# Constructing piecewise-constant models in multi-dimensional minimum-structure inversions

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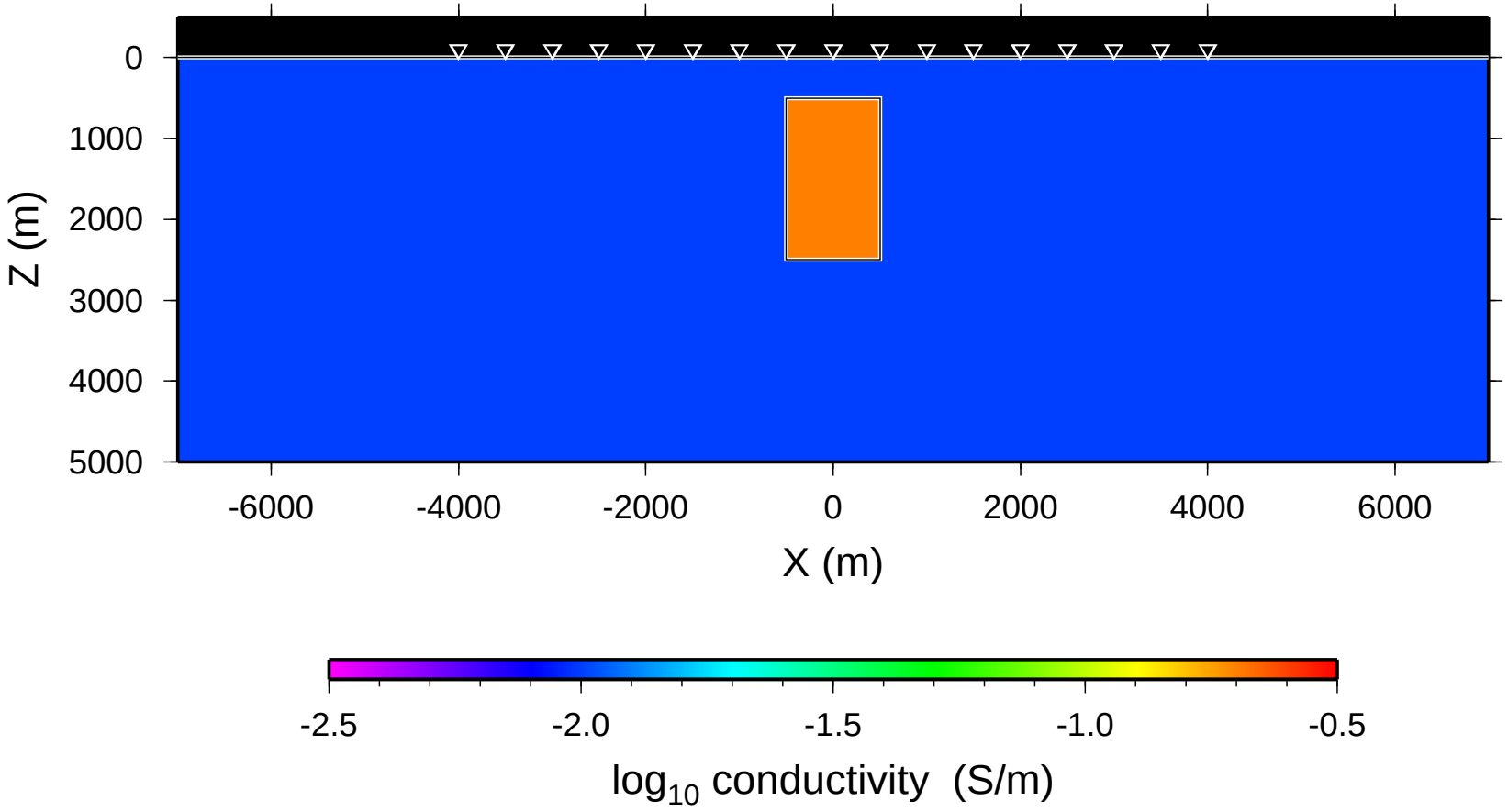
# Motivation and Conclusions

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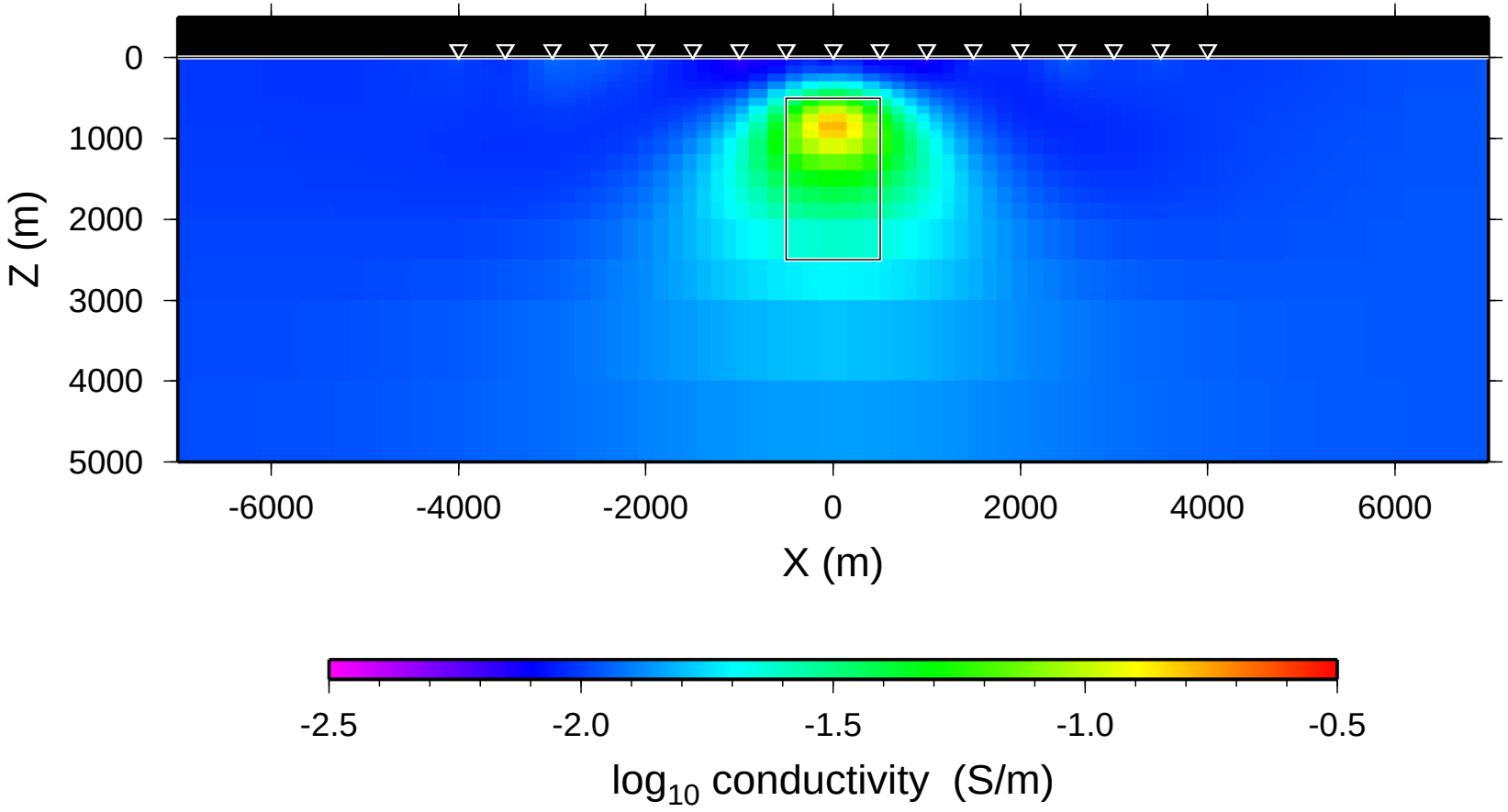


