Introduction to Seismic Imaging

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August 20, 2010

Outline

- Introduction
  - Why we image the Earth
  - How data are collected
  - Imaging vs inversion
  - Underlying physical model
- Data Model
- Imaging methods
  - Kirchhoff
  - One-way methods
  - Reverse-time migration
  - Full-waveform inversion
- Comparison of Methods

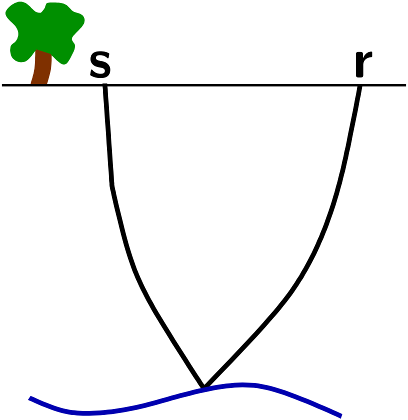
Approximate Techniques

- Kirchhoff
  - Integral technique
  - Related to X-ray CT imaging
  - Generalized Radon Transform
  - Conventionally uses ray theory
- One-way
  - Based on a paraxial approximation
  - Usually computed with finite differences

One-Way Methods
Physical Motivation

- **downward continuation**
- **imaging condition**

Claerbout 71, 85



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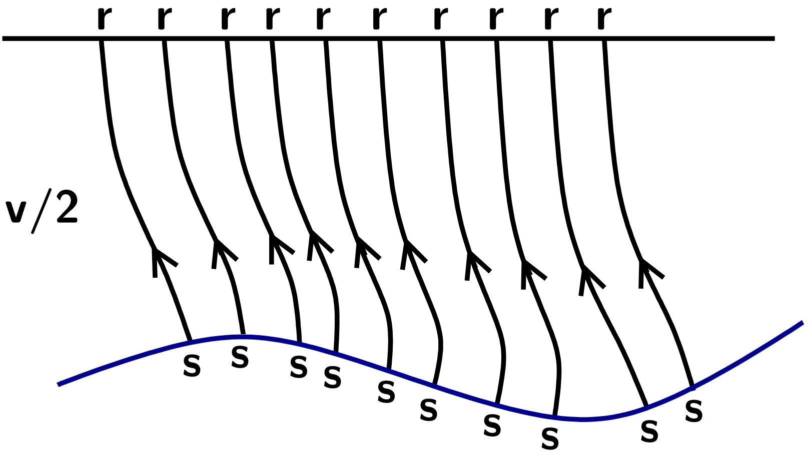

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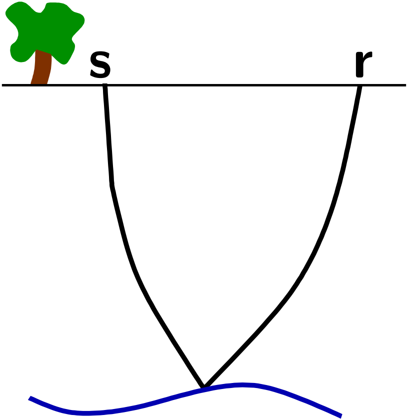

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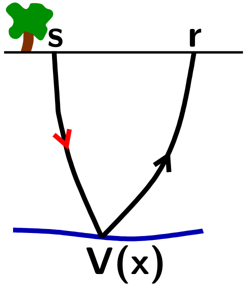
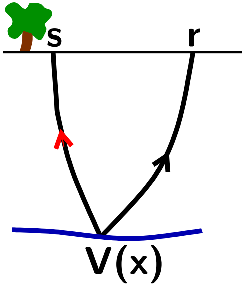
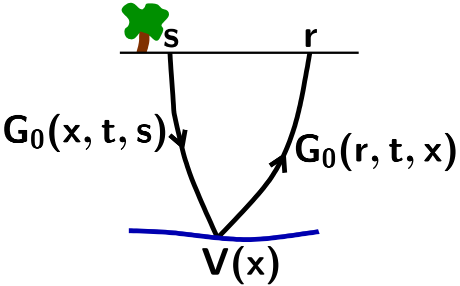




A Data Model

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) V(x) \partial_t^2 G_0(x, t_0, s) dx dt_0$$

$$\delta G(s,r,\omega) = -\int_X \omega^2 G_0(r,\omega,x) V(x) G_0(x,\omega,s) dx$$



A Data Model

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) V(x) \partial_t^2 G_0(x, t_0, s) dx dt_0$$

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$$\Downarrow$$

$$\int_X \omega^2 G_-(r,\omega,x) G_-(s,\omega,x) \widetilde{V}(x) dx$$

One-Way Methods
Approximating the Wave Equation

Idea (1D, c constant):

$$(\partial_x^2 - \partial_t^2)u = (\partial_x - \partial_t)(\partial_x + \partial_t)u$$

c not constant:

$$(c(x)^2\partial_x^2 - \partial_t^2)u = (c(x)\partial_x - \partial_t)(c(x)\partial_x + \partial_t)u - c(x)(\partial_x c(x))\partial_x u$$

c(x) smooth $\Rightarrow$ better approximation

One-Way Methods
Approximating the Wave Equation
Taylor (81), Stolk & de Hoop (05)

