Level set method in seismic inversion: 2D reconstruction of boundaries

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Abstract

There are many features in the Earth's crust that involve a jump in physical property across a sharp boundary, for example, an ore deposit in host rocks. Such well defined boundaries are often of interest to geophysicists, however traditional minimum structure inversion methods produce blurred images of the sub-surface. In this paper we explore the application of a level set inversion method to recover a sharp boundary between two slowness values, one characterizing an inclusion, e.g. an ore body, the other characterizing a background, e.g. host rock, from first arrival travel time data. The scenario considered is that of cross-borehole tomography in two dimensions.

Motivation

- Typical geophysical inversions discretize the Earth into many cells and seek smoothly varying models by minimizing the $L2$ norm of the gradient.
- In contrast, geologists' interpretations about the Earth typically involve contacts between distinct rock units. There are benefits to performing fundamentally different inversions that seek the interfaces between proposed rock units.
- Possible application: more precise delineation of massive sulphides for resource estimation and mine planning after the initial drilling and logging.

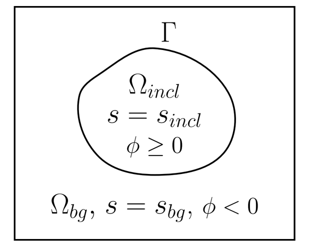
Level set method

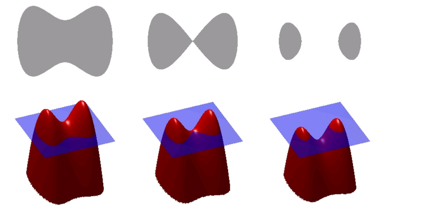
- Originated as a method to track propagating fronts with curvature dependent propagation speed, in such application as crystal growth and flame propagation (Osher & Sethian, 1988).
- Application to inverse problems was first proposed by Santosa (1996).
- Applications to many inverse problems have been developed in medical imaging, inverse scattering, electromagnetics, image segmentation and geophysics. In many application the real innovation is in determining how to evolve the level set function.
- Other methods to parametrize and recover interfaces include: interface preserving regularization norms; explicit boundary parametrization, e.g. with B-splines and spherical harmonics; hybrid methods, e.g. level sets of Hermite interpolants drawn through points on the boundary.

Level Set Parametrization

- An interface (a contact) Γ is parametrized as the 0-level set of a Lipschitz continuous function
- The model values on an underlying mesh are determined by the level set function ϕ as follows:

  $\phi > 0$, in the inclusion
  $\phi < 0$, in the background
  $\phi = 0$, on the interface





An illustration of the concept of the level set method: the intersection of the 0-level (blue) with the level set function (red) generates the lower dimensional bodies (grey) (with permission of Oleg Alexandrov).

- The slowness model can be represented as

$$s(\mathbf{x}) = s_{incl}H(\phi(\mathbf{x})) + s_{bg}(1 - H(\phi(\mathbf{x}))), \qquad H - \text{ 1-D Heaviside function}$$

- The interface changes as the level set function evolves to minimize the objective function.
- The level set method naturally handles topology changes (merges, separations) without adding algorithmic complexity.

Problem formulation

- **Forward problem**. Given piecewise constant slowness $s(\mathbf{x})$ compute the first arrival travel times from sources to receivers. Travel times are found by computing the viscosity solution of the eikonal equation:

$$|\nabla\tau(\mathbf{x})| = s(\mathbf{x}), \qquad \tau(\mathbf{x}_{source}) = 0$$

- **Inverse problem**. Given observed first arrival times τ^{obs} from sources to receivers, and slowness values s_{incl}, s_{bg} inside inclusion and background inferred from sonic logs, find a level set function $\phi(\mathbf{x})$ that minimizes the objective function:

$$F[\phi(\mathbf{x})] = M[\phi(\mathbf{x})] + \beta R[\phi(\mathbf{x})]$$

- M is square misfit

$$M[\phi(\mathbf{x})] = \frac{1}{2}\sum_{data\ points}\left(\tau[\phi(\mathbf{x})] - \tau^{obs}\right)^2$$

- Regularization

$$R[\phi(\mathbf{x})] = \text{ length of interface } = \int_\Omega \delta^\epsilon(\phi(\mathbf{x}))|\nabla\phi(\mathbf{x})|d\mathbf{x}$$

- The inverse problem is solved by an optimization routine, e.g. steepest descent method:

$$\phi^{(k)} = \phi^{(k-1)} - \alpha^{(k)}\frac{\partial F}{\partial\phi}\left(\phi^{(k-1)}\right)$$

