Introduction to Seismic Imaging

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August 20, 2010

Outline

- Introduction
  - Why we image the Earth
  - How data are collected
  - Imaging vs inversion
  - Underlying physical model
- Data Model
- Imaging methods
  - Kirchhoff
  - One-way methods
  - Reverse-time migration
  - Full-waveform inversion
- Comparison of methods

Some organized references

Reviews:

- **Symes (09)**
- **Etgen et al. (09)**

Imaging Book:

- **Bleistein et al. (01)**

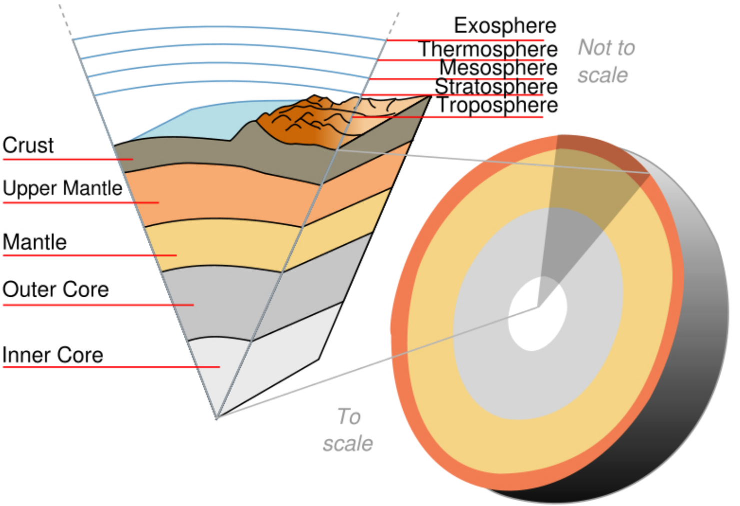
Microlocal Analysis of Reflection Seismology:

- **Stolk (00,01,04,05,06) . . . (+ co-authors)**
- `http://www.math.purdue.edu/~mdehoop/10_topics/`

Background:

- **Treves (80a,b), Hörmander (83,85), Sjöstrand & Grigis (94), Duistermaat (96), Maslov & Fedoruik (81)**

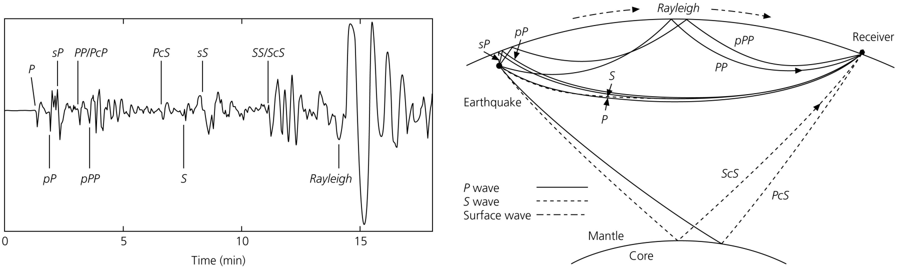
What is imaged?



from `http://en.wikipedia.org/wiki/Mantle_(geology)`

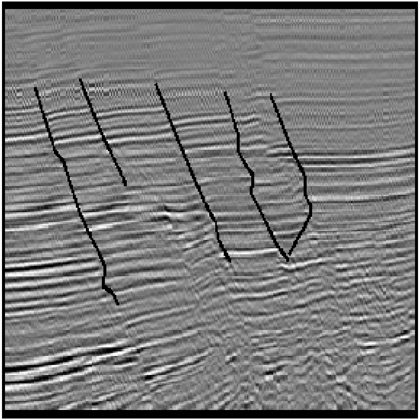
What is imaged?

Figure 1.1-3: Example of seismogram, showing accompanying ray paths.



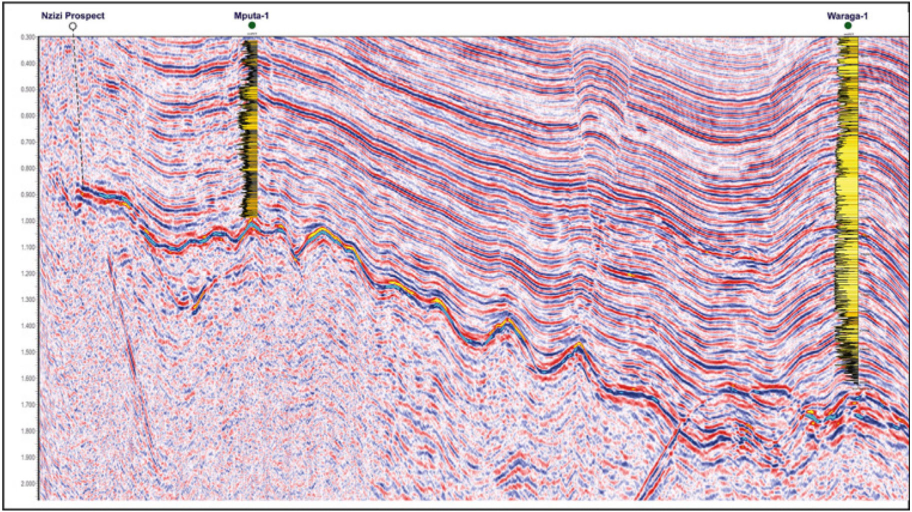
from Stein & Wysession (2003)