Multi-dimensional:
$\mathbf{Lu} = \mathbf{f}$

$$\partial_z \begin{pmatrix} u \\ \partial_z u \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -A & 0 \end{pmatrix} \begin{pmatrix} u \\ \partial_z u \end{pmatrix} + \begin{pmatrix} 0 \\ -f \end{pmatrix}$$

$$A(z, x, \partial_x, \partial_t) = \partial_x^2 - c(z, x)^{-2} \partial_t^2$$

One-Way Methods

Approximating the Wave Equation

Taylor (81), Stolk & de Hoop (05)

Diagonalize in smooth background

$$\partial_z \begin{pmatrix} u_+ \\ u_- \end{pmatrix} = \begin{pmatrix} iB_+ & 0 \\ 0 & iB_- \end{pmatrix} \begin{pmatrix} u_+ \\ u_- \end{pmatrix} + \begin{pmatrix} f_+ \\ f_- \end{pmatrix}$$

$$b_\pm(\xi, x, \omega) = \pm\sqrt{\xi^2 - c_0(x)^{-2}\omega^2}$$

$B_\pm$ FIOs

One-Way Methods
Approximating the Wave Equation
Taylor (81), Stolk & de Hoop (05)

Diagonalize in smooth background

$$\partial_z \begin{pmatrix} u_+ \\ u_- \end{pmatrix} = \begin{pmatrix} iB_+ & 0 \\ 0 & iB_- \end{pmatrix} \begin{pmatrix} u_+ \\ u_- \end{pmatrix} + \begin{pmatrix} f_+ \\ f_- \end{pmatrix}$$

Solution:

$$G_0 = \begin{pmatrix} G_+ & 0 \\ 0 & G_- \end{pmatrix}$$

G_0 propagator in c_0

One-Way Methods
Approximating the Wave Equation
Taylor (81), Stolk & de Hoop (05)

Relate $\mathbf{u}_\pm$ to $\mathbf{u}$ and $\partial_z \mathbf{u}$

$$\begin{pmatrix} \mathbf{u}_+ \\ \mathbf{u}_- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} (\mathbf{Q}_+)^{-1} & -\mathcal{H}\mathbf{Q}_+ \\ (\mathbf{Q}_-^*)^{-1} & \mathcal{H}\mathbf{Q}_- \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \partial_z \mathbf{u} \end{pmatrix}$$

$$\mathbf{q}_\pm = \left[\left(\frac{\omega}{\mathbf{c}(\mathbf{z}, \mathbf{x})} \right)^2 - \|\xi\|^2 \right]^{-1/4}$$

$\mathbf{Q}_\pm$ differ in sub-principal parts
$\mathbf{Q}_\pm$ ψDOs

One-Way Methods
Modeling Data Stolk & de Hoop (05)

$$\delta G(s, r, \omega) \approx$$
$$\int_X \omega^2 Q_-^*(r) G_-(r, \omega, x) V(x) G_+(x, \omega, s) Q_+(s) dxdz$$

$$V(x) = \frac{1}{4} Q_-(z, x) \delta c(z, x) c_0(z, x)^{-3} Q_+^*(z, x)$$

One-Way Methods

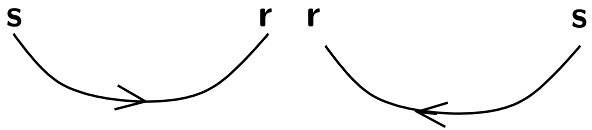
Modeling Data Stolk & de Hoop (05)

$$\delta G(s, r, \omega) \approx$$

$$\int_X \omega^2 Q_-^*(r) G_-(r, \omega, x) V(x) G_+(x, \omega, s) Q_+(s)\, dx dz$$

$$V(x) = \frac{1}{4} Q_-(z, x)\, \delta c(z, x)\, c_0(z, x)^{-3} Q_+^*(z, x)$$

Reciprocity:



One-Way Methods
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$$\delta G(s, r, \omega) \approx$$

$$\int_X \omega^2 Q_-^*(r) G_-(r, \omega, x) V(x) G_+(x, \omega, s) Q_+(s) dxdz$$

$$V(x) = \tfrac{1}{4} Q_-(z, x) \delta c(z, x) c_0(z, x)^{-3} Q_+^*(z, x)$$

Drop Qs

$$\int_X \int_Z \omega^2 G_-(0, r, \omega, z, x) G_-(0, s, \omega, z, x)$$

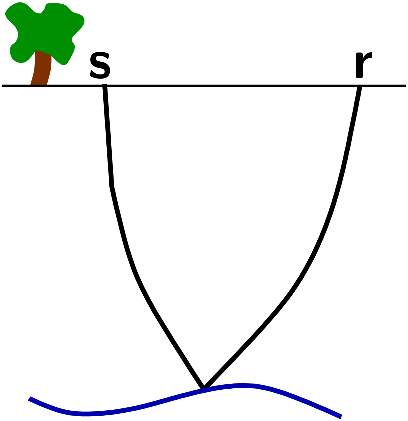
$$(E_2 E_1 a)(z, x) dzdx$$

$$E_2 : b(z, r, s) \mapsto \delta(t) b(z, r, s)$$
$$E_1 : a(z, x) \mapsto \delta(r - s) a(z, \tfrac{r+s}{2})$$

One-Way Methods

Imaging Claerbout (78) Stolk & de Hoop (06)

- **downward continuation**
- **imaging condition**



One-Way Methods
Imaging Claerbout (78) Stolk & de Hoop (06)

