Propagation of uncertainty from effective-diffusion flow to microseismicity

Oleg V. Poliannikov†, Michael Prange‡, Hugues Djikpesse‡, Alison E. Malcolm†, and Michael Fehler†*
† *Earth Resources Laboratory, Department of Earth, Atmospheric, and Planetary Sciences, MIT*
‡ *Department of Mathematics and Modeling, Schlumberger-Doll Research*

SUMMARY

We present the problem of predicting hydraulically-induced microseismic events in the field from geo-mechanical and flow data. Understanding this problem is key for formulating and solving the inverse problem of constraining fluid flow using observed microseismic events. We consider a diffusion-based fluid flow model and discuss some of the factors that cause rock failure and thus induce microseismicity. Accurate prediction requires knowledge of many input parameters, most of which are not well known in field applications. Careful uncertainty analysis is thus required if the resulting model is to be used in practice.

INTRODUCTION

Hydraulic fracturing is the primary method of increasing the production potential of unconventional hydrocarbon and geothermal reservoirs. Fluid under high pressure is injected into the subsurface, causing rock failure that creates new fractures and/or reactivates existing fractures. These fractures serve as additional pathways that facilitate the extraction of reservoir fluids. This fracturing of the rock is usually accompanied by microseismic events. These events emit waves that are then recorded by a receiver array located at the surface and/or inside one or more nearby well bores. The recorded microseismic data are used to invert for the locations, initiation times, and source mechanisms of the microseismic events, and thus to understand the fracture system that has been created. Traditionally, the locations and moment tensors of the microseismic events are used to infer the most basic properties of the fracture system like the fracture length, height, spacing. This type of information, while invaluable, is insufficient to fully understand the flow that results from the fracturing. In this paper, we discuss the problem of connecting the flow model to the induced microseismicity, paying particular attention to the propagation of uncertainty through the steps of the analysis. Understanding the forward model would pave the way to formulating, and possibly solving, the inverse problem of constraining the fluid flow given observed microseismicity, which is one of the ultimate goals of microseismic monitoring.

THEORY

Microseismic-to-flow

The probability to observe some data, d, given a particular underlying model, m, is given by the likelihood function, $p(d \mid m)$. In our context, the likelihood function predicts observed microseismicity given our assumptions about the initial distribution of stress, permeability, velocity model, etc. As will be elaborated below, induced fracturing is a complex process that is not amenable to deterministic modeling. Stresses are redistributed and the permeability field changes as old fractures are reactivated and new ones are created. Additionally, some underlying parameters, such as physical properties (friction, cohesion,…) of the rock, are not known exactly well known, and thus forecasts of fracturing events are intrinsically uncertain. The probabilistic modeling therefore becomes necessary.

What we are really interested in is solving the inverse problem of constraining the fluid flow using observed microseismicity. In the statistical Bayesian lingo, we seek the posterior distribution, $p(m \mid d)$ of the model parameters given the data. Within the Bayesian inversion, the posterior estimate of the model (diffusivity, stresses) given observed microseismic data is obtained from the likelihood function using Bayes' rule:

$$p(m \mid d) = \frac{p(d \mid m)\, p(m)}{\displaystyle\int p(d \mid m)\, p(m)\, \mathrm{d}m}. \tag{1}$$

Evaluating the left-hand side of Equation 1 numerically for a complex likelihood function is a challenge by itself that needs to be addressed separately. In the remainder of this paper, we focus in detail on individual components of the forward model. Our aim is to summarize the necessary steps that must be undertaken before a physically meaningful inverse problem can be formulated.

Effective diffusion flow

Several models have been proposed in the literature for fluid flow in the subsurface. Among them are diffusion (Shapiro et al., 2002) and discrete fracture networks (McClure and Horne, 2011; Detring and Williams-Stroud, 2013). In this paper we will consider the effective diffusion model. Under this model, fluid pressure diffuses away from the injection point. We use the words *effective diffusion* to emphasize that the diffusion in question is not limited to diffusion through the rock matrix. Effective diffusion may be observed on the macroscopic scale even when the matrix permeability is zero, because fluid is also flowing through fractures. If the pre-existing fractures have a dominant orientation then the effective diffusion may be anisotropic. Of course, diffusion through the rock matrix is also a special case of the effective diffusion so this model covers a wide range of physical scenarios.

We propose to model this effective diffusion by the diffusion equation. It has the form

$$\frac{\partial P(t,x)}{\partial t} = \nabla \cdot \left[D(t,x,P(x))\, \nabla P(t,x) \right],\ x \in \mathbb{R}^3, \tag{2}$$

where P is the fluid pressure, and the effective diffusivity D may be time and space dependent and may also be a function of

DOI http://dx.doi.org/10.1190/segam2014-1340.1

the current pressure at that point. If the diffusion is anisotropic, then D becomes a tensor.

The effective diffusivity, D, is the parameter that controls the flow. The fluid pressure, P, could be computed numerically for any assumed diffusivity, D. If the diffusivity is known only approximately, then the calculated solution of Equation 2 will deviate from the true flow in the subsurface. The uncertainty in the calculated flow must be taken into account at later steps of the forward problem.