What is imaged?



from `http://utam.gg.utah.edu/stanford/node5.html`

What is imaged?



from `http://www.searchanddiscovery.com/documents/2009/10183abeinomugisha/images/fig05.htm`

Data Collection

from `www.litho.ucalgary.ca/transect_info/snorcle/photos/`

Data Collection

from `www.litho.ucalgary.ca/transect_info/snorcle/photos/`

Data Collection

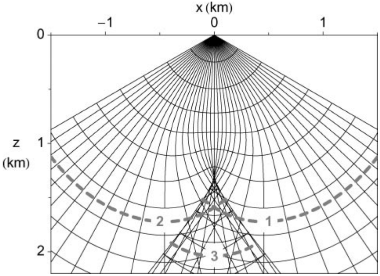
from `http://www.geop.ubc.ca/Lithoprobe/transect/snore97.html`

Why is this hard?

sedimentary rocks:	2-4 km/s
igneous rocks:	2-7 km/s
metamorphic rocks:	1-4 km/s
at depth:	8-10 km/s

travel distance: tens of wavelengths

wavepaths: Stolk & Symes (2004)



Imaging vs Inversion

Imaging: Locating the singularities in structure.

$$\mathbf{A}\mathbf{m} = \mathbf{d}$$
$$\mathbf{m} \approx \mathbf{A}^*\mathbf{d}$$

we will discuss when $\mathbf{A}^*$ correctly locates singularities

Inversion: Determining the physical properties of the Earth.

$$\mathbf{m} \approx (\mathbf{A}^*\mathbf{A})^{-1}\mathbf{A}^*\mathbf{d}$$

(least squares)

Mathematical Model

e.g. Achenbach (73), Landau & Lifshitz (86), Aki & Richards (02)

Conservation of momentum ($F = ma$):

$$\rho \frac{Dv_j}{Dt} = \rho f_j + \partial_i \sigma_{ij}$$

$$\frac{Da}{Dt} = \frac{\partial a}{\partial t} + v \cdot \nabla a$$

Hooke's Law (linearly elastic, isotropic material):

$$F = -kx$$

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij}$$

σ_{ij} stress tensor

$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$ strain tensor

Mathematical Model

Assumptions:

- long wavelength compared to amplitude
- linear elasticity
- smooth displacement
- constant density

Conservation of momentum ($F = ma$):

$$\rho \frac{Dv_j}{Dt} = \rho f_j + \partial_i \sigma_{ij}$$

Mathematical Model

Elastic Wave Equation:

$$\rho \frac{\partial^2 \mathbf{u_j}}{\partial \mathbf{t}^2} = (\lambda + \mu)\partial_{\mathbf{j}}\partial_{\mathbf{k}}\mathbf{u_k} + \mu \nabla^2 \mathbf{u_j}$$

Mathematical Model

Elastic Wave Equation:

$$\rho \frac{\partial^2 \mathbf{u_j}}{\partial \mathbf{t}^2} = (\lambda + \mu)\partial_{\mathbf{j}}\partial_{\mathbf{k}}\mathbf{u_k} + \mu \nabla^2 \mathbf{u_j}$$

Helmholtz decomposition: $\vec{\mathbf{u}} = \nabla\phi + \nabla \times \psi$

Mathematical Model

Elastic Wave Equation:

$$\rho \frac{\partial^2 u_j}{\partial t^2} = (\lambda + \mu)\partial_j \partial_k u_k + \mu \nabla^2 u_j$$

Helmholtz decomposition: $\vec{u} = \nabla\phi + \nabla \times \psi$

$$\partial_t^2 \phi = c_p^2 \nabla^2 \phi$$
$$\partial_t^2 \psi = c_s^2 \nabla^2 \psi$$

$$c_p = \sqrt{(\lambda + 2\mu)/\rho}$$
$$c_s = \sqrt{\mu/\rho}$$

Contrast formulation

Acoustic (really P-wave only) assumption ($\mathbf{u} = \nabla\phi$)

$$\nabla^2 \mathbf{u} - \frac{1}{c^2}\partial_t^2 \mathbf{u} = \mathbf{f}$$

$$\mathbf{u} = 0 \qquad t < 0$$

$$\partial_z \mathbf{u}|_{z=0} = 0$$

Contrast formulation

Acoustic (really P-wave only) assumption ($\mathbf{u} = \nabla \phi$)

$$\nabla^2 \mathbf{u} - \frac{1}{c^2} \partial_t^2 \mathbf{u} = f$$

Theorem (Lions 72)

Suppose that $\log \rho, \log c \in L^\infty(\Omega)$, $f \in L^2(\Omega \times \mathbb{R})$. Then weak solutions of the Dirichlet problem exist; initial data $\mathbf{u}(\cdot, 0) \in H_0^1(\Omega)$, $\partial_t \mathbf{u}(\cdot, 0) \in L^2(\Omega)$ uniquely determine them.