- downward continuation

$$\hat{d}(z, s, r, t) = H(z, 0)^* Q^{-1*} d(s, r, t)$$

$$(H(z, z_0))(s, r, t, s_0, r_0) = (G_-(z, z_0))(r, t, r_0) * (G_-(z, z_0))(s, t, s_0)$$

H is an FIO $H * H$ is ΨDO

- imaging condition

One-Way Methods

Imaging Claerbout (78) Stolk & de Hoop (06)

- downward continuation
- imaging condition

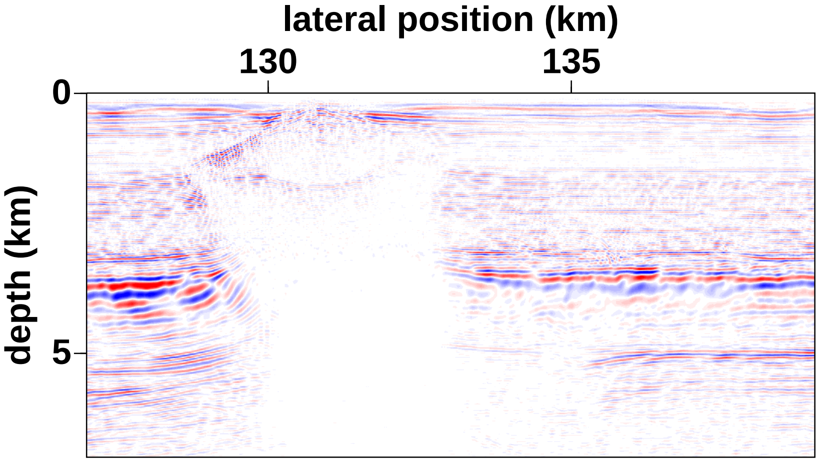
$$V(z,x) \approx \hat{d}(z,x,x,0)$$

$$V(x) = \frac{1}{4}Q_-(z)\delta c(z,x)c_0(z,x)^{-3}Q_+^*(z)$$

Recall Q are ΨDOs & H^*H is ΨDO

We have again located the singularities

One-Way Methods
Example



Approximate Techniques

- **Kirchhoff**
  - Integral technique
  - Related to X-ray CT imaging
  - Generalized Radon Transform
  - Conventionally uses ray theory
- **One-way**
  - Based on a paraxial approximation
  - Usually computed with finite differences

‘Exact’ Techniques

- Reverse-time migration (RTM)
  - Run wave-equation backward
  - Usually computed with finite differences
  - “No” approximations (to the acoustic, isotropic, linearized wave-equation, for smooth media assuming single scattering)
- Full-waveform inversion (FWI)
  - Iterative method to match the entire waveform
  - Gives smooth part of velocity model

Reverse-Time Migration

Modeling Data

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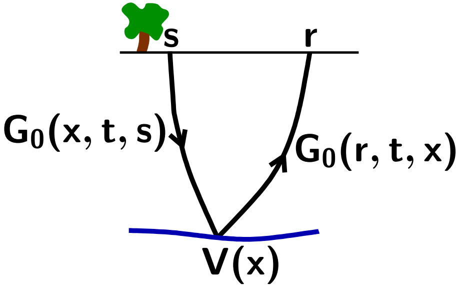

Reverse-Time Migration
Modeling Data

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) V(x) \partial_t^2 G_0(x, t_0, s)\, dx\, dt_0$$

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$F : \delta c \mapsto \delta G$ is again an FIO

Reverse-Time Migration
Forming an image

$$\delta G(s, r, \omega) = -\int_x \omega^2 G_0(r, \omega, x) V(x) G_0(x, \omega, s) dx$$

$$F : \delta c \mapsto \delta G \quad \text{is again an FIO}$$

$$F^* : \delta G \mapsto \delta c \quad \text{is again an FIO}$$

F^*F is ΨDO for complete coverage

Stolk et al. (09)

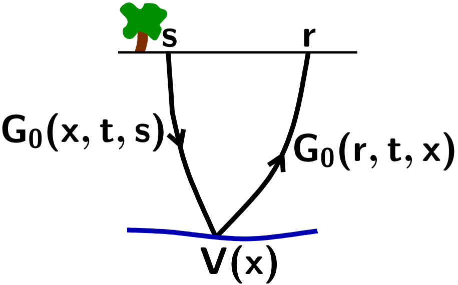
- no direct rays (transversal intersection)
- no source-side caustics
- $c_0 \in C^\infty$

Reverse-Time Migration
Forming an Image

Procedure:

Whitmore (83), Loewenthal & Mufti (83), Baysal et al (83)

- **back propagate in time**
- **imaging condition**



Reverse-Time Migration
an Adjoint State Method

Lailly (83,84), Tarantola (84,86,87) Symes (09)