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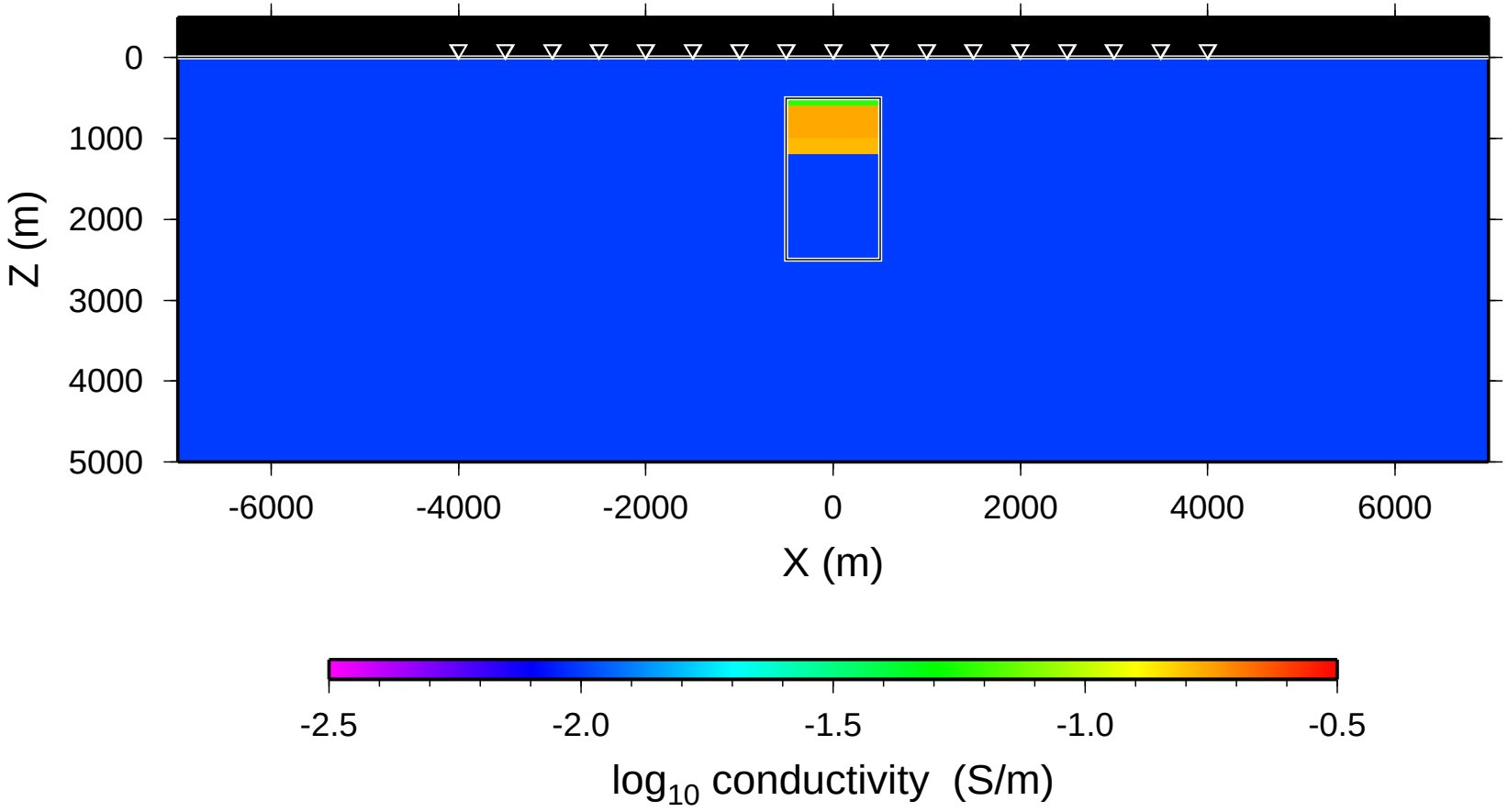


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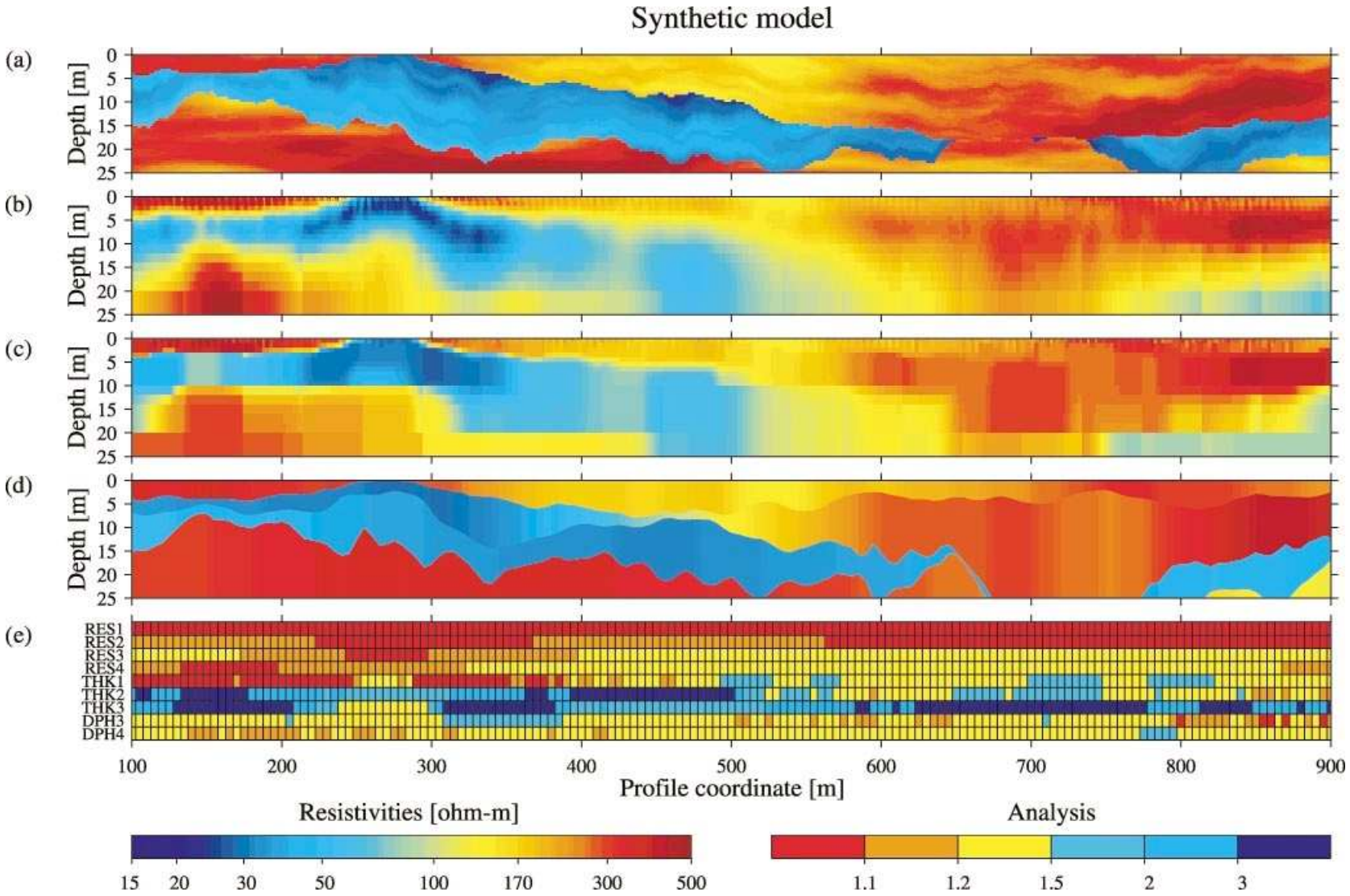
Auken & Christiansen (2004, Geophysics, 69, p752–761):

