Abstract

There are many features in the Earth’s crust that involve a jump in physical property across a sharp boundary, for example, an ore deposit in host rocks. Such well-defined boundaries are often of interest to geophysicists, however traditional minimum structure inversion methods produce blurred images of the sub-surface. In this paper we explore the application of a level set inversion method to recover a sharp boundary between two slowness values, one characterizing an inclusion, e.g. an ore body, the other characterizing a background, e.g. host rock, from first arrival travel times. The scenario considered is that of cross-borehole tomography in two dimensions.

Problem formulation

- **Forward problem**: Given known sources and receivers, compute the first arrival travel times from sources to receivers.
- **Inverse problem**: Given observed first arrival times from sources to receivers, and slowness values for the inclusion and background derived from sonic logs, a level set function $\phi$ that minimizes the objective function:

  $F(M) = \| R_{data} - R_{model} \|_2$, where:

  - $R_{data}$ is the observed travel times from the data set.
  - $R_{model}$ is the travel times computed with the level set model.

- If $\phi$ is a smooth surface, $\nabla \phi$ is the surface normal, the model $M$ can be expressed as:

  $M(x) = \begin{cases} 
  \text{inclusion} & \phi > 0 \\
  \text{background} & \phi < 0 
  \end{cases}

- **Regularization**: A regularization term is added to the objective function to control the complexity of the solution:

  $\frac{\lambda}{2} \int \phi^2 dx$, where $\lambda$ is a regularization parameter.

Motivation

- Geophysical inversions discretize the Earth into many cells and seek smoothly varying models by minimizing the L2 norm of the gradient.
- In contrast, geologists’ interpretations of seismic data involve contacts between distinct rock units. There are benefits to performing fundamentally different inversion algorithms that seek the interface between proposed rock units.
- Possible application: more precise delineation of reservoirs suitable for reservoir extraction and mine planning after the initial drilling and staking.

Level set method

- **Discretization of the surface**: A surface is represented by a level set function $\phi(x)$.
- **Level sets**: The level sets of $\phi$ are planes defined by $\phi = c$.
- The interface $\Gamma$ is defined as the level set $\phi = 0$.
- **Hyperbolic instabilities**: Geometric information on the boundary of the phase is preserved.
- **Flexibility**: Level sets can be easily modified and animated.

Level set Parametrization

- **An interface (a contact)** is parameterized as the 0-level set of a Lipschitz continuous function $\phi$.
- The model values on an underlying mesh are determined by the level set function $\phi$ as follows:

  $\phi < 0 \quad \Rightarrow \quad \text{inclusion}$

  $\phi = 0 \quad \Rightarrow \quad \text{interface}$

  $\phi > 0 \quad \Rightarrow \quad \text{background}$

- The slowness model can be represented as $\phi(x) = \phi(x) + \phi(x) \cdot \phi(x) + \phi(x) \cdot \phi(x) + \phi(x) \cdot \phi(x)$.

- **Level set method**: The level set method blends topology changes (images, separation) without adding algorithms completely.

- **Derivatives calculation**: Sensitivity studies with respect to the errors in the slowness values, and uncertainty analysis of the recovered inclusion shapes are necessary to determine our confidence in the performance of the level set method.

References and further reading

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Examples with synthetic data: slow inclusion

- **Slow inclusion**: 4.5 km/s, fast background: 6.3 km/s

Examples with synthetic data: fast inclusion

- **Fast inclusion**: 6.3 km/s, slow background: 4.5 km/s

Reconstruction from a real data set

- **True model**: Data, 1% noise
- **Solution from final model**: Difference

Conclusions

- The method is rather insensitive with respect to the starting inclusion shape.
- Sensitivity studies with respect to the errors in the slowness values, and uncertainty analysis of the recovered inclusion shapes are necessary to determine our confidence in the performance of the level set method.

Acknowledgements

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