For a fixed source, s,

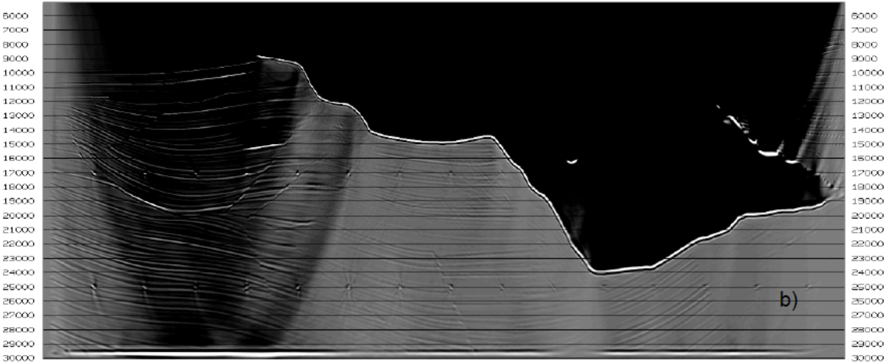
$$\begin{aligned}(c^{-2}\partial_t^2 - \nabla^2)q(x,t;s) &= \int_{R_s} \delta G(r,t;s)\delta(x-r)dr \\ q(\cdot,t,\cdot) &= 0 \text{ for } t > T\end{aligned}$$

receivers act as sources, reversed in time

$$Im(x) = \frac{2}{c^2(x)} \int\int q(x,t;s)\partial_t^2 G_0(x,t,s)dtds$$

Reverse-Time Migration

Example Liu et al (07)



Reverse-Time Migration
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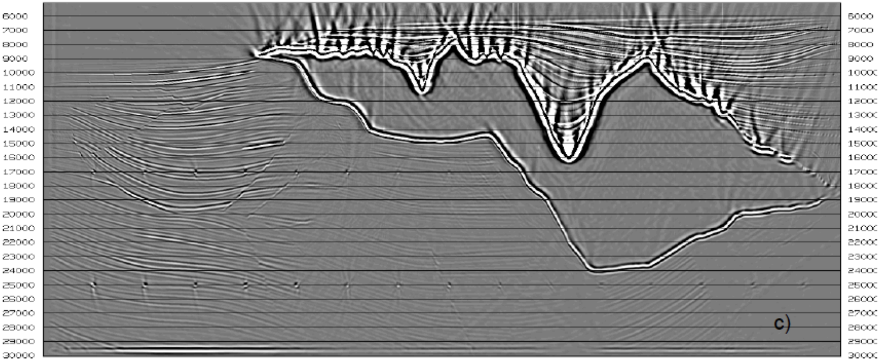

Reverse-Time Migration

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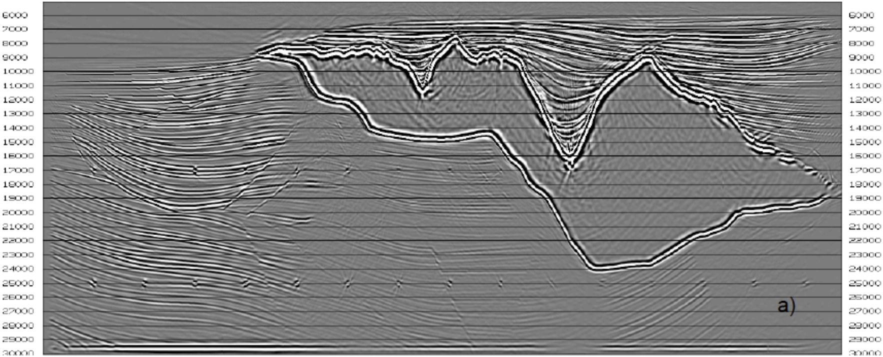

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- **Full-waveform inversion (FWI)**
  - Iterative method to match the entire waveform
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Full-waveform Inversion

Problem Setup Tarantola (87), Virieux & Operto (09)

Recall our initial formulation:

$$Lu := (\nabla^2 - \frac{1}{c^2}\partial_t^2)u = f$$

$$u = 0 \qquad t < 0$$

$$\partial_z u|_{z=0} = 0$$

FWI attempts to solve for c directly given u,f

no splitting of c

but band limited data $\Rightarrow$ smooth solution

Full-waveform Inversion
Problem Setup Tarantola (87), Virieux & Operto (09)

Recall our initial formulation:

$$\mathbf{Lu} := (\nabla^2 - \frac{1}{c^2}\partial_t^2)\mathbf{u} = \mathbf{f}$$

Compute:

$$\min_c \|Lu - d\|_{L^2((S,R)\times[0,T])}$$

Full-waveform Inversion

Some Issues Symes (09)

Compute:

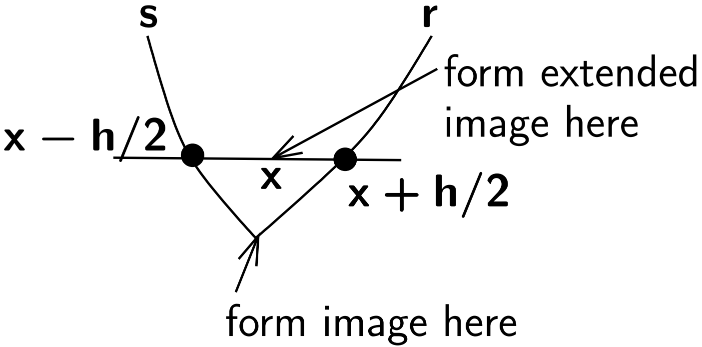
$$\min_c \|Lu - d\|_{L^2((S,R)\times[0,T])}$$

- **Objective function appears non-convex Can we restrict the domain of models so that it is? e.g. $c_{min} < c < c_{max}$**
- **What space must c be in for objective function to be differentiable?**
- **Data are finite bandwidth – we cannot resolve structures on all scales**

Full-waveform Inversion
Work-around 1: Partial Linearization Symes (09)

