Joint microseismic event location with uncertain velocity

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SUMMARY

We study the problem of the joint location of seismic events using an array of receivers. We show that locating multiple seismic events simultaneously is advantageous compared to the more traditional approaches of locating each event independently. Joint location, by design, includes estimating an uncertainty distribution on the absolute position of the events. From this can be deduced the distribution on the relative position of one event with respect to others. Many quantities of interest, such as fault sizes, fracture spacing or orientation, can be directly estimated from the joint distribution of seismic events. Event relocation methods usually update only the target event, while keeping the reference events fixed. Our joint approach can be used to update the locations of all events simultaneously. The joint approach can also be used in a Bayesian sense as prior information in locating a new event.

INTRODUCTION

Locating seismic events is an important problem both in global seismology and in reservoir exploration. Applications of this problem vary in scale from earthquake characterization to hydraulic fracture monitoring. Traditionally events are located individually, for example, from variants of Geiger's method by ray-tracing them from receiver locations using their respective arrival time and polarization estimates. Important information that couples data from different events and thus ties them together is ignored (Richards et al., 2006; Hulsey et al., 2009; Kummerow, 2010).

Event locations are usually understood in either absolute or relative terms (Slunga et al., 1995). Absolute event locations are defined globally with respect to a fixed coordinate system. Relative location is the location of an event relative to other events in the vicinity. Consider, in the context of hyrdofracture monitoring, microseismic events from the same fracture. If we move the fracture by moving all events in it a constant distance in a specified direction, then the absolute locations of those events will change. However, the relative location of any given event in this fracture with respect to all the rest will remain the same. The primary advantage of relative location over absolute location is that it is less sensitive to the uncertainties in the velocity model that lie between the cluster of sources and the receiver array, since these uncertainties tend to reposition the cluster as a whole, with a much smaller impact on the relative locations within the cluster (Waldhauser and Ellsworth, 2000; Zhang and Thurber, 2003; Poliannikov et al., 2011, 2013).

The joint location that we advocate in this paper is a way to recover the absolute as well as relative positions of all recorded events. Given recorded arrival-time data we will construct a joint location estimator that is a multi-dimensional joint distribution of all recorded events. This distribution contains the complete statistical description of the events including individual event locations as well as existing correlations between the locations of different events. It may then be used in a Bayesian sense as prior information in the location of a new event.

In most situations, event location is not the final goal but a step towards a more complete description of geophysical features such as fractures, faults, pressure fronts, etc. Physical quantities such as fracture spacing or fault orientation can be inferred from the estimated event locations. Fracture spacing, for example, can be thought of as the average distance between the events in neighboring fractures. Fracture size is related to the distance between events in the same fracture. Having the joint location distribution, we can compute the entire statistical distribution or some statistics of any function of those events, like the mean fracture spacing.

THEORY

Problem setup

Consider N_s seismic events, $\mathbf{s} = \{\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathrm{s}}\}$ originating inside a domain $\mathscr{D}$ in the Earth model. We assume that the possibly heterogeneous seismic velocity, V, inside $\mathscr{D}$ is uncertain. Mathematically we assume that V belongs to some family of admissible velocity models $\mathscr{V}$. The probability distribution, $p(V)$, determines the likelihood of any given velocity model.

Direct arrivals from all events are recorded at receiver locations $\mathbf{r}_j$, and arrival times, $\hat{\mathbf{T}} = \{\hat{T}_{\alpha,i,j}\}$, are picked. Here $\alpha \in \{P, S, \ldots\}$ denotes the recorded phase, $i \in \{1, \ldots, N_\mathrm{s}\}$ the event number, and $j \in \{1, \ldots, N_\mathrm{r}\}$ is the receiver number. In addition to picking direct arrival times, we may also correlate arrivals from events i and i', and pick correlation lags, $\hat{\tau} = \{\hat{\tau}_{\alpha,i,i',j}\}$.

