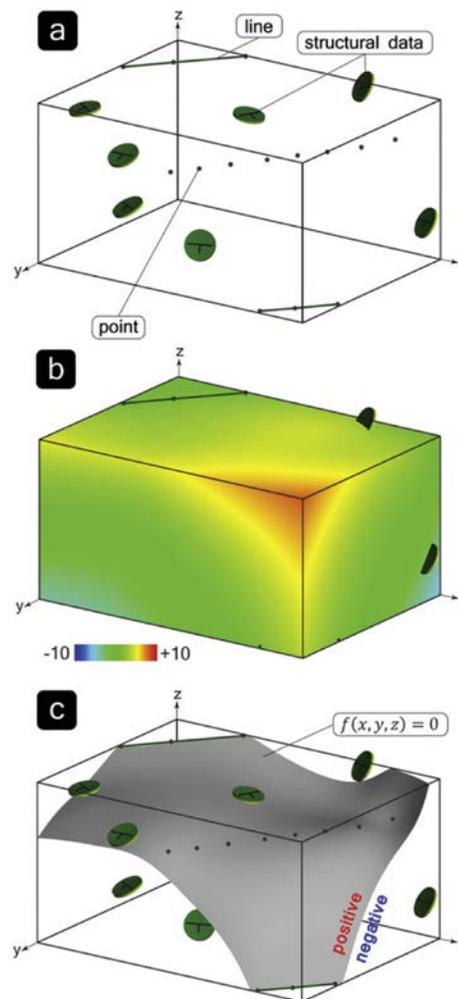
Guillen et al. (2008) and Schreiber et al. (2010) present an inversion method for gravity and magnetic data which, as far as the end results are concerned, is just like a level-set approach, although their motivation comes very much from a geological, practical viewpoint. Guillen et al. and Schreiber et al. build cell-based versions of their 3D subsurface geology map. Each cell is

prescribed the density or susceptibility that is being assumed for each particular rock unit. The inversion approach then involves randomly selecting a cell from along an interface between units, flipping the density or susceptibility value in that cell to the value for the unit on the other side of the interface, and determining the effect that this change has on the fit to the observed survey data. The change in this cell, which effectively corresponds to incorporating this cell into the neighbouring unit and hence to moving the interface, is then accepted if the fit to the data is improved or, if the fit is worsened, accepted based on probabilistic sampling criteria. This approach is particularly effective because a full modelling to synthesize new data every time a cell is changed is not required: only the contribution from the old version of the cell removed and the contribution from the new version of the cell added. Uieda and Barbosa (2012) also present fixed-mesh gravity inversion approaches that result in models comprising distinct, uniform regions. In their approaches, however, the regions of uniform, prescribed anomalous densities are steadily grown in the model until the observations are adequately reproduced.

There have also been methods presented for constructing geology map-like models which use a fixed-mesh model and cast the inverse problem as a minimum-structure-like optimization problem, but which consider the unit type or unit identity of each cell as the unknowns to be determined by the inversion (see, for example, Krahenbuhl and Li, 2006, Bijani et al., 2017). Because these unknowns are now essentially integers and do not correspond to a continuous range of a physical property, gradient-based methods are not appropriate for solving the optimization problem; a random sampling (MCMC) or global optimization technique (such as a genetic algorithm) is needed instead. Such “lithology-based inversions” or “litho-type inversions” possess the enticing potential to construct directly from our geophysical survey data a model that is essentially a 3D geological map of the subsurface. However, because these methods rely on random sampling (e.g., Markov chain Monte Carlo – MCMC) or global optimization methods for their solution, they suffer from the usual bane of such optimization methods: namely, they require an inordinately large number of forward modellings to find the global minimum and hence are, at the moment, prohibitively expensive.