Idea: Extend $\delta c(x)$ from X to $\hat{X}$

Example: $\delta c(x) \mapsto \delta\hat{c}(x,h) = \delta(h)\delta c(x)$



Claerbout (85) suggested this extension

Full-waveform Inversion
Work-around 1: Partial Linearization Symes (09)

Idea: Extend $\delta c(x)$ from X to $\hat{X}$

Example: $\delta c(x) \mapsto \delta \hat{c}(x, h) = \delta(h)\delta c(x)$

Now extend

$$F[\delta c] \mapsto \delta G$$

to

$$\hat{F}[\delta \hat{c}(x, h)] \mapsto \delta G$$

s.t. $\hat{F}$ is 'invertible' ($I - \hat{F}^{\dagger}\hat{F}$ is smoothing)

Find c_0 s.t. $\mathrm{supp}\left(\delta \hat{c}(x, h)\right) \subset \{(x, h) | h = 0\}$

Stolk & de Hoop (2001), Stolk et al (2005) show invertibility

Full-waveform Inversion

Work-around 2: Separation of Scales

Pratt (99), Virieux (09)

$\min_{\mathrm{c}} \|\mathrm{Lu} - \mathrm{d}\|_{\mathrm{L}^2((\mathrm{S},\mathrm{R})\times[0,\mathrm{T}])}$ **not convex**

$\Rightarrow \|\mathsf{c}_{\mathrm{initial}} - \mathsf{c}_{\mathrm{true}}\|$ **must be small**

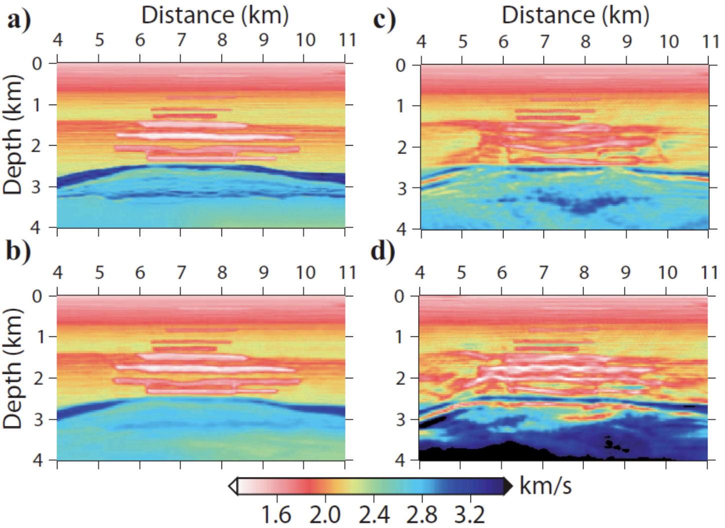
Solve for G(ω) from $\omega_{\min}$ to $\omega_{\max}$

No proof optimization converges

Full-waveform Inversion

Work-around 2: Separation of Scales, example

Virieux (09)



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  - Kirchhoff
  - One-way methods
  - Reverse-time migration
  - Full-waveform inversion
- Comparison of Methods

Comparison of Methods
Kirchhoff vs One-way Fehler & Huang (02)

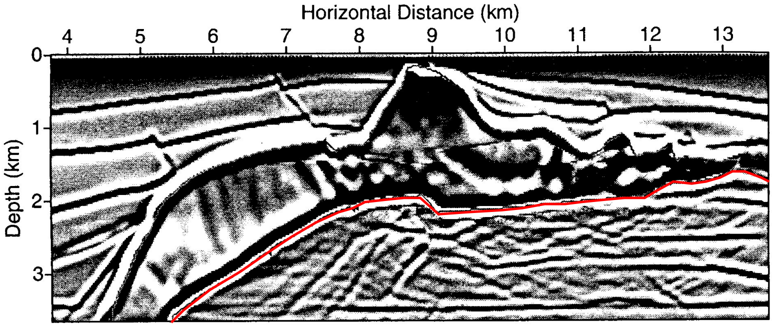


Figure 5 Kirchhoff migration image obtained by migrating all common-shot gathers for the model in Figure 1. Solid line shows boundary of salt body in Figure 1.

Kirchhoff

Comparison of Methods
Kirchhoff vs One-way Fehler & Huang (02)

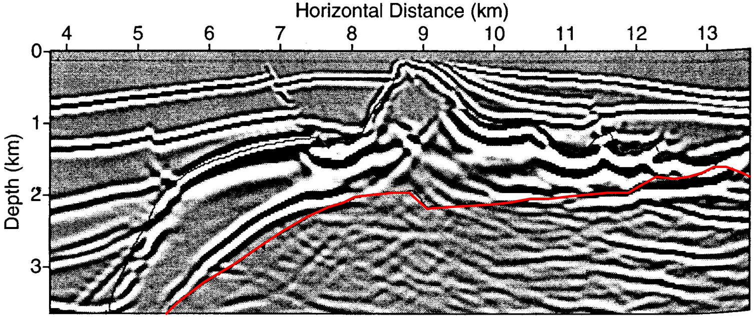


Figure 6 Phase-shift migration image obtained by migrating all common-shot gathers for the model in Figure 1. Solid line shows boundary of salt body in Figure 1.