More info: Symes (09); elastic case: Stolk (00)

But we have discrete data and singular sources!

Contrast formulation

Acoustic (really P-wave only) assumption ($\mathbf{u} = \nabla\phi$)

$$\nabla^2 \mathbf{u} - \frac{1}{\mathbf{c}^2}\partial_t^2 \mathbf{u} = \mathbf{f}$$

Linearize: $\mathbf{c}(\mathbf{x}) = \mathbf{c}_0(\mathbf{x}) + \delta\mathbf{c}(\mathbf{x})$

$$\mathbf{L}\mathbf{u} = \mathbf{f}$$
$$\mathbf{L}_0\mathbf{u}_0 = \mathbf{f}$$

$\mathbf{L}_0$ **and** $\mathbf{u}_0$ **use** $\mathbf{c}_0(\mathbf{x})$

Contrast formulation

Acoustic (really P-wave only) assumption ($\mathbf{u} = \nabla\phi$)

$$\nabla^2 \mathbf{u} - \frac{1}{\mathbf{c}^2}\partial_t^2 \mathbf{u} = \mathbf{f}$$

Linearize: $\mathbf{c(x) = c_0(x) + \delta c(x)}$

$$\mathbf{Lu = f}$$
$$\mathbf{L_0 u_0 = f}$$

$\mathbf{L_0}$ and $\mathbf{u_0}$ use $\mathbf{c_0(x)}$
subtract

$$\mathbf{L_o}\delta\mathbf{u} = \delta\mathbf{L}\phi$$

Symes 09 and Stolk 00 give estimates on linearization error

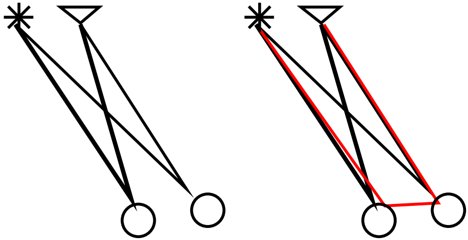
Contrast formulation

Born approximation

$$\mathbf{L_0 \delta u = \delta L u_0}$$

$$\nabla^2 \delta u - {\frac{1}{c_0}}^2 \partial_t^2 \delta u = \frac{2\delta c}{c_0^3} \partial_t^2 u_0$$

δu is called the scattered field



this will re-appear next week in the radar tutorial...

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- Comparison of methods

A Data Model

Born Approximation

$$L_0 \delta u = \delta L u_0$$

$$\nabla^2 \delta u - \frac{1}{c_0}^2 \partial_t^2 \delta u = \frac{2\delta c}{c_0^3} \partial_t^2 u_0$$

Given source $s(x, t) = \delta(x - s)\delta(t)$

$$\begin{aligned} u_0(x, t) &= \int_X \int_T G_0(x, t - t_0, x')s(x', t_0)dx'dt_0 \\ &= G_0(x, t, s) \end{aligned}$$

$\delta(x - s)$ is a good approximation on the scale of the wavelength

$\delta(t)$ is not; we assume (optimistically) that the source-time signature can be deconvolved

A Data Model

Given source $s(x,t) = \delta(x - s)\delta(t)$

$$
\begin{aligned}
u_0(x,t) &= \int_X \int_T G_0(x, t - t_0, x')s(x', t_0)dx'dt_0 \\
&= G_0(x,t,s)
\end{aligned}
$$

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x)V(x)\partial_t^2 G_0(x, t_0, s)dxdt_0$$

$$\delta G(s,r,\omega) = -\int_X \omega^2 G_0(r,\omega,x)V(x)G_0(x,\omega,s)dx$$

$$V(x) = \frac{2\delta c(x)}{c_0(x)^3}$$

A Data Model

$$\delta G(s, r, t) = \int_X \int_T G_0(r, t - t_0, x) V(x) \partial_t^2 G_0(x, t_0, s) dx dt_0$$

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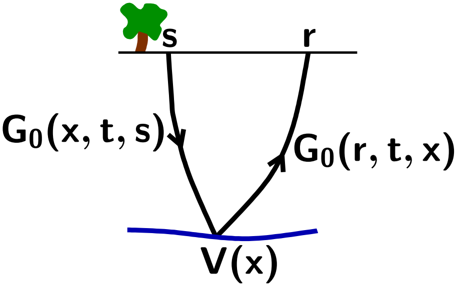

A Data Model

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) V(x) \partial_t^2 G_0(x, t_0, s)\, dx\, dt_0$$

$$\delta G(s,r,\omega) = -\int_X \omega^2 G_0(r,\omega,x) V(x) G_0(x,\omega,s)\, dx$$

forward map: $F_{c_0} : \delta c \mapsto \delta G$

- **data (s, r, t) – 5 dimensions**
- **model x – 3 dimensions**
- **redundancy is used to find $c_0(x)$**

linearization is most accurate when c_0 is smooth and δc rough or oscillatory (all relative to the wavelength)