One way in which the interfaces in a geological model are constructed is by a process known as “implicit modelling” (see, for example, Hillier et al., 2014, and Figure 15). The data that go into this process are structural geology observations: known locations of the interface determined from outcrop (or from boreholes), measured strikes and dips of the interface, and information about the units on either side of the interface. A surface is then constructed that satisfies these data and the rules and expectations of structural geology. In implicit modelling, the surface is constructed as an isosurface of a scalar potential function in the 3D space of the model, in a manner reminiscent of level-set inversion for geophysics mentioned above. In fact, the process of constructing a surface in a geology model that is consistent with the known structural geology observations is completely analogous to our geophysical inverse problem: it is a structural geology inverse problem. Furthermore, this structural geology inverse problem shares the same fundamental characteristics as our geophysical inverse problems: the

observations are invariably sparsely located and few in number, perhaps even more so than for geophysical data-sets. This structural geology inverse problem is therefore arguably even more non-unique than geophysical inverse problems, even moving-interface geophysical inversions. The natural next step is to consider whether these inverse problems can help each other out: perhaps a joint geology–geophysics inversion for interface-based models is possible, whether using an implicit level-set approach or an explicit approach like that illustrated in Figure 13?



**Figure 15:** An illustration of constructing an interface in a geological model via “implicit modelling”, i.e., a structural geology inverse problem. (a) The point and line locations of the interface, and point measurements of the dip of the interface and neighbouring units, that are the observations going into the implicit modelling. (b) The variation of the potential function in all three spatial dimensions that is constructed during the implicit modelling. (c) The resulting interface, which is the zero contour surface of the potential function. (Adapted from Vollgger et al., 2015).

The possibility of joint geology–geophysics inversion has been proposed before. Jessell et al. (2010) consider a synthetic test

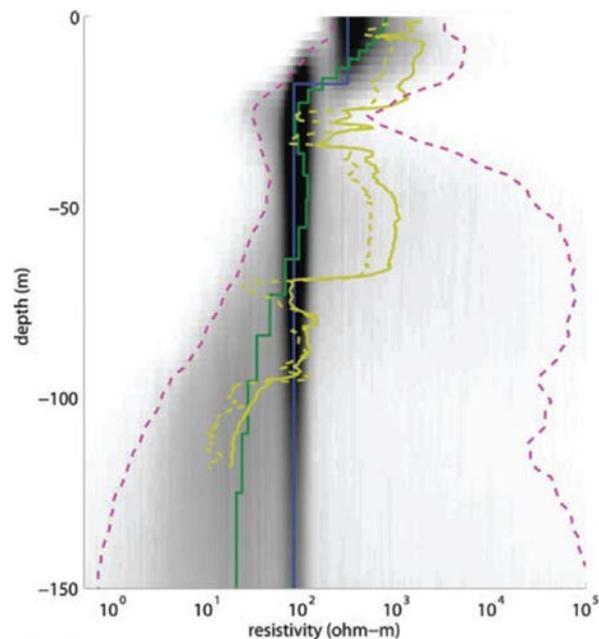
case in which they build interface-based geological models using an implicit surface modelling approach, convert these to cell-based models, assign physical property values to the cells, and then synthesize gravity and magnetic data-sets. Jessell et al. then perturb the interface-based geological model to an extent consistent with the uncertainty in the structural data driving the implicit surface modelling, redo the transformation to a cell-based geophysical model and synthesize new geophysical data. They then quantify the misfit between the original geophysical data and that for the perturbed model by the kinds of measure of data misfit familiar to us from geophysical inversion. Jessell et al. then suggest that their approach, in particular the linkages between geology and geophysics, could be the basis and inspiration for joint geology–geophysics inversion.

Lindsay et al. (2014) perform a somewhat similar analysis to that of Jessell et al. (2010) for real data from the Ashanti Greenstone Belt in Ghana, exploring the uncertainty in their geology model arising from the limited nature (discrete, sparse) of the geological observations. Lindsay et al. noted that the most uncertain parts of their geological models were the parts that were then subsequently altered the most in gravity inversions, and hence suggest that uncertainty in geological modelling could be used to localize and target subsequent geophysical modelling and inversion.

A recent trend in computational structural geology is to treat the automated process of constructing models more and more like the kind of inverse problem familiar to us in geophysics. Cherpeau and Caumon (2015) and de la Varga and Wellmann (2016) present statistics-based inversion approaches to constructing interface-based geological models. The quantitative information about the uncertainty in the constructed models that can be obtained from these statistical approaches is considered particularly useful. It is possible, of course, to use the same approaches for, and generate corresponding uncertainty information from, moving-interface geophysical inversion (see Figure 13).