Phase-shift

Comparison of Methods
Kirchhoff vs One-way Fehler & Huang (02)

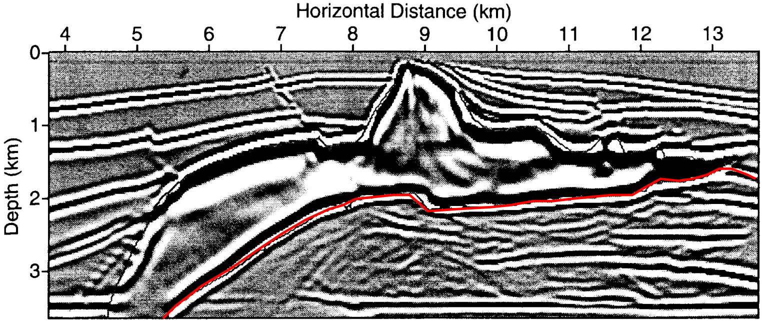


Figure 7 Split-step Fourier migration image obtained by migrating all common-shot gathers for the model in Figure 1. Solid line shows boundary of salt body in Figure 1.

Split-step

Comparison of Methods
Kirchhoff vs RTM Zhu & Lines (98)

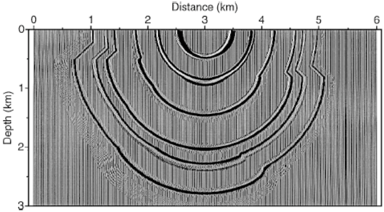


FIG. 1. Kirchhoff migration impulses. The migration is based on a blocked velocity model with a normal fault developed throughout the depth range. The velocity in each block linearly increases with depth.

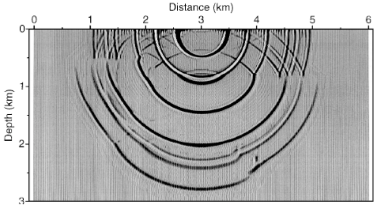


FIG. 2. Reverse-time migration impulses. The migration is based on the same input data and the same velocity model as used in Figure 1.

Comparison of Methods
Kirchhoff vs RTM Zhu & Lines (98)

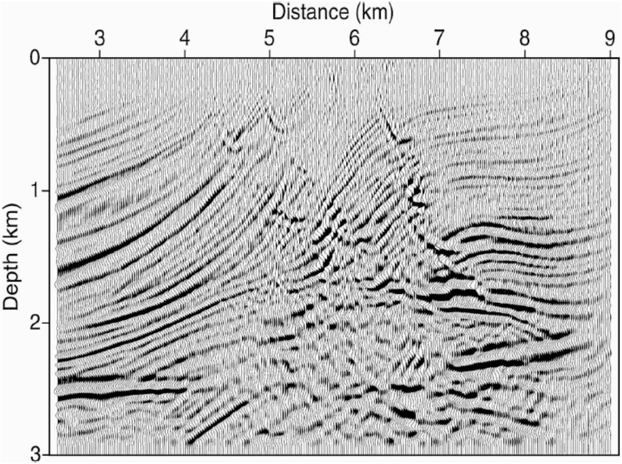


FIG. 5. The final migration section of the Marmousi model data set produced by the Kirchhoff integral method.

Kirchhoff

Comparison of Methods
Kirchhoff vs RTM Zhu & Lines (98)

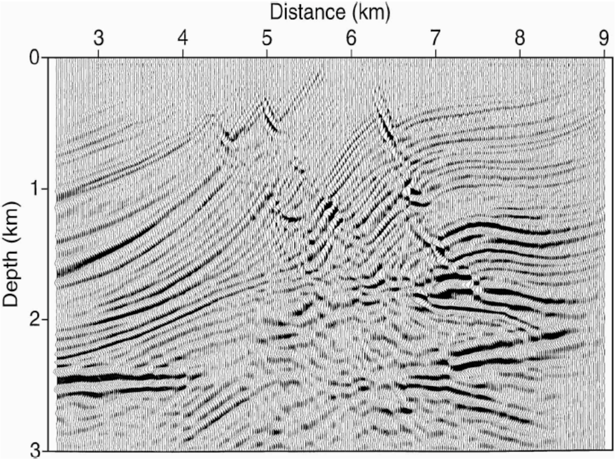
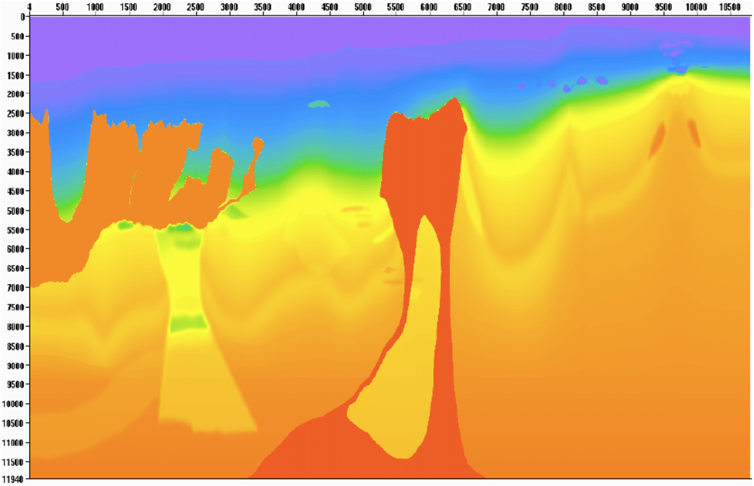


FIG. 6. The final migration section of the Marmousi model data set produced by the reverse-time migration.