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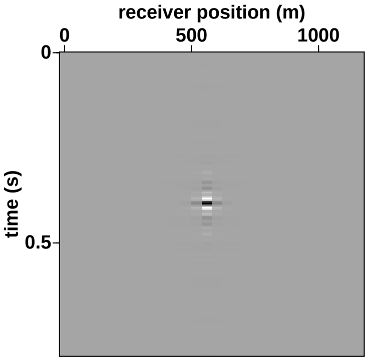
Approximate Techniques

- **Kirchhoff**
  - Integral technique
  - Related to X-ray CT imaging
  - Generalized Radon Transform
  - Conventionally uses ray theory
- **One-way**
  - Based on a paraxial approximation
  - Usually computed with finite differences

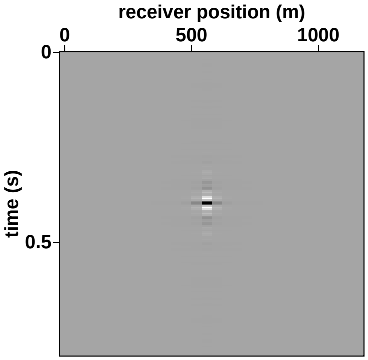
‘Exact’ Techniques

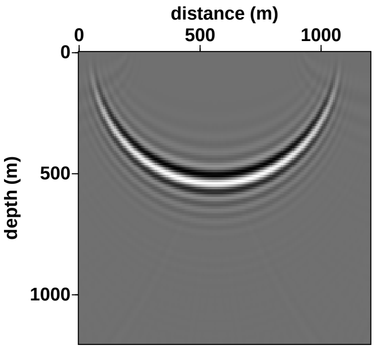
- Reverse-time migration (RTM)
  - Run wave-equation backward
  - Usually computed with finite differences
  - “No” approximations (to the acoustic, isotropic, linearized wave-equation, for smooth media assuming single scattering)
- Full-waveform inversion (FWI)
  - Iterative method to match the entire waveform
  - Gives smooth part of velocity model

Kirchhoff Migration
Physical Motivation 1



Kirchhoff Migration
Physical Motivation 1





Kirchhoff Migration
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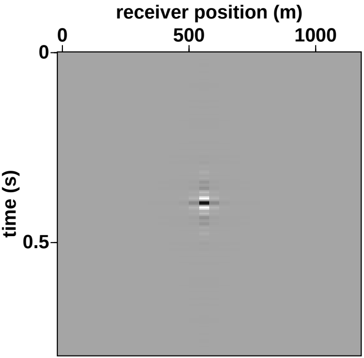

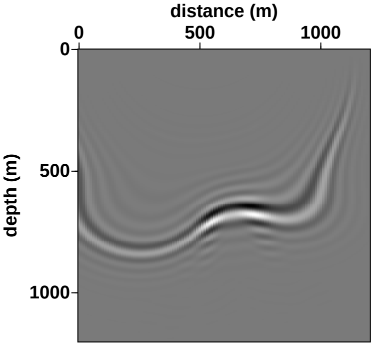

Kirchhoff Migration
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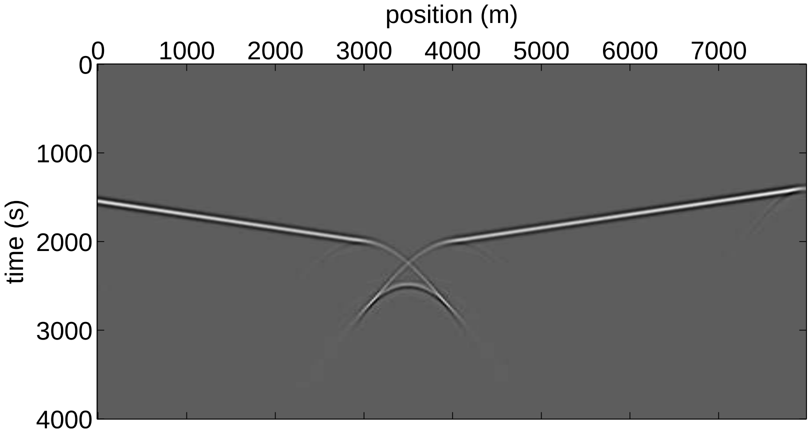
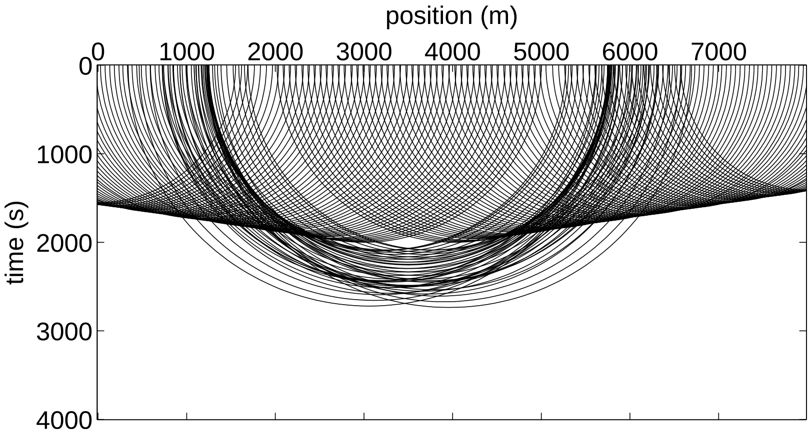
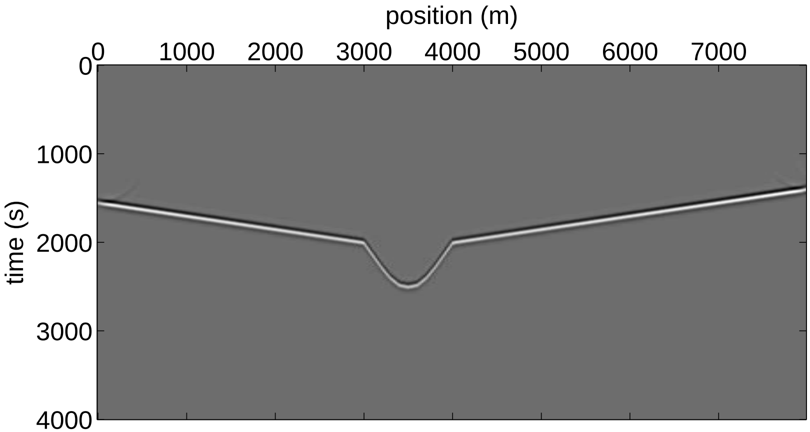