Mohr-Coulomb theory

Mohr-Coulomb theory is a mathematical model that describes the response of brittle rocks to effective stresses. This theoretical model has a long history, and it has been supported by numerous lab experiments. Our interest here is in its applicability to the reservoir scale.

Effective stress is a stress that is carried by the rock (von Terzaghi, 1943). It is calculated using the matrix stress and the fluid pressure in the pores and/or the pre-existing fractures. In the stress-principal-component reference frame, the effective stress is written as:

$$\sigma_{\text{effective}} = \sigma_{\text{matrix}} - P. \tag{3}$$

In situ stresses prior to the injection, σ_{matrix}, are estimated from density, sonic, and other measurements, and the pore pressure, P, is modeled numerically as described above. The uncertainty in the estimated stresses and the pore pressure propagate to the calculated values for the effective stresses.

Mohr's circle

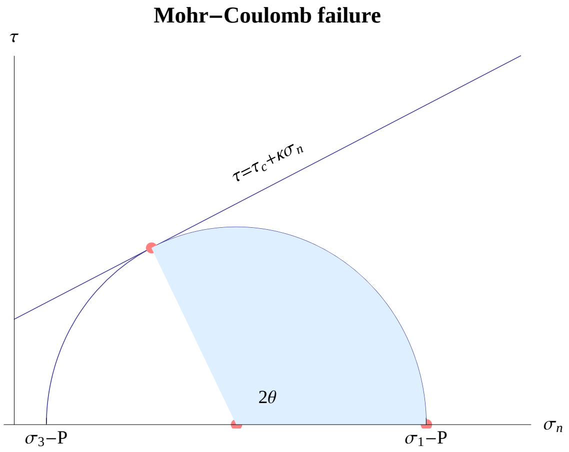

Figure 1: *Mohr's circle and failure envelope.*

Mohr's circle is a graphical representation of the normal and shear stresses for any reference plane passing through the point in question. If σ_1 and σ_3 are the maximal and minimal principal stresses respectively estimated at a particular point, then the Mohr's circle is constructed as shown in Figure 1.

Failure envelope

The rock failure envelope is a plot of the shear stress at which a rock will fail (with or without a pre-existing fracture) *versus* the applied normal stress across the reference failure plane (Marshak and Mitra, 1988,Chapter 10). The failure envelope is approximately linear for compressive normal stress, with significant non-linear plastic effects when the normal stress is very large (Figure 2). When the normal stress is tensile, significant nonlinearity is also observed (Griffith, 1921).

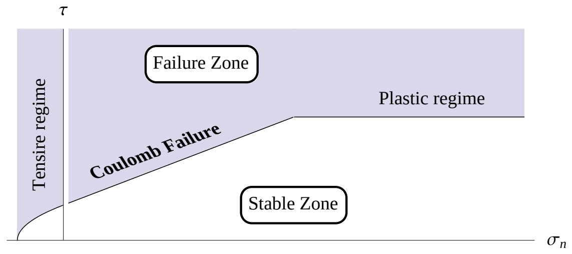

Figure 2: *Rock failure envelope.*

We will focus on characterizing the failure envelope directly from microseismic and log measurements. The equation for the linear failure envelope in the compressive regime is $\tau_c + \kappa\sigma_n$, where τ_c is the cohesion, κ is the internal friction of the rock, and σ_n is the applied normal stress (Byerlee, 1978).

The two physical parameters, τ_c and κ, are experimentally derived. They are often much better known in laboratory conditions where the rock sample can be carefully studied; however, these quantities should be estimated in the field. Because the rock in the field is highly heterogeneous, we can expect to estimate the cohesion and the internal friction only approximately. This means that both the slope and the intercept of the failure envelope carry uncertainty, which propagates into seismic estimates of flow.

Mohr-Coulomb failure criterion

The Coulomb criterion for brittle rock failure—one of most commonly used failure criteria—states that fracturing occurs when the shear stress along the reference plane, τ, satisfies (Byerlee, 1978)

$$\tau = \tau_c + \kappa\sigma_n. \tag{4}$$

Because failure occurs as soon as the Coulomb criterion is met, *i.e.*, Equation 4 is satisfied, the shear stress for a given normal stress will not exceed the failure threshold. We assume that rock failure sometimes results in a microseismic event. Once failure has occurred, the shear stress at that point is reduced.

Figure 1 summarizes the process of rock failure under pressure of the injected fluid. Higher pore pressure moves the Mohr's circles to the left until the circle intersects the failure envelope. The location and the angle, θ, of the failure plane is derived directly from Figure 1.

Given the predicted effective stress and the orientation of the failure plane (taken from the moment tensor of the seismic event), each microseismic event can be plotted on a Mohr-Coulomb diagram. The locus of such points define the effective failure envelope. In the presence of more than one system of pre-existing fractures or planes of weakness, each may be associated with its own failure envelope.