Derivatives calculation

- Iterative gradient type methods require the computation of the Frechet derivative of $\frac{\partial F}{\partial\phi}$ at each iteration. Assuming the Frechet derivative exists, it can be obtained by the chain rule:

$$\frac{\partial F}{\partial\phi} = \frac{\partial M}{\partial s}\frac{\partial s}{\partial\phi} + \beta\frac{\partial R}{\partial\phi}$$

- The derivative $\frac{\partial s}{\partial\phi}$ is computed from the level set parametrization

$$\frac{\partial s}{\partial\phi}(\mathbf{x}) = (s_{incl} - s_{bg})\delta^\epsilon(\phi(\mathbf{x})).$$

- The derivative $\frac{\partial M}{\partial s}$ can be efficiently computed for each source by the adjoint state method (Leung and Qian, 2006). Assuming that the receivers are located on a part $\partial\tilde{\Omega}$ of the domain boundary $\partial\Omega$, the adjoint problem for the linearized eikonal equation with respect to the L^2 inner product is formulated as follows: find $\lambda(x)$ such that

$$\nabla\cdot(\lambda\nabla\tau) = 0 \quad \text{in } \Omega$$

$$\mathbf{n}\cdot(\lambda\nabla\tau) = -(\tau - \tau^{obs}) \quad \text{on } \partial\tilde{\Omega}$$
$$\mathbf{n}\cdot(\lambda\nabla\tau) = 0 \quad \text{on } \partial\Omega\setminus\partial\tilde{\Omega}$$

- Once λ is found, $\frac{\partial M}{\partial s}$ is computed by

$$\frac{\partial M}{\partial s}(\mathbf{x}) = \sum_{sources}\lambda(\mathbf{x})s(\mathbf{x}).$$

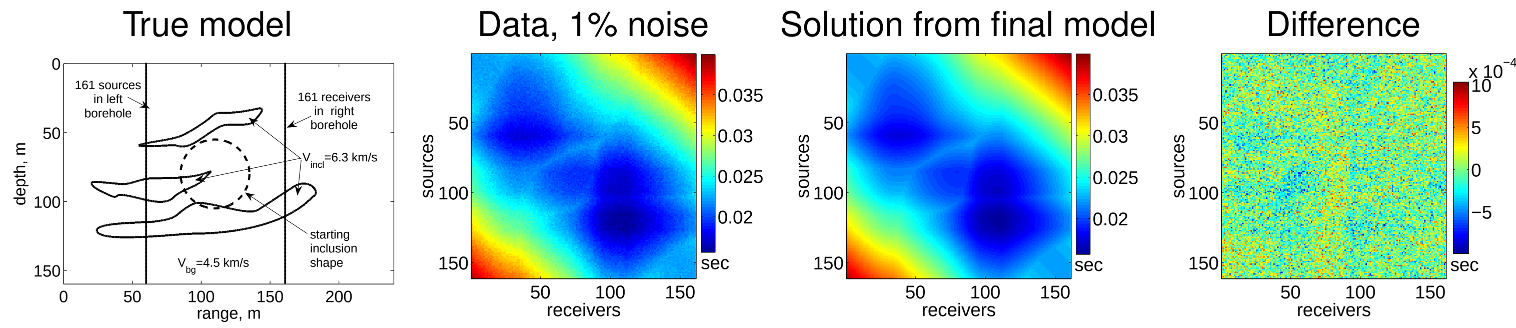
- The partial derivative of $\frac{\partial R}{\partial\phi}$:

$$\frac{\partial R}{\partial\phi}(\mathbf{x}) = -\delta^\epsilon(\phi)\nabla\cdot\frac{\nabla\phi}{|\nabla\phi|}$$

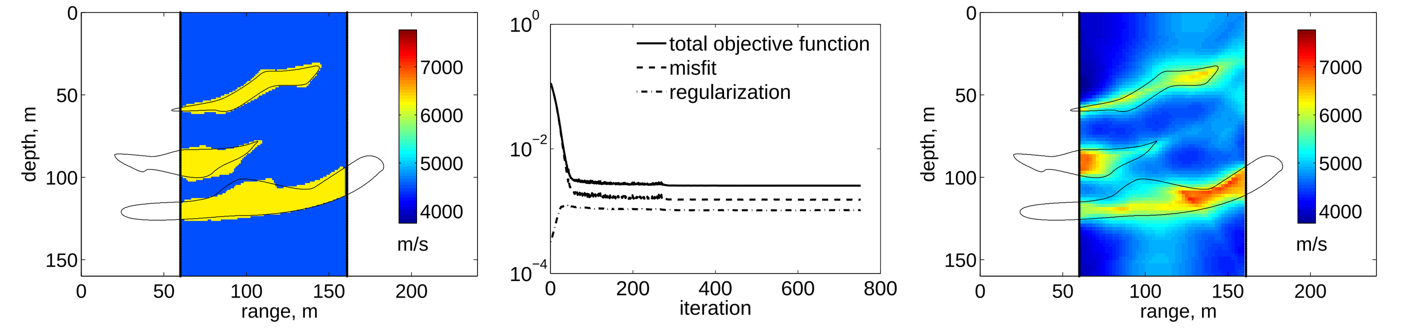
- $\delta^\epsilon(\phi)$ is a smooth approximation to δ-function such that $\delta^\epsilon(\phi) \neq 0$ only when $|\phi| < \epsilon$.