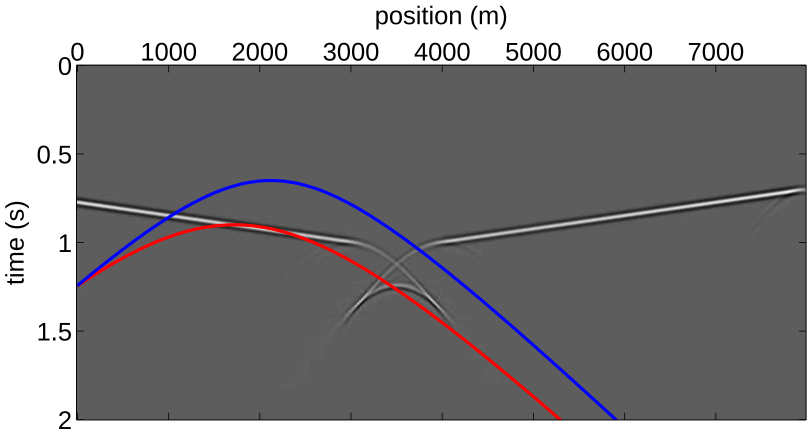
Kirchhoff Migration
Physical Motivation 1



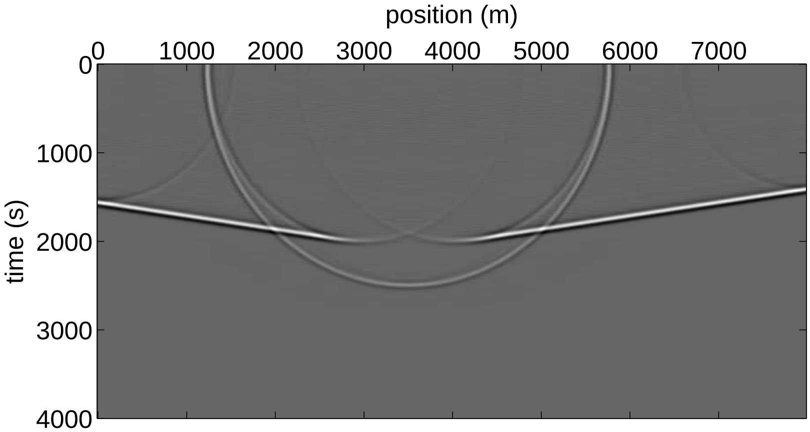
Kirchhoff Migration
Physical Motivation 1



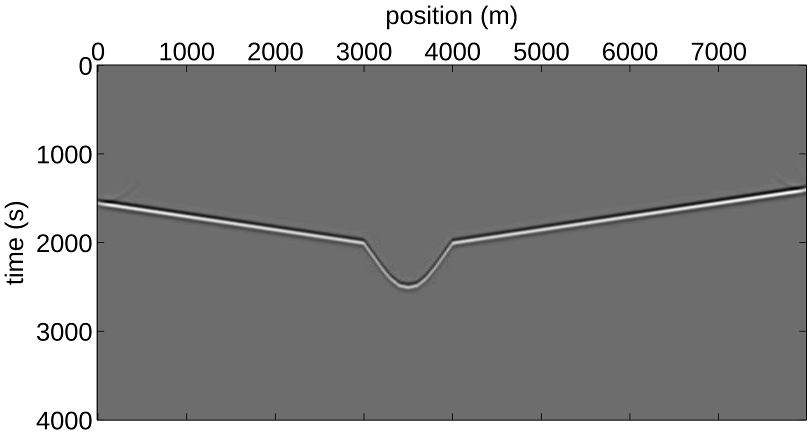
Kirchhoff Migration
Physical Motivation 2



