Data continuation in the presence of caustics: A synthetic data example

Alison E. Malcolm and Maarten V. De Hoop*
Center for Wave Phenomena, Colorado School of Mines

Summary

In data continuation, the information present in the collected data is used to estimate data at new positions. We investigate two examples of data continuation: data healing and dip moveout. By composing a modeling operator with an imaging operator we can construct an operator to perform each of the tasks. We review the theory and present an algorithm for data continuation specifically directed at these two examples. This algorithm is able to continue data in the presence of caustics. To demonstrate the performance of this algorithm, we use a synthetic example in which there are caustics.

Introduction

Often data collected in the field are not sufficient for processing. For example, it is not possible to collect zero-offset data but these data are important in attenuating surface-related multiples. Also, the data collected are sometimes incomplete; certain offsets may be missing, for example. Such gaps in the data can cause problems in imaging. Errors introduced in the computation of missing data will propagate into other procedures in which these data are used; thus, an accurate data continuation algorithm is important. A fundamental limitation to this type of data continuation is that it is not possible to compute missing data that scattered from a subsurface point not sampled in the available data.

To get around problems introduced by missing data, we propose a method of data continuation that allows missing data to be filled in from the available data (data healing) and the computation of different geometries than those collected originally, as in dip moveout (DMO) for example. The theory to do this involves the composition of an imaging/migration operator and a modeling/demigration operator to form a single operator that maps the initial data set to a second, computed, data set. This theory is discussed in detail in Malcolm *et al.* [4]. We are particularly interested in situations in which caustics occur. This puts our theory in a framework similar to that of depth migration, which goes beyond the time migration framework of Fomel [3], Bleistein *et al.* [1], and Stolt [5].

We have developed an algorithm to test this theory and illustrate its capabilities using synthetic data. To accurately fill in missing data in the presence of caustics, knowledge of the smooth velocity model is required. Our algorithm relies on this knowledge and in turn, the missing data it fills in assists in the migration and velocity analysis portions of data processing. Thus, we see the three processes of data continuation, velocity analysis, and migration as interdependent steps in the imaging process.

Theory and Algorithm

As stated in the introduction, we construct an operator to calculate missing data from the available data by composing a modeling operator with an imaging operator. It is the exact form of the modeling operator used in the composition that determines the form of the output data. In this paper we discuss two forms of this operator: data healing, for which the output data are a more complete version of the input data set, and DMO, for which the output data are exploding-reflector-data computed from a data set without near offsets. For data healing, the modeling operator is fairly general. It simply models data with source and receiver positions in the range missing in the original data set. For DMO, however, the modeling operator is more specialized, computing the single ray between each subsurface scattering point and surface location for which data are required. This distinguishes the exploding-reflector modeling operator from a true zero-offset modeling operator. A zero-offset modeling operator models two rays with coincident source and receiver positions that do not necessarily follow the same path through the subsurface.

The algorithm we present for data continuation is ray-based, wherein the rays are shot from the subsurface to the acquisition surface. We consider only acoustic waves in isotropic media. The algorithm used to compute exploding-reflector data is essentially the same as that used for data healing. For each subsurface point and migration dip (vector sum of the two ray directions), there are three steps:

- step 1: from a particular subsurface point, shoot rays to the surface in the range of the known data and extract an amplitude value from the data
- step 2: from the same subsurface point and migration dip, shoot again to the surface, now into the range of the desired output data
- step 3: apply a correction to the amplitude retrieved in step 1 and add it to data at the time computed in step 2.

In step 1 it is important to average over a large enough range of input scattering angles and migration dip directions. If this is not done, the final image will contain artifacts in the presence of caustics, as seen in Brandsberg-Dahl *et al.* [2]. It is in step 2 where data healing and

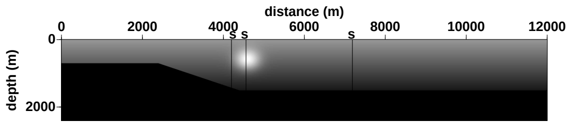

Fig. 1: Velocity model used in this test. The shading gives the velocity, with light colors being low velocity and dark higher velocity. The vertical lines indicate the location of the shot points for which shot records are shown in Figure 2.