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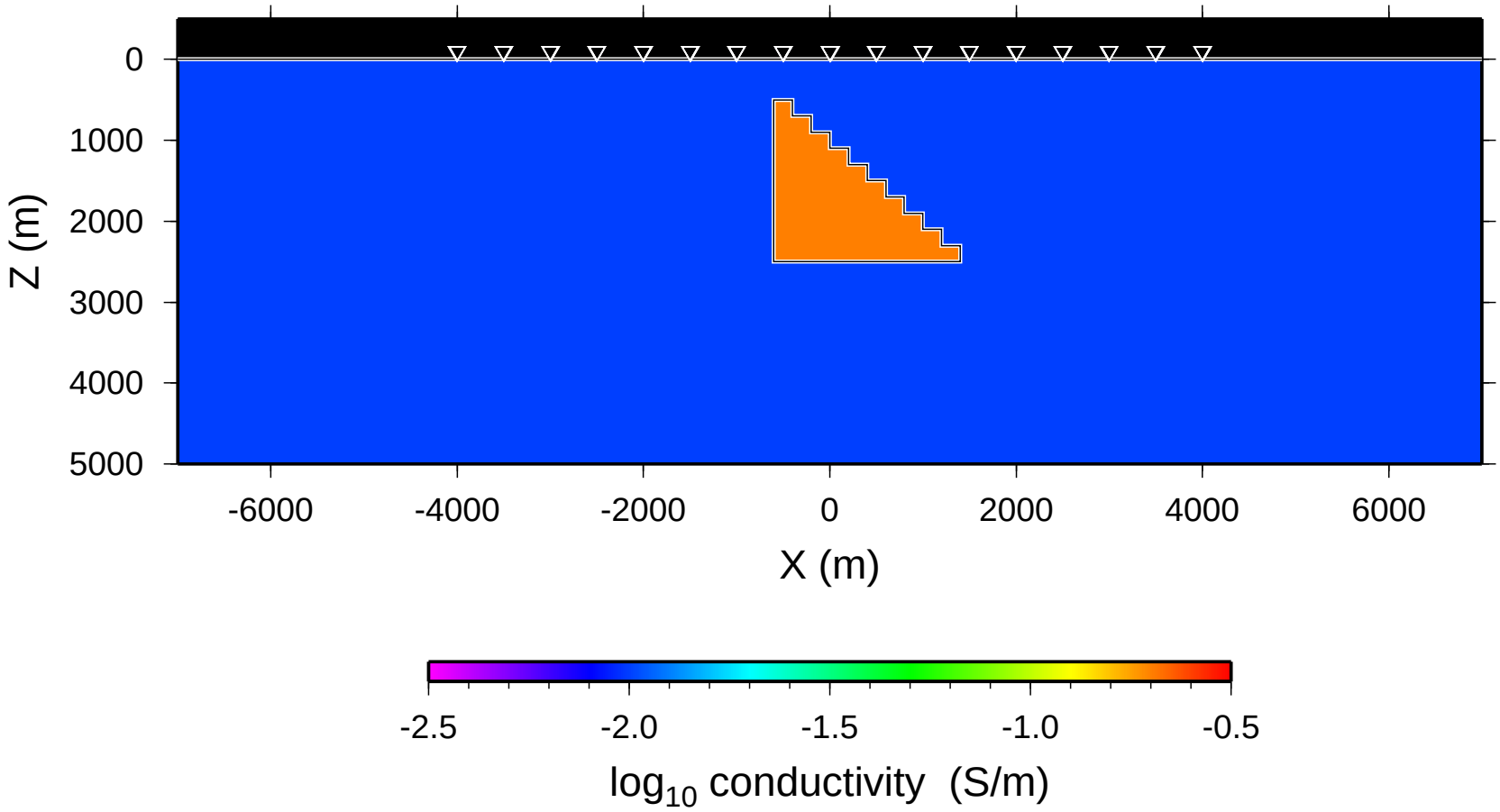


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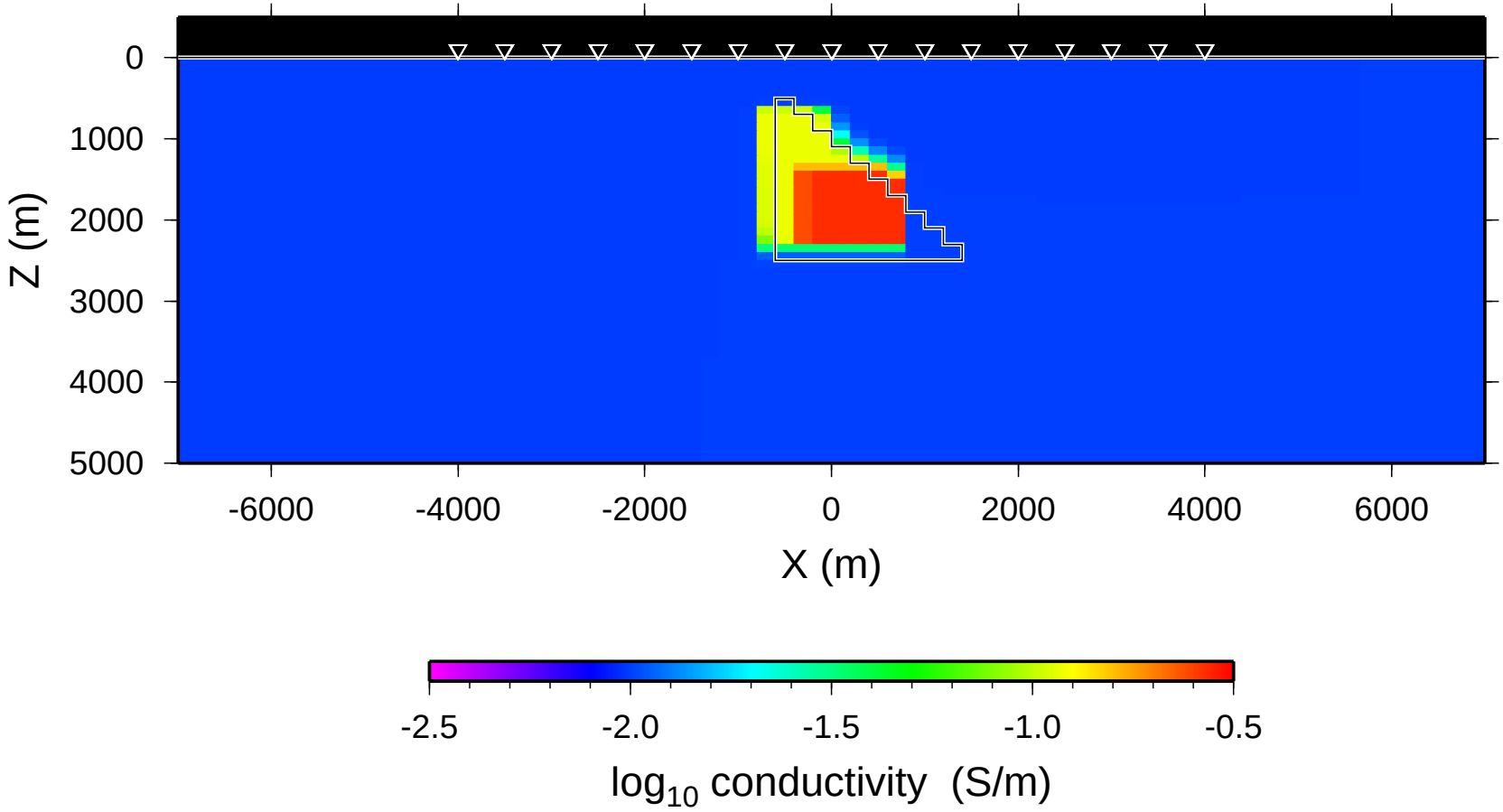


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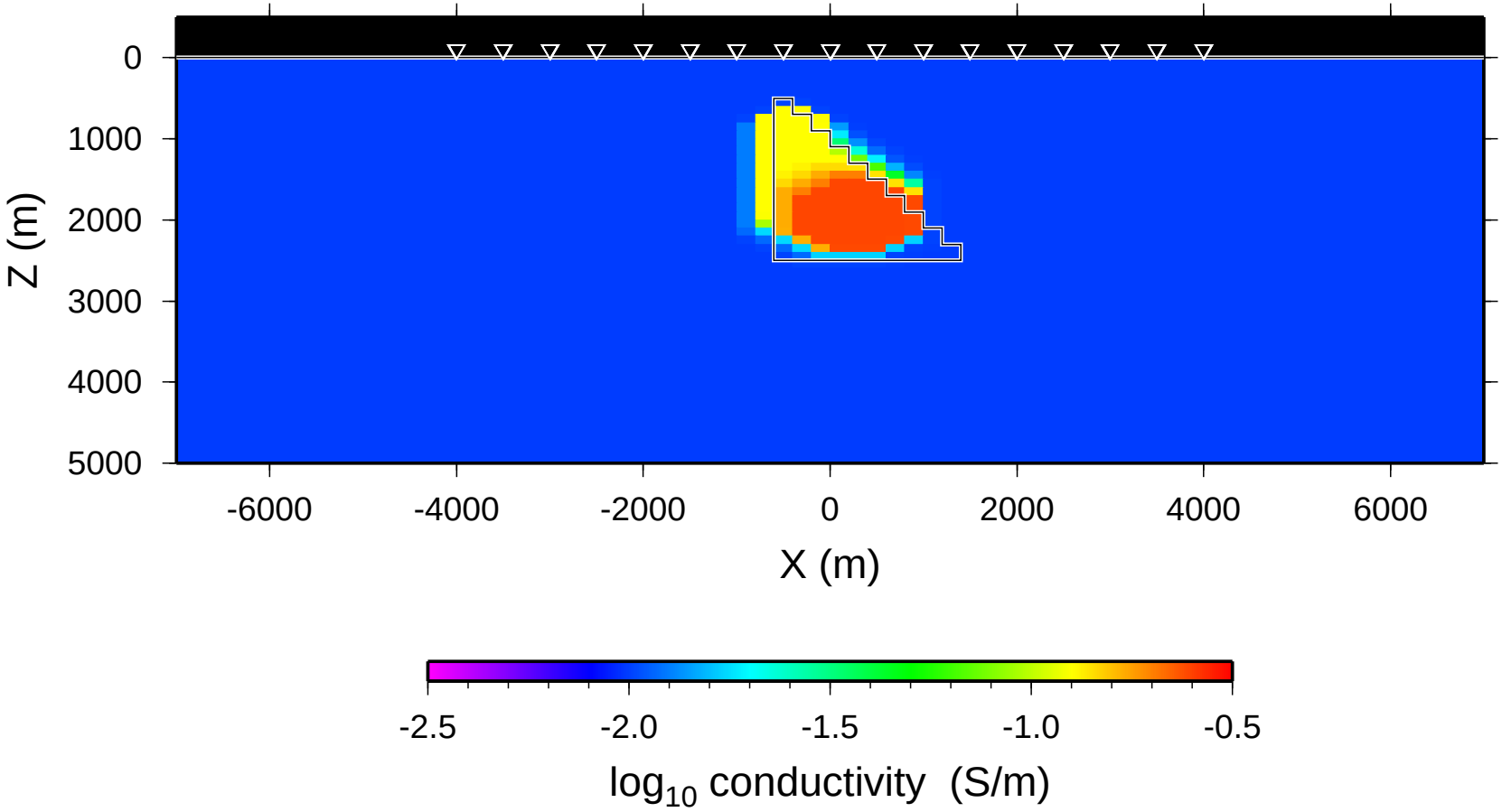