Quantification of the uncertainty in the parameters of a model constructed by geophysical inversion has been recognized as being useful for as long as we geophysicists have been constructing Earth models from geophysical survey data. However, the fixed-mesh minimum-structure style of inversion, especially for the kind of fine discretization of the model that is most appropriate for this style of inversion, does not lend itself to inversion methods that aim to rigorously quantify uncertainty because synthesizing the data for a particular model is not cheap. A number of attempts have been made to develop a statistics-based inversion process that can provide useful uncertainty information. For example, Minsley (2011) presents results from an MCMC sampling approach applied to the 1D inversion of airborne EM data (see Figure 16). The uncertainty information that Minsley obtains is probably a fair (albeit somewhat disheartening) indication of the resolution and sensitivity of airborne EM data. However, the MCMC approach, like all global optimization and sampling methods, is computationally expensive. Minsley reports that the MCMC uncertainty analysis could only be performed on a limited number of soundings from the airborne data-set: it was too computationally demanding to be applied to the whole data-set.

Being able to quantify the uncertainty in a geophysical model—e.g., error bars on the depth of a potential drill target—constructed by inversion is nevertheless so important that we expect this to be an active area of research over the next decade. However, we are not sure what methods will end up being useful. Perhaps it will simply be a matter of waiting until computers become even more powerful than they are today, especially in terms of parallel computations, such that random sampling or global optimization methods become practical for all geophysical data types and model parameterizations.



**Figure 16:** An example of generating uncertainty information from a geophysical inversion. This figure shows the results obtained by Minsley (2011) of 1D inversions of one sounding from an airborne frequency-domain EM survey from western Nebraska. Grey shading: distribution of acceptable models found via MCMC sampling. Dashed lines: 95% confidence limits from MCMC sampling. Faint grey line: best individual model obtained from MCMC method. Green line: standard 1D minimum-structure inversion. Yellow lines: resistivity logs. (Adapted from Minsley, 2011.)

As mentioned several times above, geological Earth models are made up of uniform regions separated by sharp, distinct interfaces, with the uniform regions corresponding to different rock types, just as on a geology map. However, variability within the same rock type might result in a sufficiently strong variation in physical properties that the geophysical survey data might be affected. Also, there are a number of geological features that are particularly relevant to mineral exploration that do correspond to gradual variations in both geological and physical properties, most notably, alteration halos around mineralized zones. Is it possible, therefore, to be able to construct an Earth model via geophysical inversion that comprises both sharp interfaces between distinct rock units and gradual variations such as those associated with the transitions in an alteration halo? Sun and Li (2014) present an adaptive method based on different measures of model complexity in an

attempt to do this. However, we feel this is a particularly challenging problem, perhaps a challenge too far, as the resolving capabilities of the common geophysical methods used for mineral exploration are so poor. In most cases a geophysical data-set can probably be reproduced to a level appropriate for the noise in the measurements by either a completely blocky, piecewise constant model, or a classic smooth, smeared-out minimum-structure model, or anything in between. The data themselves cannot provide definitive information about any alteration halo: that will need to come from a constraint or other form of prior or assumed knowledge that's provided to the inversion. Perhaps the question of the presence or not of an alteration halo or some other such gradual variation in a part of the subsurface is best addressed by a trial-and-error modelling study where a model is altered manually and the data response compared to the measured data. Alternatively, a hypothesis-testing experiment using inversion might attempt to move, remove, or reshape particular model features to assess the likelihood or possibility of different scenarios (see Figure 8).

Inversion is a particularly attractive process because it automatically constructs the Earth model with a minimal or modest amount of effort on our part. As we hope we have illustrated, there are many different types of inversion approach each with their own set of variations, making it possible to construct a type of model that is particularly tailored for a particular example. However, because of the poor resolution and sensitivity of geophysical data, and the still nevertheless fundamentally limited capabilities of all inversion procedures, it is not always possible to construct a model with the kinds of features that one desires. There is still most definitely a place amongst geophysical interpretation methods, therefore, for post-inversion trial-and-error modelling. To be effective and practical, this needs a powerful yet intuitive and easy-to-use graphical means of editing and modifying the Earth model and a fast, preferably real-time means of synthesizing the relevant geophysical data-set each time the model is revised. We see these two areas—software for point-and-click graphical manipulation of 3D Earth models and ever faster forward modelling software for all types of geophysical data—as important areas of research and development over the next decade.