DMO differ. For data healing, two rays are shot to the surface within the output data range; for DMO, only one ray is shot in this step. This algorithm is easily parallelizable over the subsurface point as there is no information passed between different image points. Here, we see the importance of the limitation, mentioned in the introduction, that the known data must sample a particular subsurface point for data to be generated from it. Though the underlying theory [4] does account correctly for the amplitudes, here we use an approximation of those amplitudes; we have corrected for only the obliquity factor ($|\cos\theta_i|^2$ where θ_i is the angle between the source and receiver rays at the scattering point). In general, dynamic ray-tracing is necessary to compute the amplitude correction. To do this a sufficiently accurate estimate of the velocity model is required.

The computational complexity of the theory presented is on the same order as that for depth migration, although that of the algorithm described here is greater. The operator used for continuation changes laterally, meaning that in regions where the velocity model is relatively simple it will be faster than a migration whereas, in regions of complicated structure, the cost of the algorithm increases.

Synthetic Test

We begin with the data healing example, by filling in missing data in synthetic shot records. The model, shown in Figure 1, contains a single reflector in a background consisting of a constant vertical gradient along with a low-velocity Gaussian lens. This model was introduced in Brandsberg-Dahl *et al.* [2], and the details of its properties can be found there. The synthetic data are computed for 134 source positions, beginning at 2400 m with a spacing of 36 m, and 468 receivers beginning at 1200 m with a spacing of 18 m. Data from each shot are computed at all receivers. Although this example has a regular sampling in both shot and receiver coordinates, this is not necessary for the algorithm.

To test the algorithm we reconstruct three shot gathers from different regions of the model. In each shot, offsets of 0 to 500 m are removed from the data and then reconstructed. The shot records with missing data are shown in the left column of Figure 2. The center and right columns of this Figure show the data filled in with our algorithm and the true data, respectively. The first record is far from the lens, at $s = 7188$. Here the structure is simple and so we expect to be able to easily fill in the missing data, with this theory or a simpler one. The top row of Figure 2 shows that this is the case. The middle row of Figure 2 shows the same test applied to the shot record at $s = 4200$. In contrast to the previous shot record, this one is influenced by the low-velocity lens making the reconstruction more difficult. We are still able to reconstruct the missing data, however, including the truncation of the event at about 1.8 s. Thirdly, we compute the missing data traces in the shot record directly over the lens, at $s = 4560$, showing the most complicated structure of any of the shot records. Here we have arrivals from both the horizontal and dipping portions of the reflector as well as complications due to the caustics caused by the lens. The reconstruction, shown in the middle row of the figure, remains quite good although it contains some amplitude errors.

To illustrate problems that arise when too small a neighborhood of the input scattering angle (angle between the incoming and outgoing rays at the reflection point) is used to reconstruct the data, we computed the missing traces for the second shot record using only a 2° range in the input scattering angle. The result of this computation is shown in the Figure 3. Note the artifact with particularly high amplitude appearing just below the last true event in this section. Although this shot record has problems because it was generated using so few input angles, it is much quicker to compute than the previous gather. More angles than necessary are most likely used to compute the gather in the bottom row of Figure 2. The optimal balance between cost and image quality lies somewhere in between.

Finally, to illustrate an application to DMO in the presence of caustics, we compute the exploding-reflector data from the original data set (with offsets from -6 to 7 km) with the offsets between -100 m and 100 m removed. These data are shown in Figure 4. The smallest-offset data (offset of 6 m) from the true data set are shown for comparison. The most notable difference between the two sections is the presence of additional features in the zero-offset data not visible in the exploding reflector data. These come from the further restriction in the exploding reflector data that the energy must travel upwards and downwards on the same path, whereas the zero-offset data require only that the surface source and receiver points are the same.