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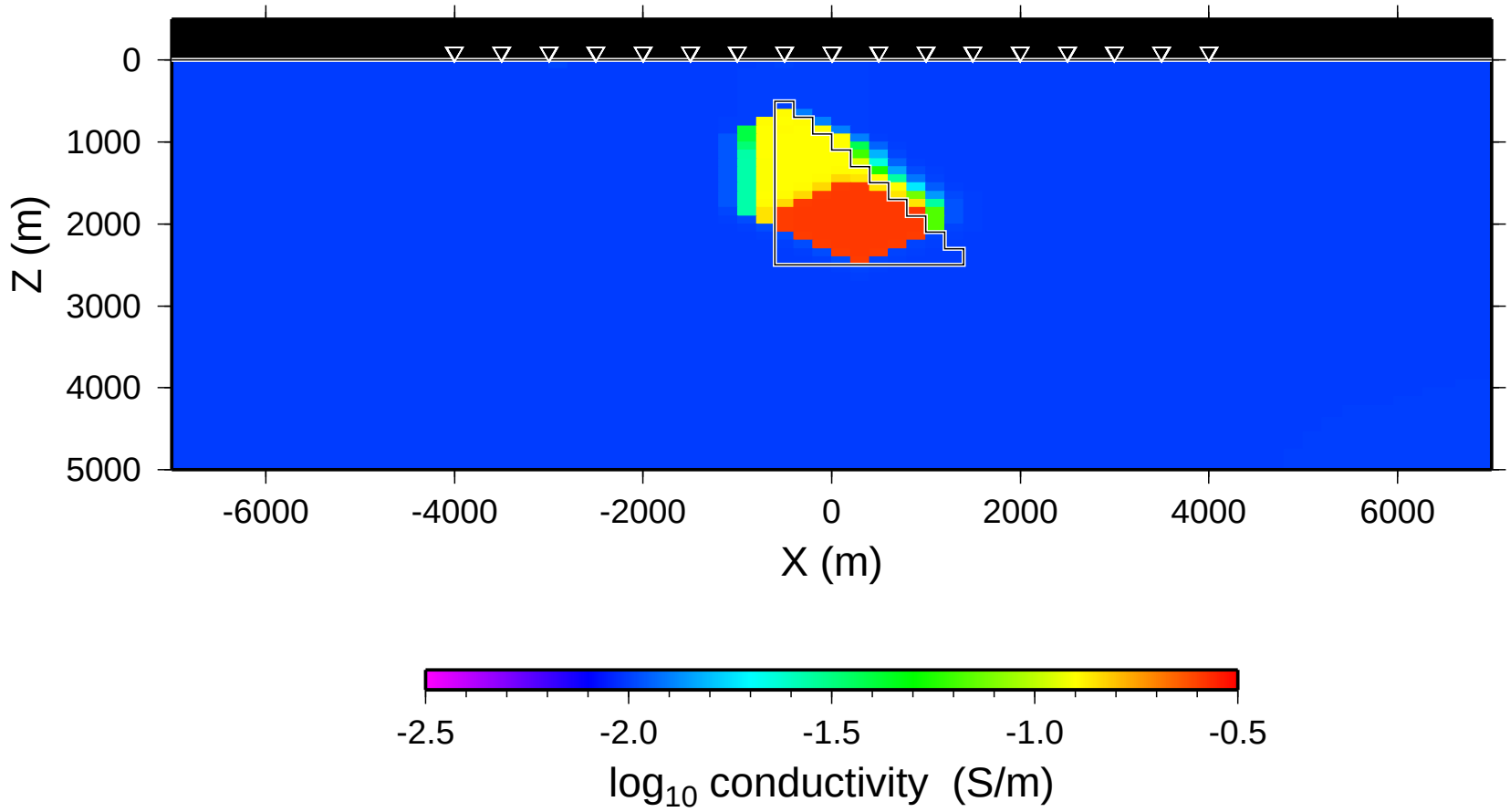


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# Outline

- Motivation and Conclusions.
- Previous work.
- General minimum-structure inversion strategy.

  General measures.

  Iterative solution procedure.

  Measure of model structure.
- Example.
- Conclusions.

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# Previous work

"True" minimum-structure algorithms:

Farquharson & Oldenburg (1998, GJI), 1-D EM;
Portniaguine & Zhdanov (1999, Geophysics), 3-D focusing;
Loke, Acworth & Dahlin (2003, Expl. Geop.), 2-D resistivity;
Farquharson & Oldenburg (2003, SEGJ), 2-D resistivity.

Laterally constrained layered inversions:

Smith et al. (1999, Geophysics), 2-D MT;
Auken & Christiansen (2004, Geophysics), 2-D resistivity;
de Groot-Hedlin & Constable (2004, Geophysics), 2-D MT.

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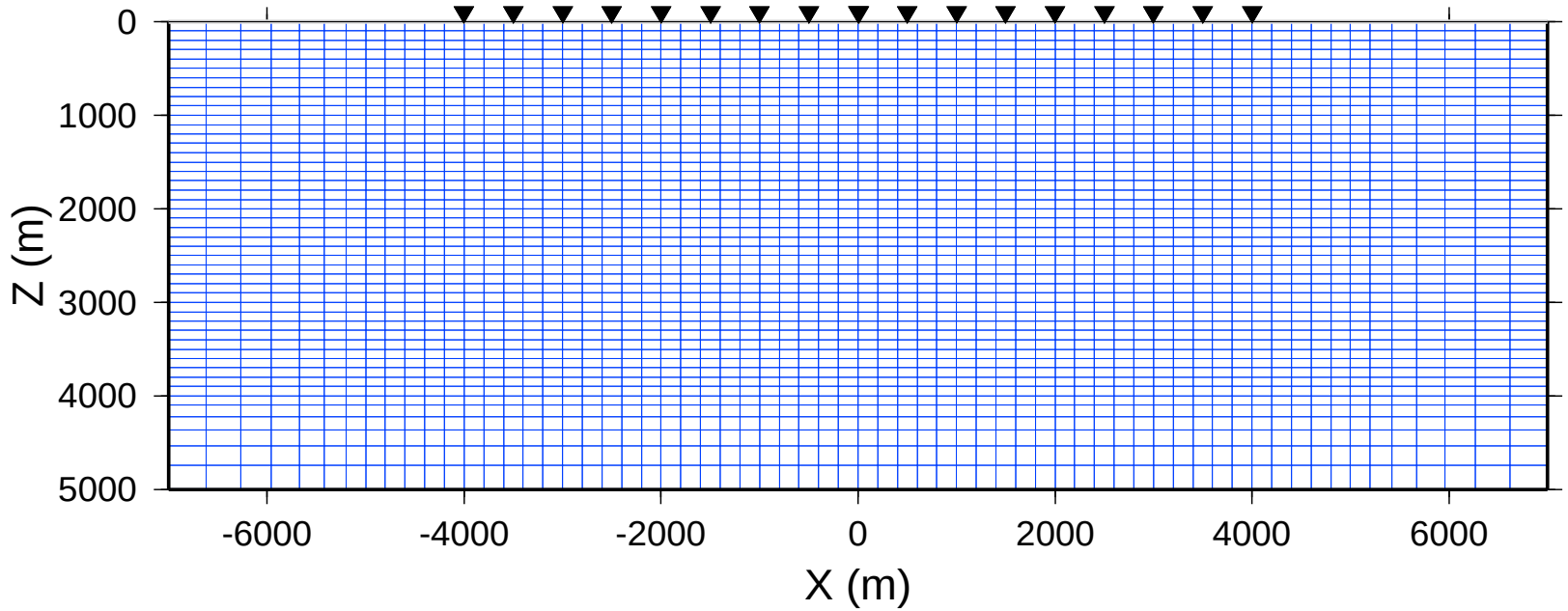
# General minimum-structure inversion strategy

- Mesh fixed during inversion; fine discretization.