Kirchhoff Migration
Physical Motivation 2



Kirchhoff Migration
Physical Motivation 2



Kirchhoff Migration
WKBJ Approximation

Assume solution form:

$$G_0(x,t) = e^{i\omega\psi(x,t)} \sum_k \frac{A_k(x,t)}{(i\omega)^k}$$

- **A_k, and ψ smooth**
  (when this is convergent is a complicated question)
- **$e^{i\omega\psi(x,t)}$ oscillatory**
- **remove frequency dependence**

Developed by Wentzel, Kramers, Brillouin, independently in 1926 and by Jeffreys in 1923.

Kirchhoff Migration
WKBJ Approximation

Assume solution form:

$$G_0(x,t) = e^{i\omega\psi(x,t)} \sum_k \frac{A_k(x,t)}{(i\omega)^k}$$

Apply Helmholtz-equation $\nabla^2 G_0 + \frac{\omega^2}{c_0(x)^2} G_0 = 0$

Eikonal equation:

$$(\nabla\psi)^2 = \frac{1}{c(x)^2}$$

Transport equations:

$$2\nabla\psi \cdot A_k + A_k \nabla^2\psi = 0$$

Kirchhoff Migration
WKBJ Approximation

Assume solution form:

$$G_0(x,t) = e^{i\omega\psi(x,t)} \sum_k \frac{A_k(x,t)}{(i\omega)^k}$$

Eikonal equation:

$$(\nabla\psi)^2 = \frac{1}{c(x)^2}$$

Transport equations:

$$2\nabla\psi \cdot A_k + A_k \nabla^2 \psi = 0$$

Nonlinear!

Solve with method of characteristics $\Rightarrow$ ray-tracing.

Kirchhoff Migration
WKBJ Modeling

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) \frac{2\delta c(x)}{c_0(x)^2} \partial_t^2 G_0(x, t_0, s) dx dt_0$$

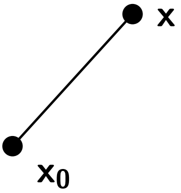
$$G_0(x, t_0, s) = \int A(x, s, \omega) e^{i\omega\psi(x, t_0, s)} d\omega$$

Kirchhoff Migration
WKBJ Modeling

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) \frac{2\delta c(x)}{c_0(x)^2} \partial_t^2 G_0(x, t_0, s) dx dt_0$$

$$G_0(x, t_0, s) = \int A(x, s, \omega) e^{i\omega\psi(x,t_0,s)} d\omega$$

- $c_0(x)$ constant $\psi(r,x) = t - \frac{|x-r|}{c}$

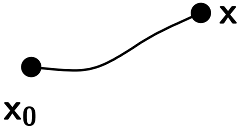


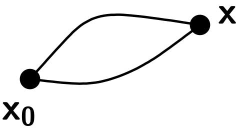
Kirchhoff Migration
WKBJ Modeling

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t-t_0, x) \frac{2\delta c(x)}{c_0(x)^2} \partial_t^2 G_0(x, t_0, s)\, dx\, dt_0$$

$$G_0(x, t_0, s) = \int A(x, s, \omega) e^{i\omega\psi(x,t_0,s)} d\omega$$

- $c_0(x)$ constant $\psi(r,x) = t - \frac{|x-r|}{c}$
- $c_0(x)$ no caustics $\psi(r,x) = t - T(r,x)$





Kirchhoff Migration
WKBJ Modeling

$$\delta G(s,r,t) = \int_X \int_T G_0(r, t - t_0, x) \frac{2\delta c(x)}{c_0(x)^2} \partial_t^2 G_0(x, t_0, s)\, dx\, dt_0$$

$$G(x, t_0, s) \approx \int A(x, s, \omega) e^{i\omega\psi(x,t_0,s)} d\omega$$

$$\delta G(s,r,t) = \int_X \int_{\mathbb{R}} \omega^2 \overbrace{A(x,s,\omega)A(r,x,\omega)\frac{2\delta c(x)}{c_0(x)^2}}^{B(x,r,s,\omega)}$$

$$e^{i\omega(t - T(x,r) - T(x,s))} dx\, d\omega$$

Kirchhoff Migration
WKBJ Modeling Formula

$$\delta G(s, r, t) = \int_X \int_{\mathbb{R}} \omega^2 B(x, r, s, \omega) e^{i\omega(t - T(x,r) - T(x,s))} dx d\omega$$