DOI http://dx.doi.org/10.1190/segam2014-1340.1

Stress redistribution

When the failure of rock occurs at a particular location, stresses are redistributed in the vicinity of the failure point. Describing the exact mechanism of stress redistribution is still controversial and a question of on-going research (Maruyama, 1968; King et al., 1994; GoertzAllmann et al., 2011; Williams-Stroud et al., 2012; Catalli et al., 2013). It is already clear, however, that even if the *in situ* stress was known before the failure, it is only approximately known after the failure. This uncertainty will propagate into the values for the effective stress, and hence affect predictions of induced failure predictions at other locations or subsequent failures at the same location.

Local changes in diffusivity

Seismic events may or may not accompany fracture growth. Creation of a new fracture or the growth of a pre-existing fracture will affect local permeability and hence the effective diffusivity coefficient in Equation 2. Subsequent modeling of fluid flow ideally should be performed with these changes incorporated in the model. Because these changes cannot be predicted precisely at this time, they may be modeled probabilistically as random perturbations to the initial diffusivity field. As with other random parameters, uncertainty in the diffusivity will propagate onward and affect fluid flow and rock failures at later times. The precise nature of this effect remains to be investigated.

DISCUSSION AND CONCLUSIONS

Microseismic monitoring may provide an important constraint on effective stress (pressure) and effective fracturing properties (failure envelope) in a hydraulic fracturing context. So far its use has been limited to extracting basic information about approximate fracture size, geometry, spacing, etc.

In this paper we have considered a problem of tying fluid flow more closely to observed microseismicity. Understanding the forward model is key for proper formulation of the inverse problem. Due to the complexity of the fracturing process, the research emphasis should likely be on understanding the effective flow parameters that could be constrained by observed microseismicity.

Each step in the forward model should be viewed through a probabilistic lens, and uncertainty propagation should be carefully modeled. Once this part is completed, the inverse problem can be attempted.

AKNOWLEDGEMENTS

We thank Stephen Brown of MIT for various fruitful discussions.

DOI http://dx.doi.org/10.1190/segam2014-1340.1

http://dx.doi.org/10.1190/segam2014-1340.1

EDITED REFERENCES

Note: This reference list is a copy-edited version of the reference list submitted by the author. Reference lists for the 2014 SEG Technical Program Expanded Abstracts have been copy edited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

REFERENCES

Byerlee, J., 1978, Friction of rock: Pure and Applied Geophysics, **116**, no. 4-5, 615–626, http://dx.doi.org/10.1007/BF00876528.

Catalli, F., M. Meier, and S. Wiemer, 2013, The role of Coulomb stress changes for injection-induced seismicity: The Basel enhanced geothermal system: Geophysical Research Letters, **40**, no. 1, 72–77, http://dx.doi.org/10.1029/2012GL054147.

Detring, J. P., and S. Williams-Stroud, 2013, The use of microseismicity to understand subsurface-fracture systems and to increase the effectiveness of completions: Eagle Ford Shale, Texas: SPE Reservoir Evaluation & Engineering, **16**, no. 4, 456–460, http://dx.doi.org/10.2118/162845-PA.

Goertz-Allmann, B. P., A. Goertz, and S. Wiemer, 2011, Stress drop variations of induced earthquakes at the Basel geothermal site: Geophysical Research Letters, **38**, no. 9, L09308, http://dx.doi.org/10.1029/2011GL047498.

Griffith, A., 1921, The phenomena of rupture and flow in solids: Philosophical Transactions of the Royal Society of London, **221**, no. 582–593, 163–198, http://dx.doi.org/10.1098/rsta.1921.0006.

King, G. C., R. S. Stein, and J. Lin, 1994, Static stress changes and the triggering of earthquakes: Bulletin of the Seismological Society of America, **84**, 935–953.

Marshak, S., and G. Mitra, eds., 1988, Basic methods of structural geology, 1st ed.: Prentice-Hall.

Maruyama, T., 1968, Stress fields in the neighborhood of a crack: Bulletin of the Earthquake Research Institute, **47**, 1–29.

McClure, M. W., and R. N. Horne, 2011, Investigation of injection-induced seismicity using a coupled fluid flow and rate/state friction model: Geophysics, **76**, no. 6, WC181–WC198, http://dx.doi.org/10.1190/geo2011-0064.1.

Shapiro, S. A., E. Rothert, V. Rathz, and J. Rindschwentner, 2002, Characterization of fluid transport properties of reservoirs using induced microseismicity: Geophysics, **67**, 212–220.

von Terzaghi, K., 1943, Theoretical soil mechanics: John Wiley & Sons, Ltd.

Williams-Stroud, S., C. W. Neuhaus, C. Telker, C. Remington, W. Barker, G. Neshyba, and K. Blair, 2012, Temporal evolution of stress states from hydraulic fracturing source mechanisms in the Marcellus shale: Presented at the SPE Canadian Unconventional Resources Conference.

DOI http://dx.doi.org/10.1190/segam2014-1340.1