Discussion

We present a method of data continuation, illustrated with examples of data healing and DMO that works in the presence of caustics. The algorithm presented can be costly, with the cost strongly dependent on the complexity of the velocity model. Because of this dependence and because of the varied applications for which this technique may be useful, evaluation of the cost-effectiveness of the method must be made on a case-by-case basis. Although the work presented here is in 2D, the theory remains valid

Data continuation examples

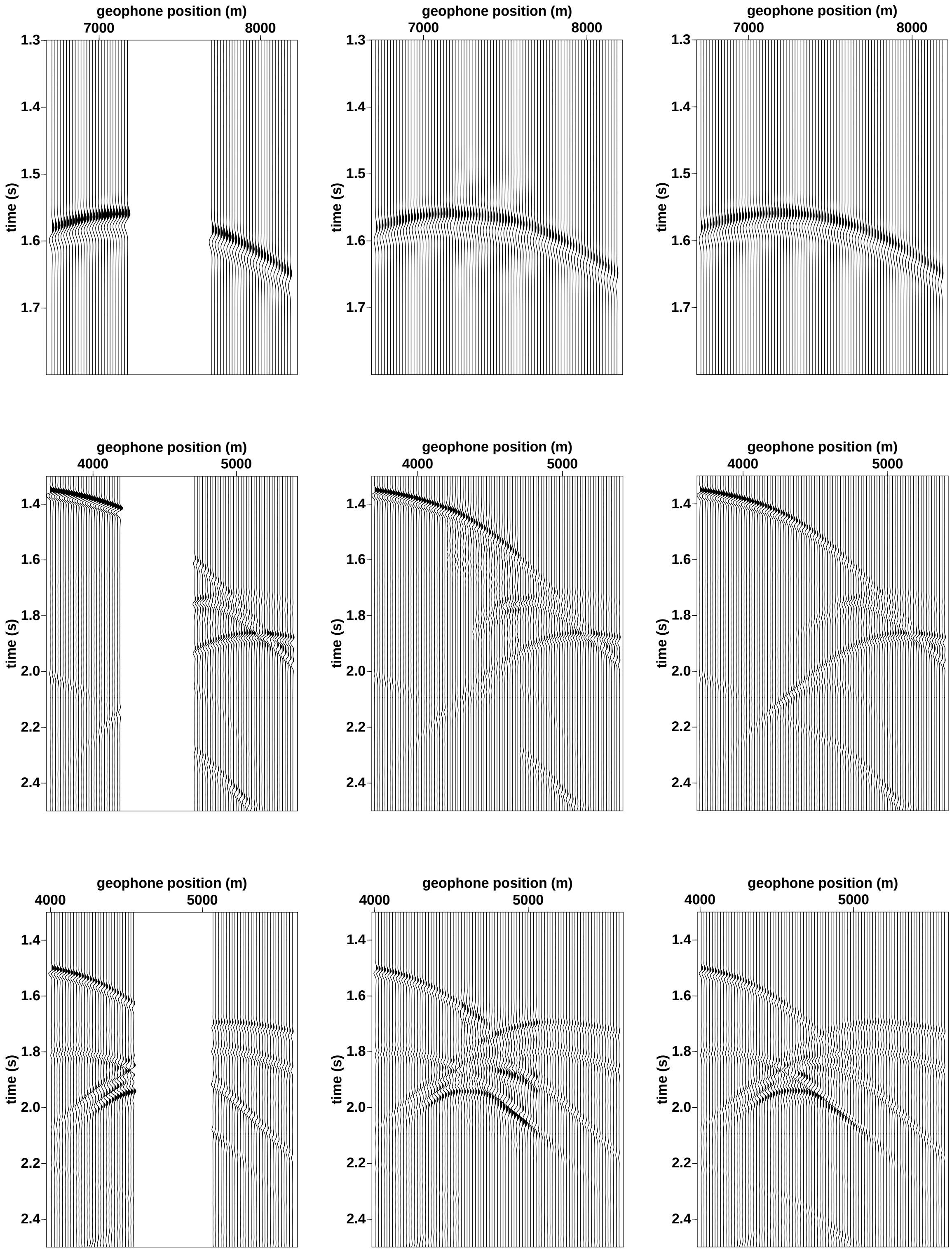

Fig. 2: In each row of this figure a different shot record has a block of traces removed and then reconstructed. The left column shows the input data, the middle column is the reconstructed data, and the right column is the actual full synthetic shot record. The position of the first shot record is $s = 7188$ m, that of the second is $s = 4200$ m and the third is located at $s = 4560$. The locations of these shots are denoted with vertical lines in the velocity model, shown in Figure 1.