$$S_\psi = \{(x, s, r, t, \omega) | t = T(x, r) + T(s, x)\}$$

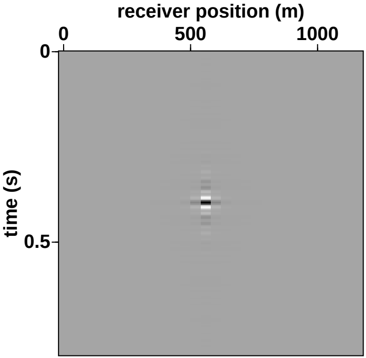
Assume B independent of ω

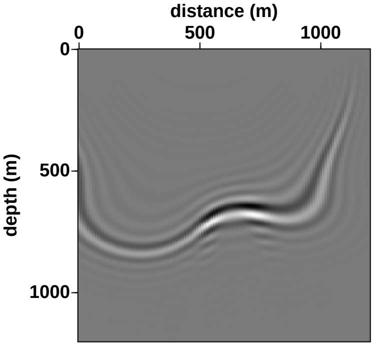
$$\delta G(s, r, t) = \int_X \int_{\mathbb{R}} \omega^2 B(x, r, s) \delta''(t - T(x, r) - T(x, s)) dx$$

This is a Generalized Radon Transform

Kirchhoff Migration
WKBJ Modeling Formula

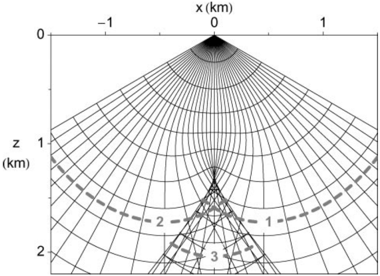
$$\delta \mathbf{G}(\mathbf{s},\mathbf{r},t) = \int_{\mathbf{X}}\int_{\mathbb{R}} \omega^2 B(\mathbf{x},\mathbf{r},\mathbf{s})\,\delta''(t - T(\mathbf{x},\mathbf{r}) - T(\mathbf{x},\mathbf{s}))\,d\mathbf{x}$$





Kirchhoff Migration
WKBJ Modeling

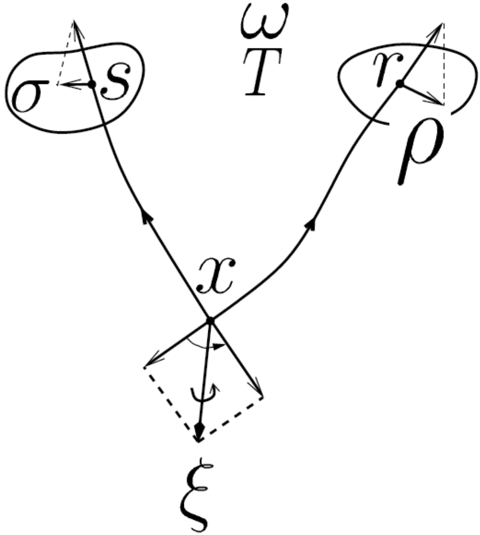
Stolk & Symes (2004)



$$(\nabla\psi)^2 = \frac{1}{c(x)^2}$$

does not have a unique solution

Kirchhoff Migration
WKBJ Modeling



Kirchhoff Migration
WKBJ Modeling

$$G(x, t_0, s) \approx \int A(x, s, \omega) e^{i\omega\psi(x,t_0,s)} d\omega$$

$$\Downarrow$$

$$G(x, t_0, s) \approx \int A(x, s, \theta) e^{i\psi(x,t_0,s,\theta)} d\theta$$

$\theta \in \mathbb{R}^{2n-1}$ (n spatial dimension)
ψ homogeneous in θ

$$S_\psi = \{(x, s, r, t, \theta) | \nabla_\theta \psi = 0\}$$

Kirchhoff Migration
WKBJ Modeling

$$\delta G(s, r, t) = \int_X \int_{\mathbb{R}} \omega^2 B(x, r, s, \omega) e^{i\omega(t - T(x,r) - T(x,s))} dx d\omega$$

$$\delta G(s, r, t) = \int_X \int_{\mathbb{R}} \omega^2 B(x, r, s, \theta) e^{i\psi(x,t_0,r,s,\theta)} dx d\theta$$

$$F : \delta c \longrightarrow \delta G$$

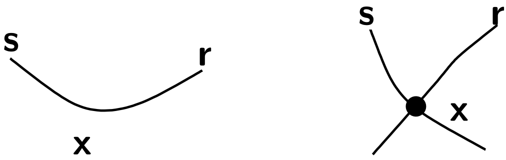
Kirchhoff Migration
WKBJ Modeling

$$\delta G(s, r, t) = \int_x \int_{\mathbb{R}} \omega^2 B(x, r, s, \theta) e^{i\psi(x, t_0, r, s, \theta)} dx d\theta$$

$$F : \delta c \longrightarrow \delta G$$

F is an FIO if: **(Beylkin 85, Rakesh 88)**

- **two rays intersect transversally**



- **no rays transversal to surface**

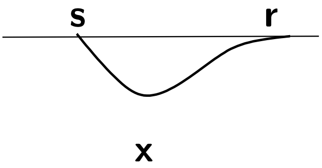
Kirchhoff Migration
WKBJ Modeling

$$\delta G(s, r, t) = \int_X \int_{\mathbb{R}} \omega^2 B(x, r, s, \theta) e^{i\psi(x,t_0,r,s,\theta)} dx d\theta$$

$$F : \delta c \longrightarrow \delta G$$

F is an FIO if: (Beylkin 85, Rakesh 88)

- two rays intersect transversally
- no rays transversal to surface



Kirchhoff Migration
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$$F : \delta c \longrightarrow \delta G$$

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Assuming only single scattering (validity of Born approximation) F models the data.