# General minimum-structure inversion strategy

- Minimize objective function:

$$\Phi = \phi_d \; + \; \beta \, \phi_m,$$

where $\phi_d$ is measure of data-misfit,

$$\phi_d = \phi_d(\mathbf{u}) \qquad \mathbf{u} = \mathbf{W}_d\big(\mathbf{d}^{\text{obs}} - \mathbf{d}^{\text{prd}}\big),$$

and $\phi_m$ is measure of structure in model,

$$\phi_m = \sum_k \alpha_k \phi_k(\mathbf{v}_k) \qquad \mathbf{v}_k = \mathbf{W}_k\big(\mathbf{m} - \mathbf{m}_k^{\text{ref}}\big).$$

# General measures

- A general form for $\phi_d$ and $\phi_m$ is:

$$\phi(\mathbf{x}) = \sum_{j=1}^{N} \rho(x_j).$$

For example, the $l_2$-norm: $\rho(x) = x^2$;

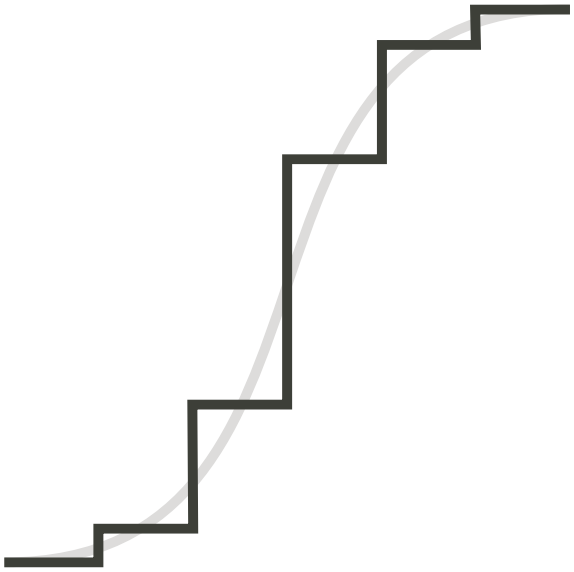
the $l_p$-norm: $\rho(x) = |x|^p$;

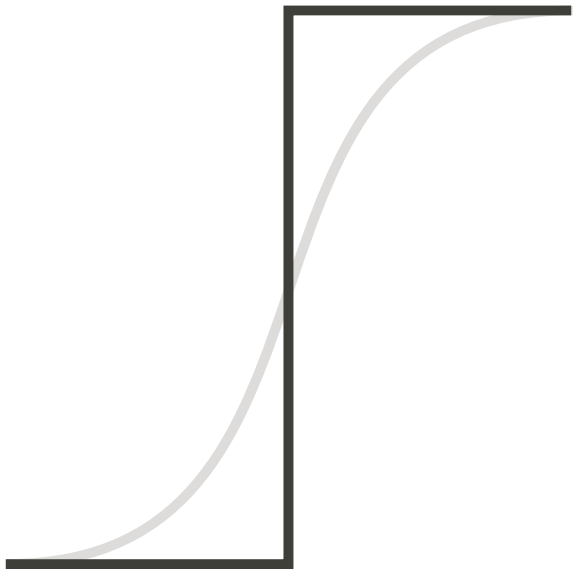
Ekblom's $l_p$-like measure: $\rho(x) = \left(x^2 + \epsilon^2\right)^{p/2}$;

Huber's $M$-measure: $\rho(x) = \begin{cases} x^2 & |x| \leq c, \\ 2c|x| - c^2 & |x| > c. \end{cases}$

# General measures

12





1 2 6 2 1 = 12

1 4 36 4 1 = 46

0 0 12 0 0 = 12

0 0 144 0 0 = 144

# Iterative solution procedure

- Differentiate $\Phi$ with respect to model parameters and equate to zero.

  Get normal system of equations:

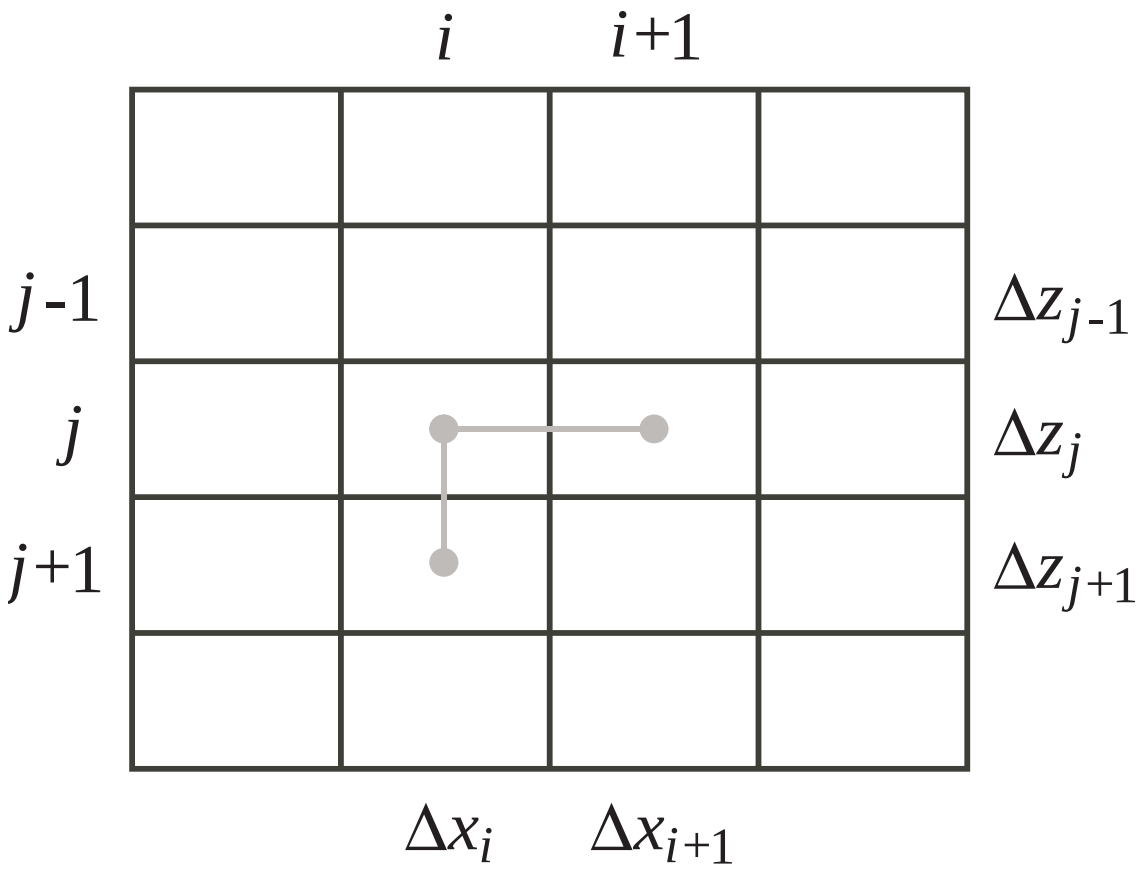
$$\left[\mathbf{J}^T\mathbf{W}_d^T\mathbf{R}_d\mathbf{W}_d\mathbf{J} \;+\; \beta^n \sum_k \alpha_k \mathbf{W}_k^T\mathbf{R}_k\mathbf{W}_k\right]\delta\mathbf{m}$$
$$= \mathbf{J}^T\mathbf{W}_d^T\mathbf{R}_d\mathbf{W}_d\left(\mathbf{d}^{\text{obs}} - \mathbf{d}^{n-1}\right) +$$
$$\beta^n \sum_k \alpha_k \mathbf{W}_k^T\mathbf{R}_k\mathbf{W}_k\left(\mathbf{m}_k^{\text{ref}} - \mathbf{m}^{n-1}\right).$$

  Update $\mathbf{R}_d$ and $\mathbf{R}_k$ as well as $\mathbf{J}$.

# Measure of model structure

- Regularization via finite-difference matrices.