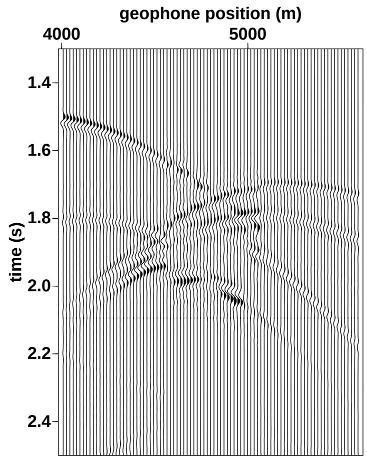

Fig. 3: This record illustrates the presence of artifacts when an insufficient range of the input scattering angle is used. For shot $s = 4560$, using only a small range of input angles, results in a poorer image than that in Figure 2 which was computed using all available data as input.

in 3D although the implementation is somewhat more difficult.

References

[1] N. Bleistein, J. K. Cohen, and J. W. Stockwell. *Mathematics of Multidimensional Seismic Imaging, Migration and Inversion.* Springer-Verlag, New York, 2000.

[2] S. Brandsberg-Dahl, M. V. de Hoop, and B. Ursin. Focusing in dip and AVA compensation on scattering-angle/azimuth gathers. *Geophysics*, 68:232–254, 2003.

[3] S. Fomel. Theory of differential offset continuation. *Geophysics*, 68:718–732, 2003.

[4] A. E. Malcolm, M. V. de Hoop, and J. H. Le Rousseau. The applicability of DMO/AMO in the presence of caustics. *submitted to Geophysics*, 2003. CWP-436P.

[5] R. H. Stolt. Seismic data mapping and reconstruction. *Geophysics*, 67:890–908, 2002.

Acknowledgments

We acknowledge the contributions to the development of the underlying theory made by J. H. Le Rousseau, and the assistance with the algorithm from H. Calandra, S.-K. Foss and S. Brandsberg-Dahl. This work was supported by the sponsors of the Consortium Project on Seismic Inverse Methods for Complex Structures at the Center for Wave Phenomena and by Total.

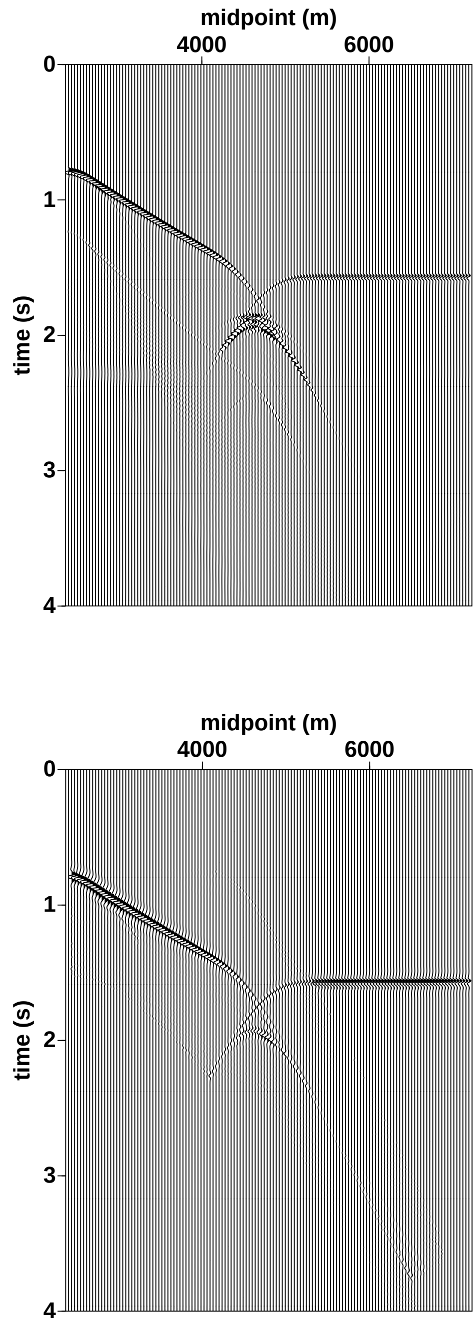

Fig. 4: The top panel shows the smallest offset in the original synthetic data set (6-m offset). The bottom panel is the exploding reflector data computed from the synthetic data set from which offsets between -100 m and 100 m were removed.