We assume that the picked times and lags so obtained are noisy, i.e.,

$$\hat{T}_{\alpha,i,j} = \mathring{T}_i + T_\alpha\left(\mathbf{s}_i, \mathbf{r}_j \mid V\right) + \mathscr{N}\left(0, \sigma^2_{\alpha,i,j}\right), \quad (1)$$

$$\hat{\tau}_{\alpha,i,i',j} = \mathring{T}_{i'} - \mathring{T}_i + \tau_\alpha\left(\mathbf{s}_i, \mathbf{s}_{i'}, \mathbf{r}_j \mid V\right) + \mathscr{N}\left(0, \zeta^2_{\alpha,i,i',j}\right), \quad (2)$$

where $\mathring{T}_i$ is the unknown origin time of the event i, $T_\alpha\left(\mathbf{s}_i, \mathbf{r}_j \mid V\right)$ is the predicted travel time in the velocity model V,

$$\tau_\alpha\left(\mathbf{s}_i, \mathbf{s}_{i'}, \mathbf{r}_j \mid V\right) = T_\alpha\left(\mathbf{s}_{i'}, \mathbf{r}_j \mid V\right) - T_\alpha\left(\mathbf{s}_i, \mathbf{r}_j \mid V\right) \quad (3)$$

is the predicted lag between the direct arrivals from events i and i', and $\mathscr{N}(\cdot,\cdot)$ is the normal distribution. We will assume that the noise in picked arrival times and lags is uncorrelated.

The problem is to estimate all event locations $\mathbf{s}$ from the observed data, $\hat{\mathbf{T}}$ and $\hat{\tau}$.

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Joint location in a known velocity model

First suppose that the velocity model, V, is known. The data likelihood function, $p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)$, determines the probability of observing $\hat{\mathbf{T}}$, $\hat{\tau}$, given prescribed event locations $\mathbf{s}$ and origin times $\mathring{\mathbf{T}}$. Under the assumptions stated in the previous section, the likelihood function has the form:

$$p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right) \propto \exp\left[-\frac{1}{2}\sum_{\alpha,i,j}\left(\frac{\hat{T}_{\alpha,i,j}-\mathring{T}_i-T_\alpha\left(\mathbf{s}_i,\mathbf{r}_j\mid V\right)}{\sigma_{\alpha,i,j}}\right)^2\right] \times\exp\left[-\frac{1}{2}\sum_{\alpha,i<i',j}\left(\frac{\hat{\tau}_{\alpha,i,i',j}-\tau_\alpha\left(\mathbf{s}_i,\mathbf{s}_{i'},\mathbf{r}_j\mid V\right)-\mathring{T}_{i'}+\mathring{T}_i}{\zeta_{\alpha,i,i',j}}\right)^2\right]. \quad (4)$$

The posterior distribution of the event locations, $\mathbf{s}$, and origin times, $\mathring{\mathbf{T}}$, given data is then obtained by Bayes' rule:

$$\begin{aligned} p\left(\mathbf{s},\mathring{\mathbf{T}}\mid\hat{\mathbf{T}},\hat{\tau},V\right) &= \frac{p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)p\left(\mathbf{s},\mathring{\mathbf{T}}\mid V\right)}{\iint p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)p\left(\mathbf{s},\mathring{\mathbf{T}}\mid V\right)d\mathring{\mathbf{T}}d\mathbf{s}} \\ &\propto \frac{p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)}{\iint p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)d\mathring{\mathbf{T}}d\mathbf{s}}. \end{aligned} \quad (5)$$

Here we assume that all locations and origin times are equally likely, i.e.,

$$p\left(\mathbf{s},\mathring{\mathbf{T}}\mid V\right)\equiv\text{const}. \quad (6)$$

If a prior distribution on reference event locations and origin times is available from a previous application of joint localization, the posterior can still be expressed in closed form if this prior is expressed as a multi-normal distribution. We do not present these expressions here.

If we are interested just in the event locations without the origin times, then we simply integrate the posterior distribution given in Equation 5 over all origin times, $\mathring{\mathbf{T}}$. We have

$$\begin{aligned} p\left(\mathbf{s}\mid\hat{\mathbf{T}},\hat{\tau},V\right) &= \int p\left(\mathbf{s},\mathring{\mathbf{T}}\mid\hat{\mathbf{T}},\hat{\tau},V\right)d\mathring{\mathbf{T}} \\ &= \frac{\overbrace{\int p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)d\mathring{\mathbf{T}}}^{I(\mathbf{s})}}{\int\underbrace{\int p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)d\mathring{\mathbf{T}}}_{I(\mathbf{s})}\,d\mathbf{s}}. \end{aligned} \quad (7)$$