  Old way:



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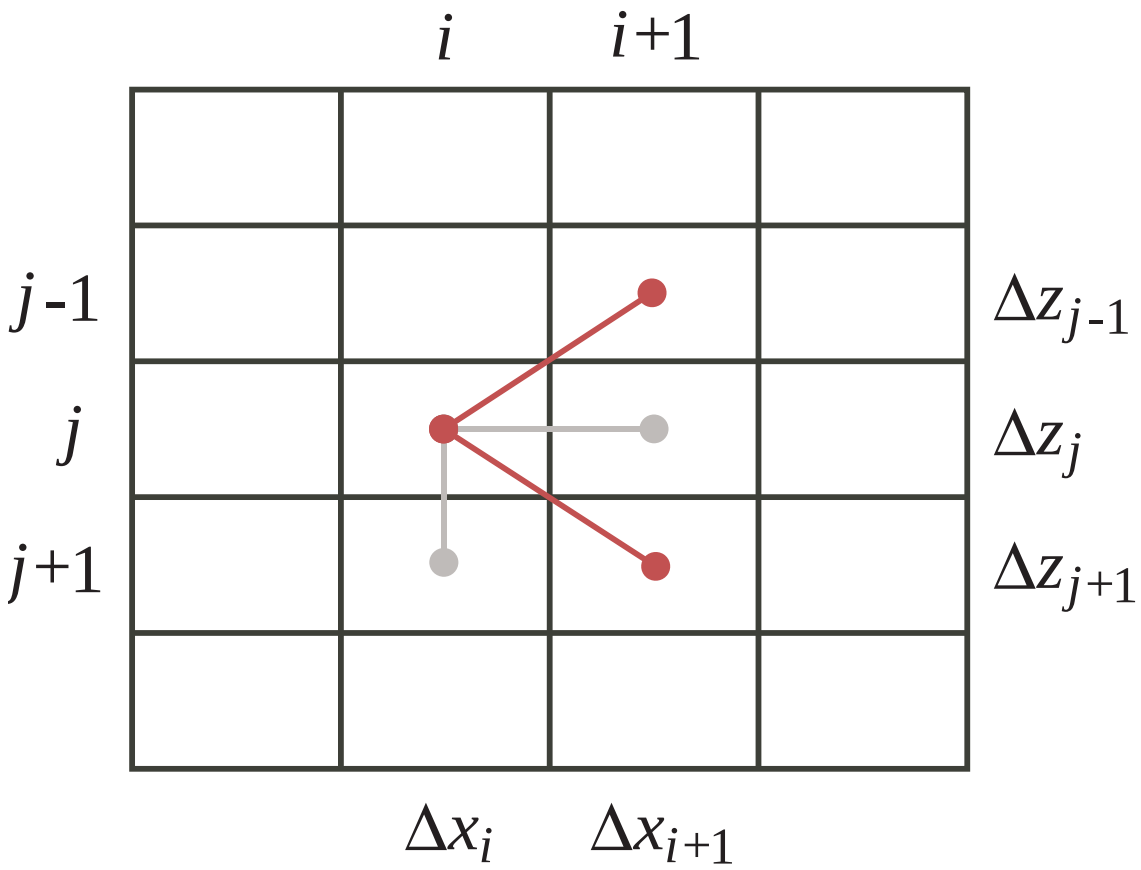

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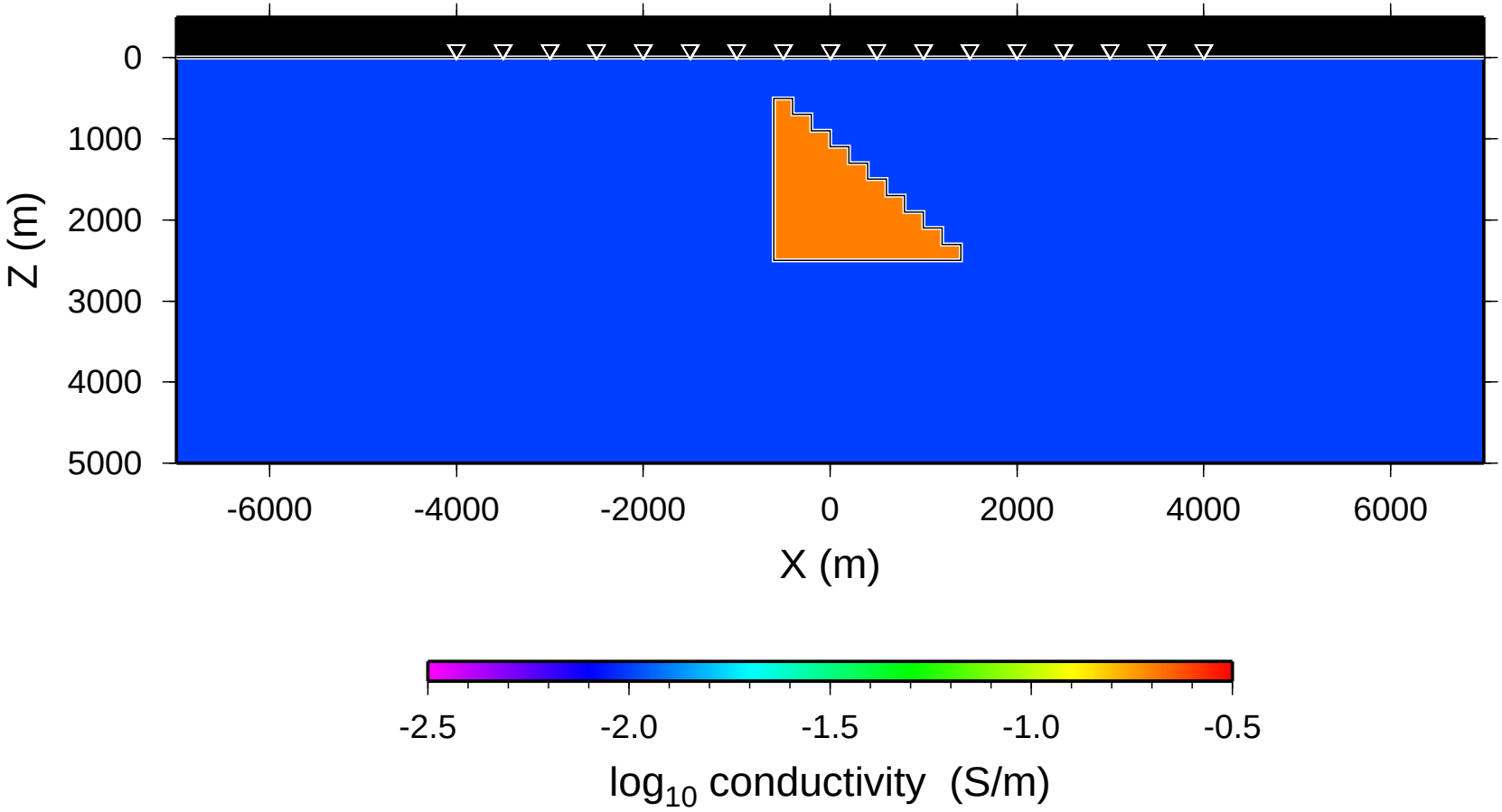


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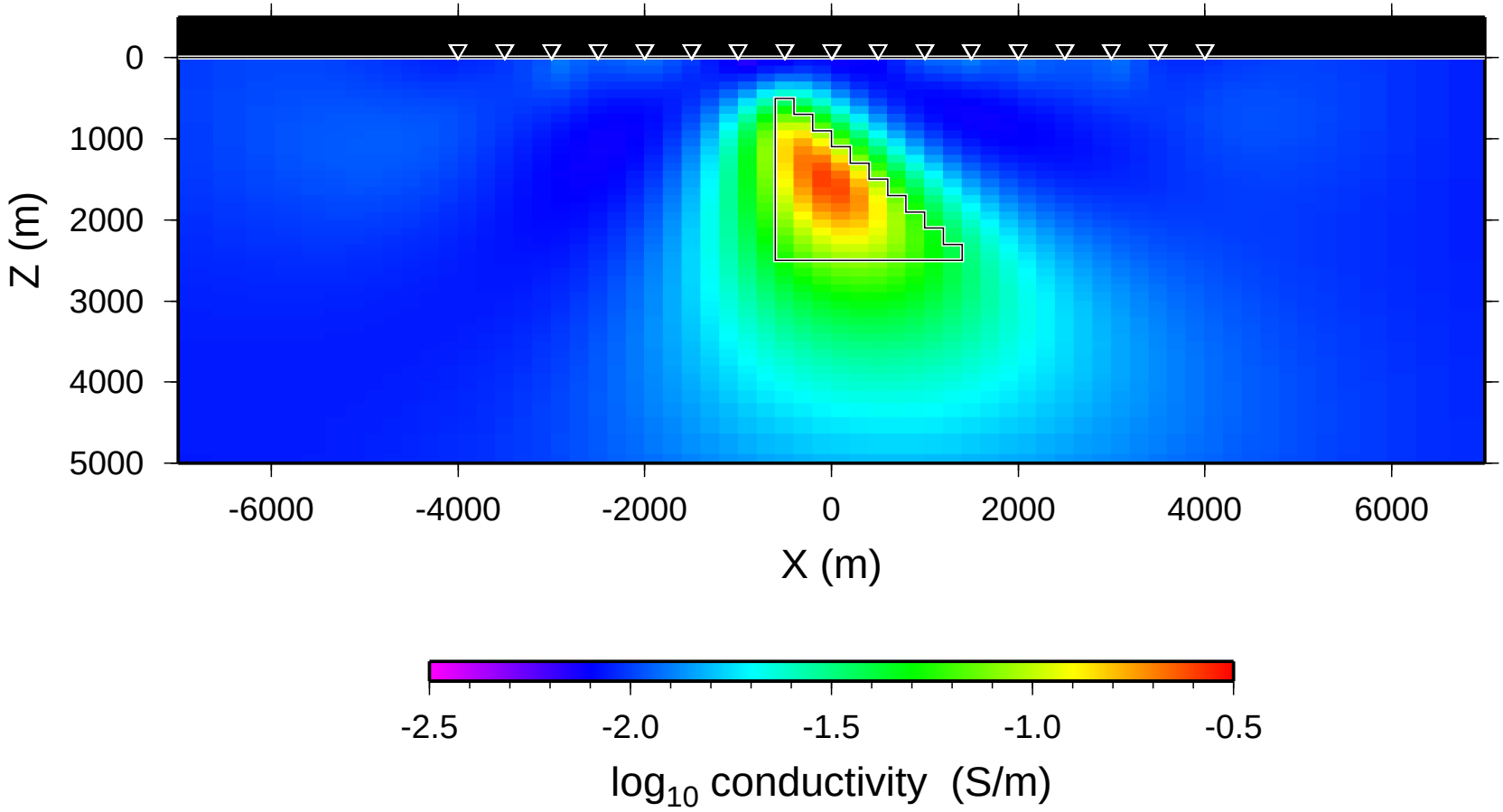