One other trend that we expect to occur over the next decade, which has been made possible by the advance in computer capabilities as much as in modelling and inversion technology, is single, multi-resolution models being constructed for whole geological provinces. Malehmir et al. (2009) considered a 30 km × 30 km area of the Skellefte Ore District around Kristineberg in Sweden, but could only manage a 400 m × 400 m discretization of their model because of computational resources available at the time. Faster, more powerful computers will enable larger numbers of cells in any one model, and hence finer discretizations over larger areas, but to represent a whole mining district or province, a Google Maps-like hierarchical, multi-level meshing strategy will be required, with meshing that makes sense for the geophysical data-sets being considered at any particular moment (e.g., airborne magnetics over the whole area; down-hole EM passing through an ore-body), and up-scaling and down-scaling of physical properties between the different meshes.

Finally, there have been considerable advances in all Earth Sciences applications of computer modelling over the last decade, not just in geophysics, including computer modelling of ore deposit formation (see, for example, Zhao et al., 2012, and Li et al., 2017). Rather than considering only purely geometrical controls on the shape of an ore-body, such as those in Figure 13, from the inversion of a geophysical data-set, maybe we should be attempting to determine the geochemical and geodynamic controls on the formation process of the ore-body, which in turn are the fundamental factors determining its location, depth and size. Such “geo-inversion” would be a true integration, in one computer modelling procedure, of geology and geophysics.

## CONCLUSIONS

Minimum-structure inversion, using rectilinear meshes, has proved very successful in constructing useful, interpretable models in the context of mineral exploration geophysics. The success and usefulness is due in no small part to the reliability and robustness of this particular approach. It is expected that minimum-structure fixed-mesh inversion will be the mainstay for modelling and inversion in mineral exploration geophysics for a long time to come. However, the last decade has seen the development of a range of more sophisticated, more complex inversion methods that offer more, or at least different, capabilities than the classic minimum-structure approach. These include: constrained inversion; integrated imaging methods such as joint inversion, cooperative inversion and post-inversion lithologic segmentation, that aim to construct a single Earth model consistent with multiple different geophysical data-types; and methods such as the level-set approach and litho-type inversion that aim to construct directly a 3D geology map of the subsurface rather than the spatial distribution of a physical property. Other developments involve the type of mesh used to parameterize the Earth model: the use of OcTree meshes which allow for local refinement while maintaining some of the computational niceness of rectilinear meshes; the use of unstructured tetrahedral meshes for geophysical Earth models in order to be completely consistent with interface-based geological models; and even using interface-based models themselves for the geophysical modelling and inversion, with no underlying mesh. In one way or another, all these methods attempt to construct a geophysical Earth model that, while still appropriately reproducing the geophysical survey data, is more in line with what we expect the subsurface of the Earth to look like than the smooth, smeared-out models produced by the classic minimum-structure approach. The downside of many of these approaches, however, is that they invariably lead to less well-behaved, less stable inverse problems that are more difficult to solve successfully, reliably and robustly. There will be a trade-off between the potential benefits in terms of the character of the model that they produce and the effort required to produce the model that will determine the usefulness of each of these new methods for real-life situations. Perhaps in a decade's time we shall be doing our geophysical modelling and inversion using exactly the same interface-based models that geologists are using, with all the geo-scientists and mining engineers working with the same integrated computer Earth model throughout the whole exploration and development process. Or perhaps the intricacies of these new inversion methods will limit their usefulness, and instead we will be

exploiting further increases in computer power to have ever finer discretizations and ever larger spatial extents of good, old-fashioned rectilinear meshes, doing tried-and-true single-property minimum-structure inversions, and using some kind of post-inversion lithologic segmentation to produce a 3D geology map to pass on to the geologists. The reality, of course, is that both of these strategies will, and probably should, find their place within an exploration workflow. The true importance of all the developments in modelling and inversion technologies that have occurred over the past decade, and that will continue to occur in the future, is that we have available an increased number and variety of powerful techniques for modelling and inverting our geophysical data, each with its own advantages and challenges, but all of which may provide benefit throughout an exploration program.

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