The integral, $I(\mathbf{s})$, appearing in the numerator and denominator of the right hand side of Equation 7 can be computed analytically. Indeed,

$$\begin{aligned} I(\mathbf{s}) &= \int p\left(\hat{\mathbf{T}},\hat{\tau}\mid \mathbf{s},\mathring{\mathbf{T}},V\right)d\mathring{\mathbf{T}} \\ &\propto \int\exp\left[-\frac{1}{2}\mathring{\mathbf{T}}^*A\mathring{\mathbf{T}}+B^*\mathring{\mathbf{T}}+C\right]d\mathring{\mathbf{T}} \\ &= \exp\left[\frac{1}{2}B^*A^{-1}B+C\right], \end{aligned} \quad (8)$$

where the matrix A is defined as follows:

$$\begin{aligned} A_{i,i'} &= \sum_{\alpha,j}\frac{1}{\sigma^2_{\alpha,i,j}} + \sum_{\alpha,i''<i,j}\frac{1}{\zeta^2_{\alpha,i'',i,j}} + \sum_{\alpha,i<i'',j}\frac{1}{\zeta^2_{\alpha,i,i'',j}} && i=i' \\ A_{i,i'} &= -2\sum_{\alpha,j}\frac{1}{\zeta^2_{\alpha,i,i',j}} && i<i' \\ A_{i,i'} &= 0 && i>i' \end{aligned} \quad (9)$$

the vector B is:

$$B_i = \sum_{\alpha,j}\frac{\hat{T}_{\alpha,i,j}-T_\alpha\left(\mathbf{s}_i,\mathbf{r}_j\mid V\right)}{\sigma^2_{\alpha,i,j}} + \sum_{\alpha,i'<i,j}\frac{\hat{\tau}_{\alpha,i',i,j}-\tau_\alpha\left(\mathbf{s}_{i'},\mathbf{s}_i,\mathbf{r}_j\mid V\right)}{\zeta^2_{\alpha,i,i',j}} - \sum_{\alpha,i<i',j}\frac{\hat{\tau}_{\alpha,i,i',j}-\tau_\alpha\left(\mathbf{s}_i,\mathbf{s}_{i'},\mathbf{r}_j\mid V\right)}{\zeta^2_{\alpha,i,i',j}}, \quad (10)$$

and the scalar C is:

$$C = -\frac{1}{2}\sum_{\alpha,i,j}\frac{\left(\hat{T}_{\alpha,i,j}-T_\alpha\left(\mathbf{s}_i,\mathbf{r}_j\mid V\right)\right)^2}{\sigma^2_{\alpha,i,j}} - \sum_{\alpha,i<i',j}\frac{\left(\hat{\tau}_{\alpha,i,i',j}-\tau_\alpha\left(\mathbf{s}_i,\mathbf{s}_{i'},\mathbf{r}_j\mid V\right)\right)^2}{\zeta^2_{\alpha,i,i',j}}. \quad (11)$$

Gaussian approximation of the joint distribution

While the posterior joint distribution of event locations can be written exactly, it may be difficult to use in practice. The joint distribution is a function of $3N_\mathrm{s}$ variables that needs to be computed numerically, which, in turn, requires the evaluation of the integral in the denominator of Equation 7. While conceptually straightforward, this computation is numerically costly when N_s becomes large. In order to simplify the computation and representation of the distribution, we will approximate the posterior distribution with a multi-variate normal distribution

$$p\left(\mathbf{s}\mid\hat{\mathbf{T}},\hat{\tau},V\right)\sim\mathscr{N}\left(\mathbf{s}^0,\Sigma_\mathbf{s}\right). \quad (12)$$

Following standard Gaussian analysis, the mean, $\mathbf{s}^0$, and the covariance matrix, $\Sigma_\mathbf{s}$, of the normal distribution are found as follows. The mean, $\mathbf{s}^0$, is found by solving the maximization problem

$$\mathbf{s}^0=\arg\max_\mathbf{s} I(\mathbf{s}), \quad (13)$$

and a local estimate of the covariance about $\mathbf{s}^0$ is given by

$$\left(\Sigma_\mathbf{s}^{-1}\right)_{m,n}\approx-\left.\frac{\partial^2\log I(\mathbf{s})}{\partial s_m\partial s_n}\right|_{\mathbf{s}=\mathbf{s}^0}, \quad (14)$$

where s_m and s_n span all $3N_\mathrm{s}$ coordinates of all event locations.