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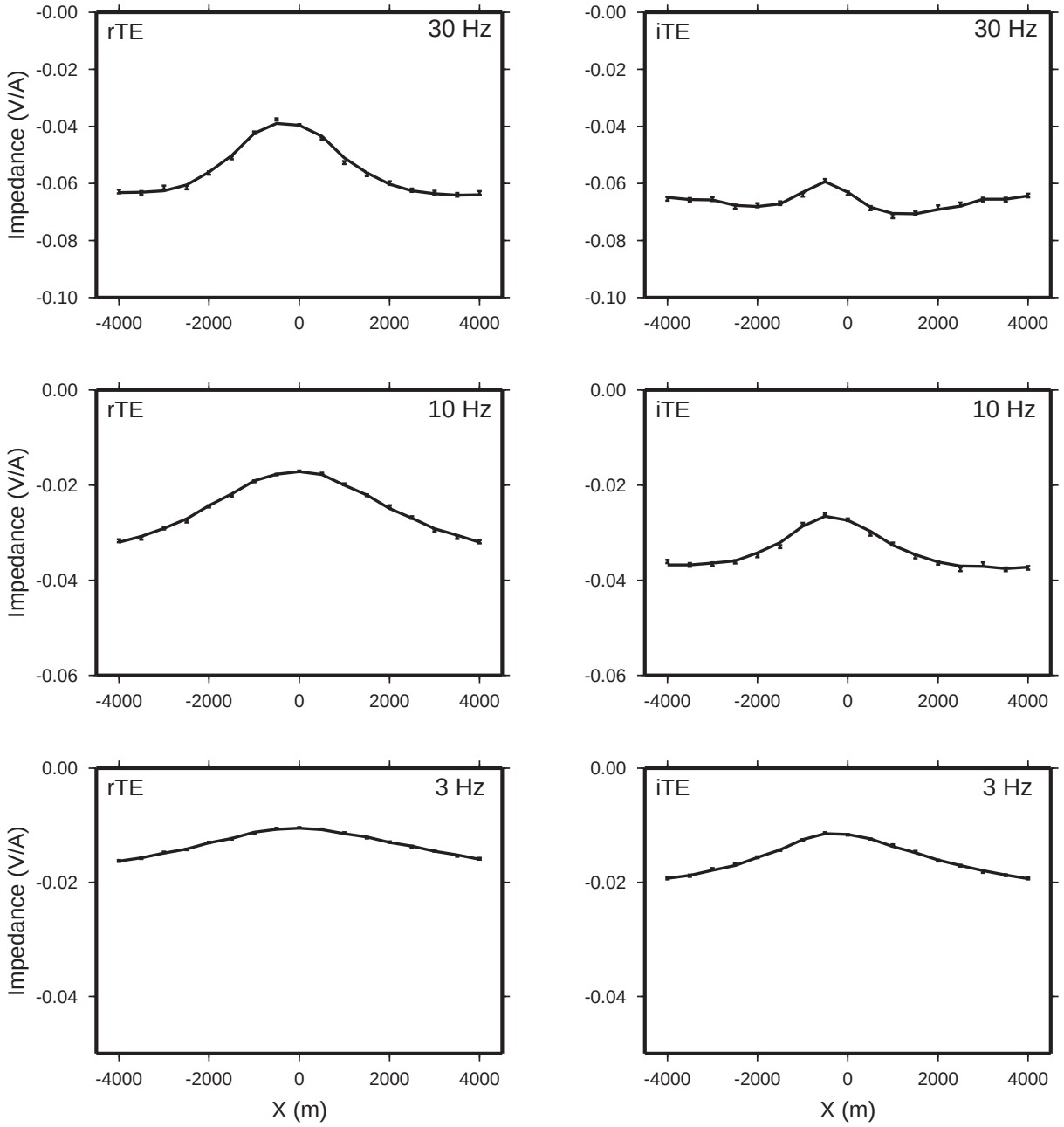


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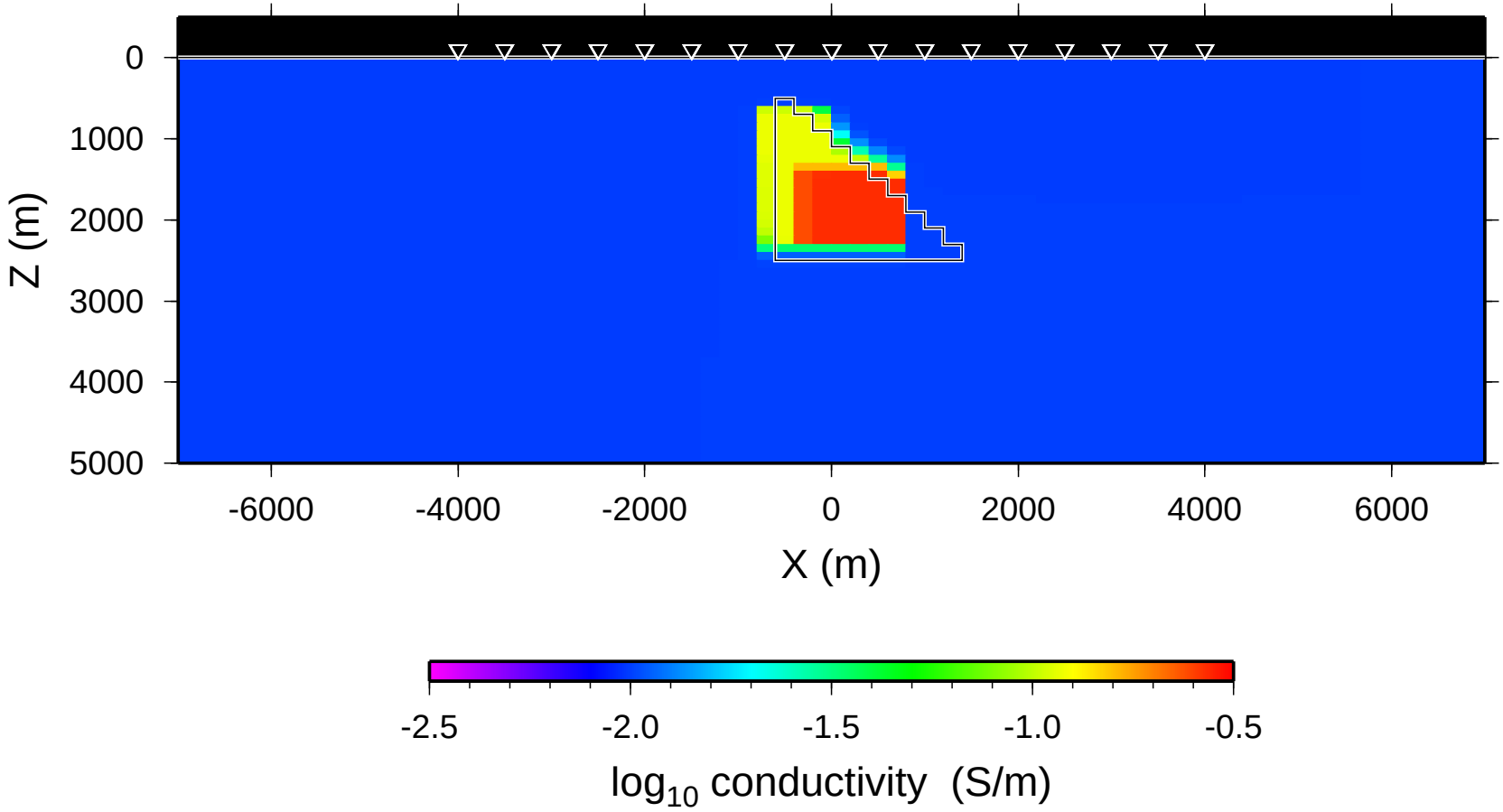