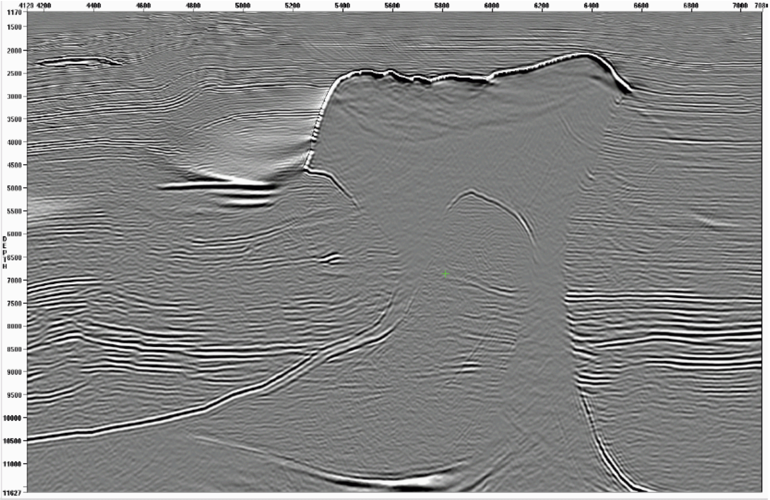
Reverse-time

Comparison of Methods
One-Way vs RTM Farmer (06)



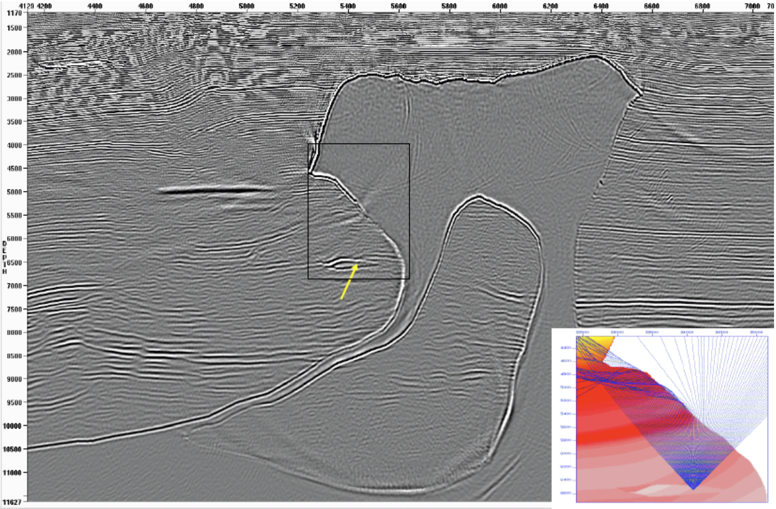
Model

Comparison of Methods
One-Way vs RTM Farmer (06)



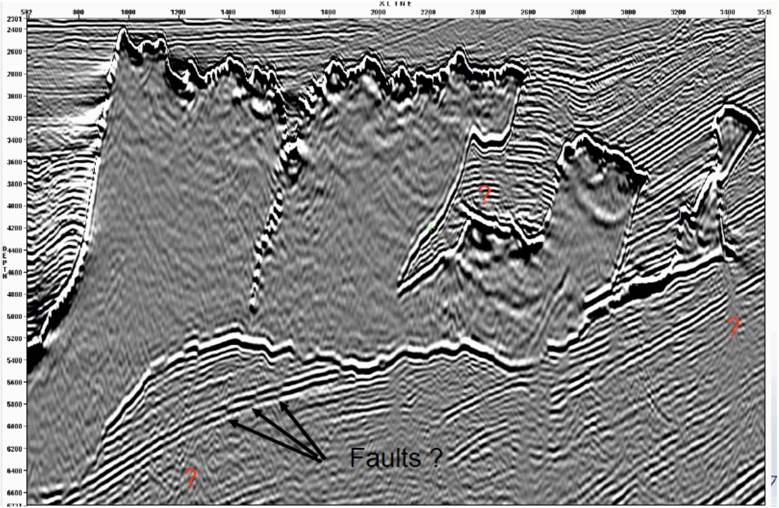
One-way

Comparison of Methods
One-Way vs RTM Farmer (06)



Reverse-time

Comparison of Methods
One-Way vs RTM Farmer (06)