Joint location in uncertain velocity model

Equations 7 and 12 provide expressions for the posterior distribution given a known velocity model. When the velocity

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model is uncertain, i.e., it is sampled from a family of admissible velocity models, $\mathscr{V}$, we can use the Total Probability Theorem to write the velocity-independent form of the posterior:

$$p\left(\mathbf{s} \mid \hat{\mathbf{T}}, \hat{\tau}\right) = \int_{\mathscr{V}} p\left(\mathbf{s} \mid \hat{\mathbf{T}}, \hat{\tau}, V\right) p(V)\, dV \tag{15}$$

In order to compute the velocity-independent distribution numerically, we generate L velocity models V_l from $\mathscr{V}$ and compute the conditional posterior distributions in parallel. Then

$$p\left(\mathbf{s} \mid \hat{\mathbf{T}}, \hat{\tau}\right) \approx \frac{1}{L} \sum_{l=1}^{L} p\left(\mathbf{s} \mid \hat{\mathbf{T}}, \hat{\tau}, V_l\right). \tag{16}$$

Quantities of interest

Estimated locations of seismic events are not the final goal of seismic monitoring. Our interest is typically in geological features that the estimated seismic event locations can help to reveal (Michaud et al., 2004; Huang et al., 2006; Bennett et al., 2006). Assuming that most microseismic events originate in fractures, clouds of microseismic events reveal the fracture size, position, orientation, etc. Such quantities of interest can be written as functions of the estimated event locations, $f(\mathbf{s})$. Because $\mathbf{s}$ is a random vector, $f(\mathbf{s})$ becomes a random variable. We can use probability theory to compute the distribution of $f(\mathbf{s})$ or estimate its statistics.

The statistics can be written analytically, e.g.,

$$\mathbb{E} f(\mathbf{s}) = \int f(\mathbf{s}) p(\mathbf{s})\, d\mathbf{s}, \tag{17}$$

or

$$\operatorname{Var} f(\mathbf{s}) = \int \left(f(\mathbf{s}) - \mathbb{E} f(\mathbf{s})\right)^2 p(\mathbf{s})\, d\mathbf{s}. \tag{18}$$

Alternatively, if the joint distribution of $\mathbf{s}$ is approximated with a multi-variate Gaussian vector, then the entire distribution of $f(\mathbf{s})$ can be computed numerically by sampling joint locations and applying the function f.

NUMERICAL EXAMPLE

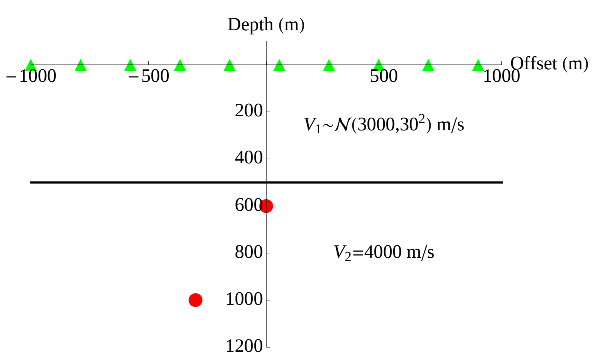

Figure 1: *A numerical setup with two layers, two source events, and ten surface receivers.*

We illustrate the proposed methodology with a simple two-dimensional numerical example. We consider a two-layer medium (Figure 1). The velocity in the top layer, dubbed "near-surface", is uncertain and has Gaussian distribution: $V_1 \sim \mathcal{N}\left(3000, 30^2\right)$ m/s. The velocity in the bottom layer, $V_2 \sim 4000$ m/s is assumed known.

Ten receivers are placed at the surface at offsets ranging from -1000 m to 1000 m. Two seismic events are located in the bottom layer at $(0, 600)$ and $(-300, 1000)$ m. We assume that direct travel times from both events are picked with errors that are normal with zero mean and standard deviation 10^{-4} s. We do not use additional correlation picks in this example in order to show the gain that the joint location brings. Using correlation picks would improve our results even further.