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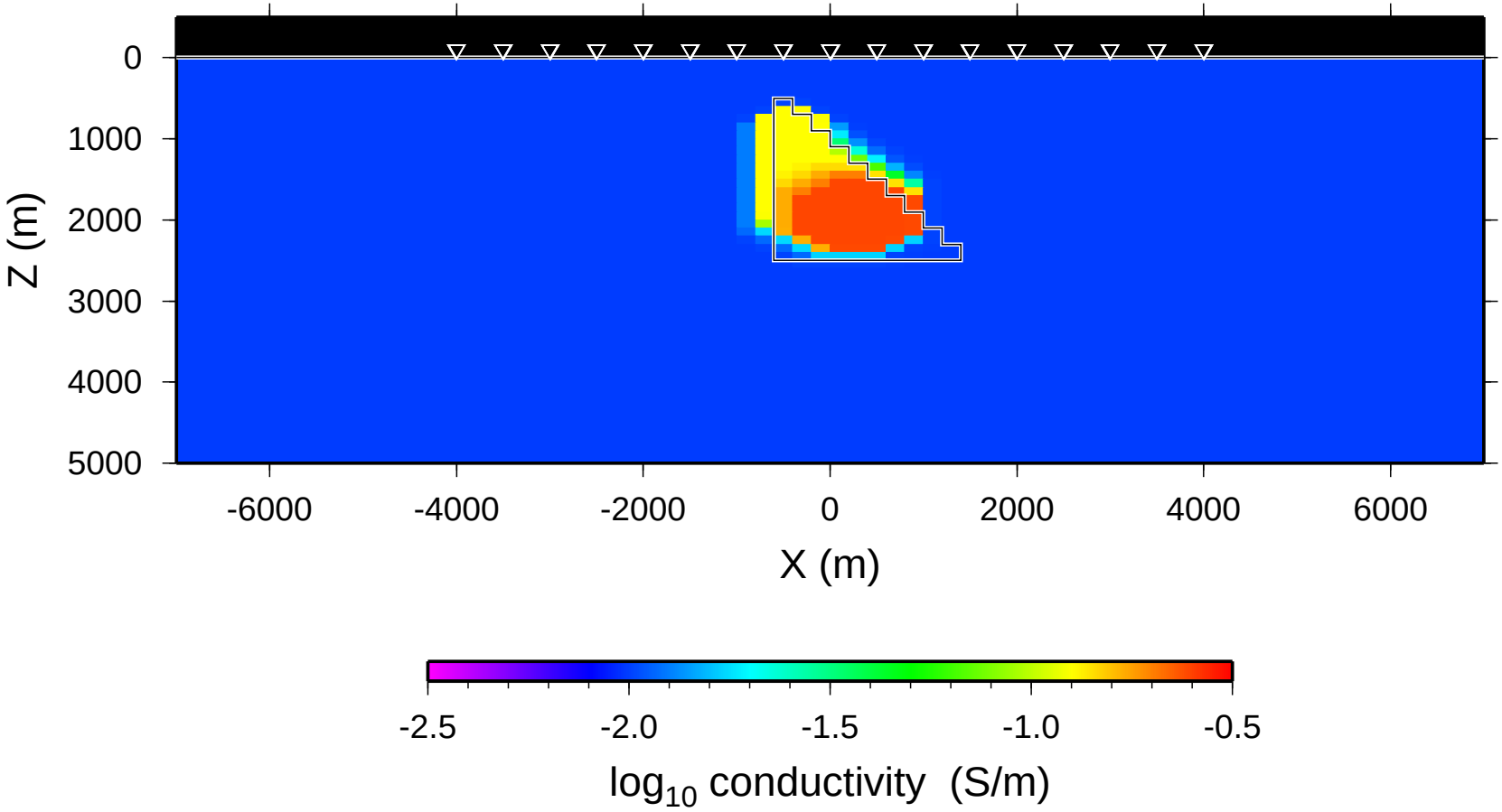


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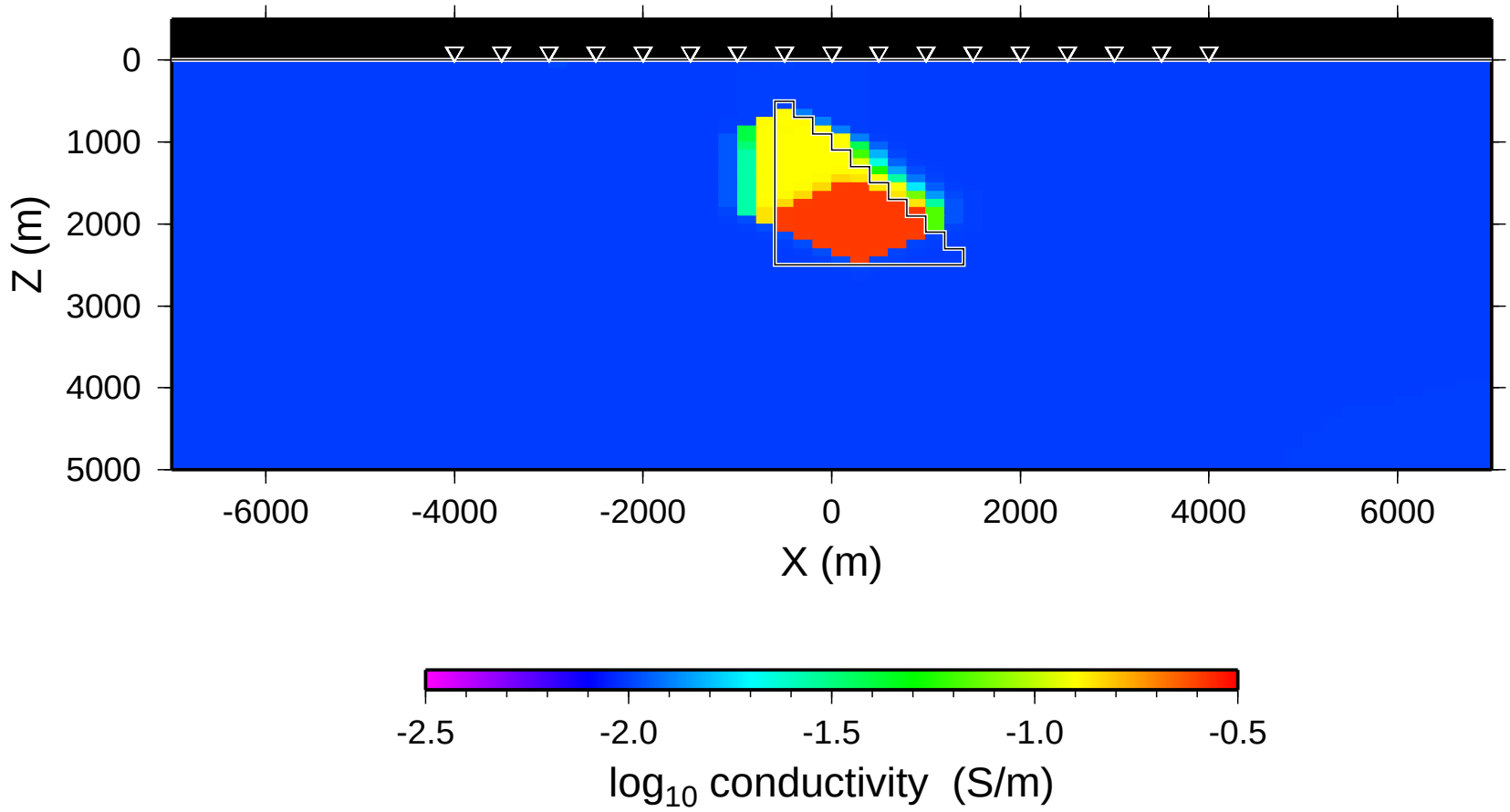


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# Conclusions

- General measures and IRLS can be used to give blocky models in minimum-structure inversion algorithms.

- Diagonal differences in the regularization term give dipping interfaces.