One-way

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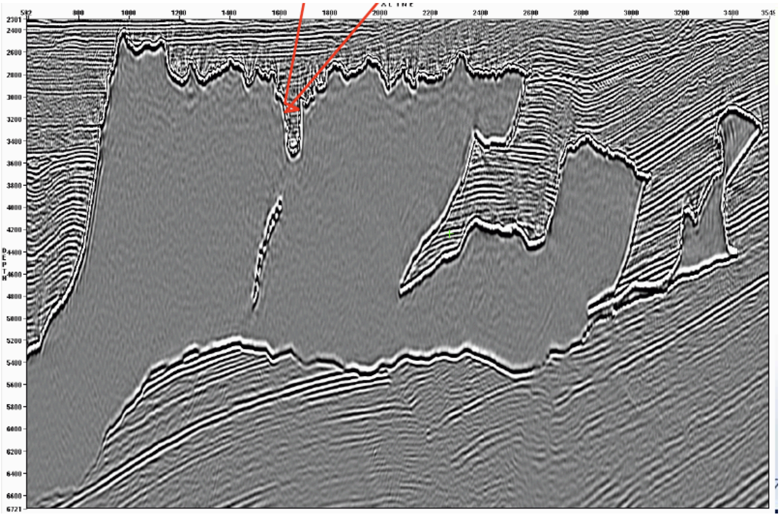
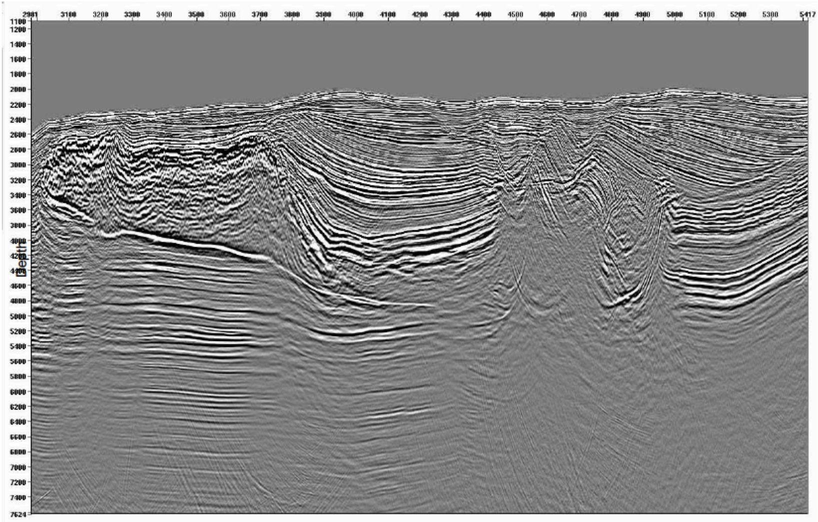
One-Way vs RTM Farmer (06)



Reverse-time

Comparison of Methods
Real Data Farmer 2006



Kirchhoff

Comparison of Methods
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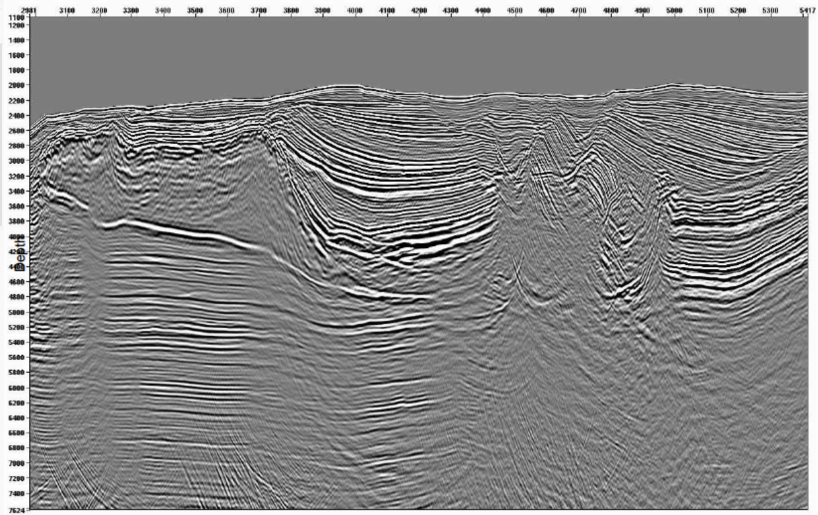
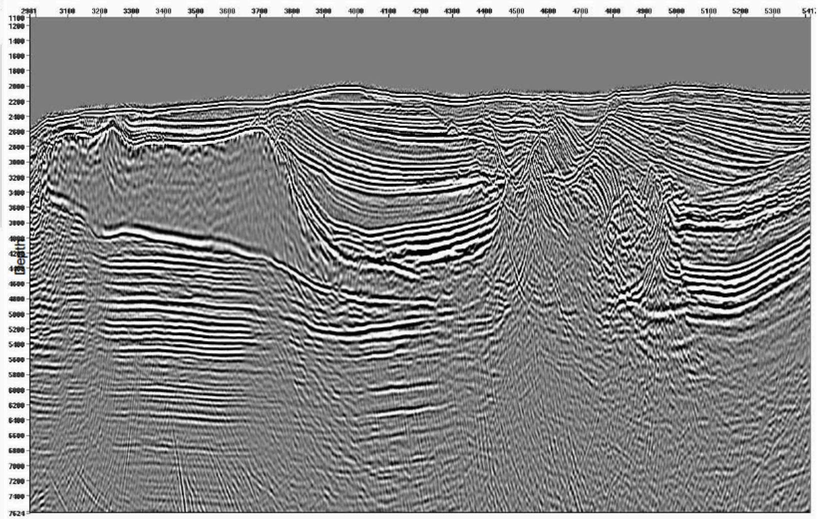


One-way

Comparison of Methods
Real Data Farmer 2006



Reverse-time

Summary of Methods

- Kirchhoff requires a lot of rays (or a simple model)
- One-way doesn't handle turning waves
- RTM separation of up/down-going waves is challenging

c_0 is key

- FWI optimization doesn't convergence from an arbitrary starting model

... and we never spoke about the source ...

... or multiple-scattering ...

Edip Baysal, Dan D. Kosloff, and John W. C. Sherwood.
Reverse time migration.
Geophysics, 48(11):1514–1524, 1983.

J. F. Claerbout.
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