Event locations

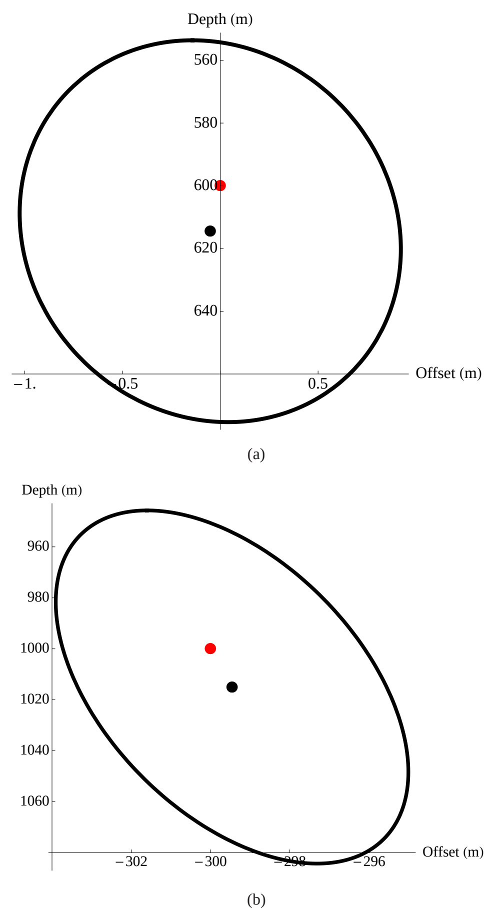

Figure 2: *The reconstructed events and their 95% error ellipses. Red dots denote the true event locations. Black dots denote the estimated event locations*

The four-dimensional joint distribution of the event locations is approximated with a Gaussian according to Equations 13 and 14. Figure 2 shows the reconstructed event locations and the 95% error ellipses. Notice that the error ellipses serve as useful indicators of the error in the location of the individual events. However, they do not contain any information about

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the correlation between those errors. The small value of the standard deviation of the time picking errors, $\sigma = 10^{-4}$ s, is associated with depth uncertainties of less than 0.5 m. This indicates that the bulk of the location uncertainty is due to the velocity uncertainty in the overburden.

	x_1	z_1	x_2	z_2
x_1	1.00	0.09	−0.61	0.08
z_1	0.09	1.00	0.50	1.00
x_2	−0.61	0.50	1.00	0.48
z_2	0.08	1.00	0.48	1.00

Table 1: *The correlation matrix of the joint distribution of the two event locations,* $\mathbf{s}_1 = (x_1, z_1)$ *and* $\mathbf{s}_2 = (x_2, z_2)$. *Observe the strong correlation between the depths of the two events.*

Table 1 shows the correlation matrix of the joint vector **s**. Observe the high correlation between the depths of the two events. This correlation is due to the fact that both events are ray-traced through the same "near-surface".

Estimating distance between events

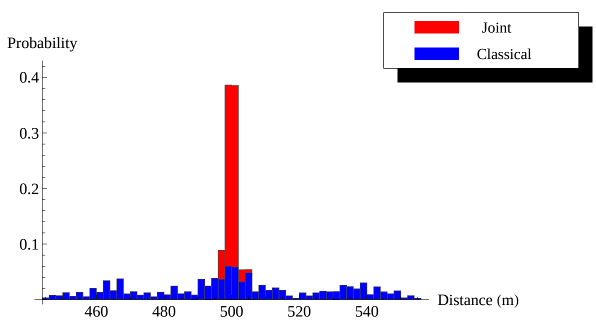

Figure 3: *The distribution of the distance between the two events as computed from the joint distribution (red) and the two marginal distributions for each event (blue). Because of high correlation between the location errors, the distance between the two events is very stable. When each event is localized separately, the correlation is lost, and the distance is recovered with a large error.*

Let us assume that the two events came from the same fracture and view the distance between the two events as a simple proxy for the fracture size. Given the joint distribution of the event locations, we can compute the distribution of the distance between the two events. Toward that end, we generate a sample from the joint distribution and compute the distance between two points for each sample. Figure 3 shows the resulting distribution of the distance in red. We then emulate a classical location approach by computing, from the joint distribution, the marginal distributions for $\mathbf{s}_1$ and $\mathbf{s}_2$. From these two distributions, we individually sample $\mathbf{s}_1$ and $\mathbf{s}_2$. The histogram of the distances between these samples is displayed in blue in Figure 3.

We can see that individual event locations, particularly depths, have significant uncertainties (standard deviation is around 30 m). However, the depths of the two events are highly correlated. Consequently, the uncertainty of the distance between the jointly located events is very small (standard deviation is around 2 m). By comparing these two histograms, we see that joint location provides an order of magnitude improvement in the distance measurement.

CONCLUSIONS

In this paper, we propose a framework for jointly locating seismic events in the presence of velocity uncertainty and signal noise. This problem is pervasive in global seismology and on the reservoir scale, e.g., in hydrofracture monitoring. Joint location better reveals geological features such as faults or fractures. In a simple numerical example we see a reduction of the error in estimated fracture size by approximately one order of magnitude.

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EDITED REFERENCES

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