Kirchhoff Migration
WKBJ Modeling

$$\delta G(s, r, t) = \int_X \int_{\mathbb{R}} \omega^2 B(x, r, s, \theta) e^{i\psi(x, t_0, r, s, \theta)} dx d\theta$$

Recall:

$$B(x, r, s, \theta) = A(x, s, \theta) A(r, x, \theta) \frac{2\delta c(x)}{c_0(x)^2}$$

Remember from Tanya:

$$\text{singsupp}(F_{c_0}\delta c) \subset S_\phi \circ \text{singsupp}(\delta c)$$

F maps singluarities in δc along bicharacteristics to singularities in δG

Hörmander (85), chapter 21

Kirchhoff Migration

Goal: Locate the singularities of δc from δG

Requires F^{-1}

Recall: data are redundant

Least Squares: $F_{LS}^{-1} = (F^*F)^{-1}F^*$

$$F^*[\delta G](x) = \int_R \int_S \int_{\mathbb{R}^{2n-1}} \omega^2 \overline{B(x, r, s, \theta)} e^{-i\psi(x,t_0,s,r,\theta)} d\theta ds dr$$

Kirchhoff Migration

$$F^*[\delta G](x) = \int_R \int_S \int_{\mathbb{R}^{2n-1}} \omega^2 \overline{B(x, r, s, \theta)} e^{-i\psi(x,t_0,s,r,\theta)} d\theta ds dr$$

- F^* also an FIO
- F^*F usually ψDO

  (Beylkin (85), Rakesh (88), Symes (95))

$$\widehat{\delta c(x)} := F^*[\delta G](x)$$

$$WF((F^*F)^{-1}\widehat{\delta c(x)}) \subset WF(\overline{\delta c(x)})$$

F^* correctly positions singularities

Kirchhoff Migration
When F^*F is not ΨDO

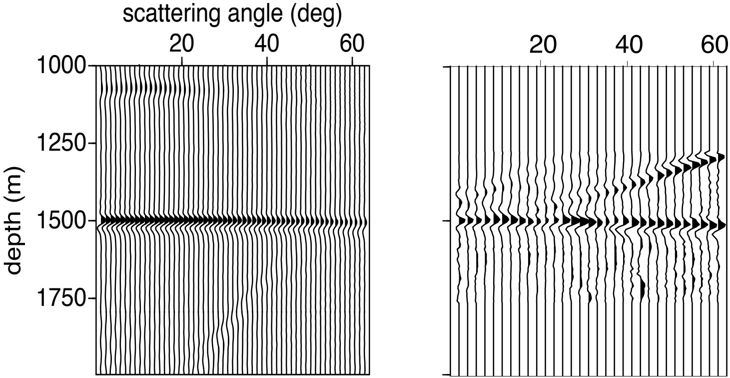
For F^*F to be ΨDO ten Kroode et al. (98)

- **complete data coverage (s, r form an open 4D manifold)**
- **traveltime injectivity condition ((s, σ, r, ρ, t) determine x uniquely)**

When F^*F not ΨDO there will be artifacts

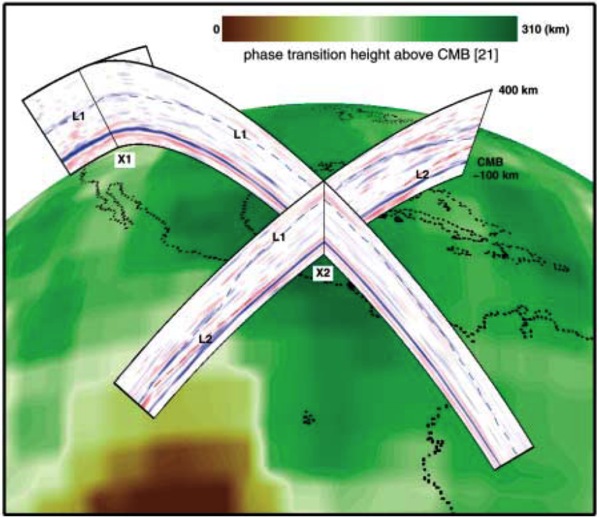
More detail: Stolk (00a), Symes (09), Nolan & Symes (97), de Hoop et al. (03), Stolk & Symes (04)

Kirchhoff Migration
Artifact Example



A Deep-Earth Example

van der Hilst et al (2007)



These waves travelled at least 6000 km
(most much more)!!

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