Examples with synthetic data: fast inclusion

Fast inclusion: 6.3 km/s, slow background: 4.5 km/s



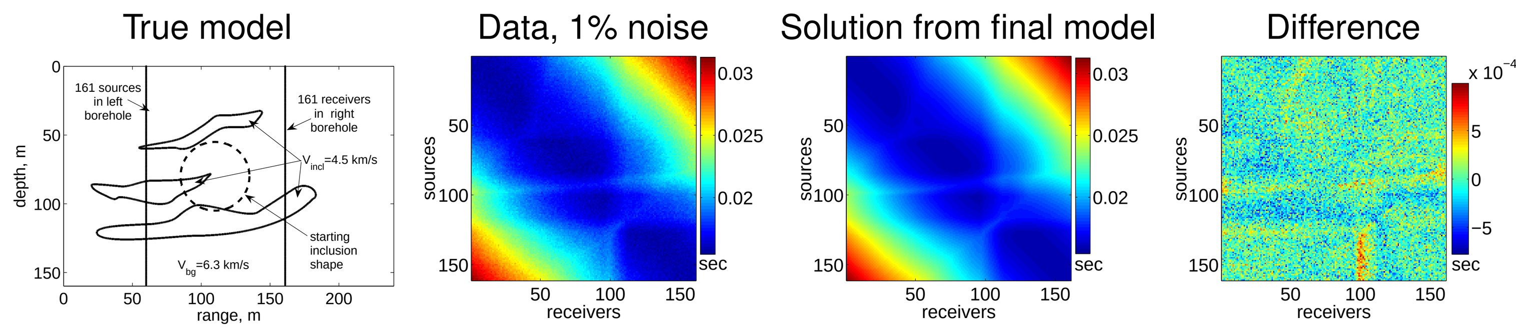
Fast inclusion example: (a) true model; (b) synthetic data with added 1% Gaussian noise; (c) forward solution from the final model; (d) difference between the noisy data and forward solution from the final model. In images (b)-(d) each pixel represents the travel time between one source-receiver pair. The color scale represents travel times in sec.



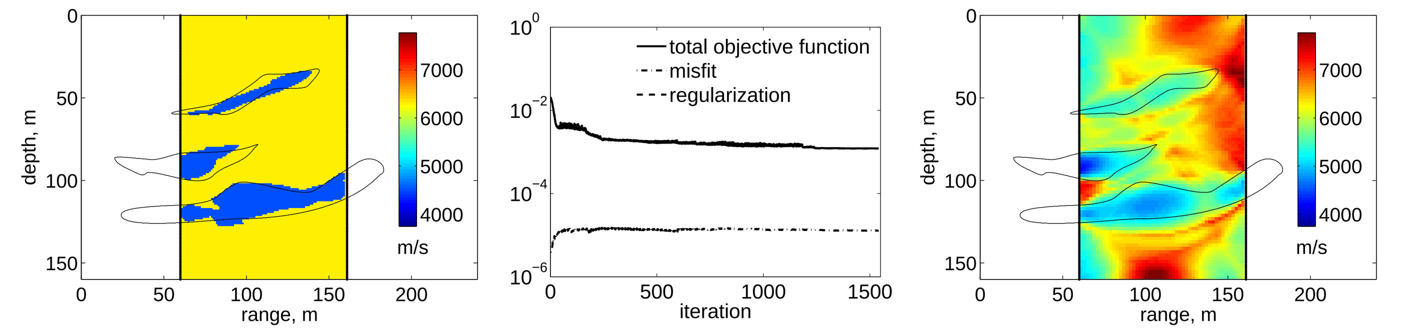
(a) Reconstruction of the 3 elongated fast inclusions with the level set inversion method. The true shapes of the inclusions are outlined in a black line. The color scale represents the P-wave speeds in m/sec. (b) Total objective function F, data misfit M and regularization βR during the inversion. (c) Model computed by a minimum structure inversion of the same data set.

Examples with synthetic data: slow inclusion

Slow inclusion: 4.5 km/s, fast background: 6.3 km/s

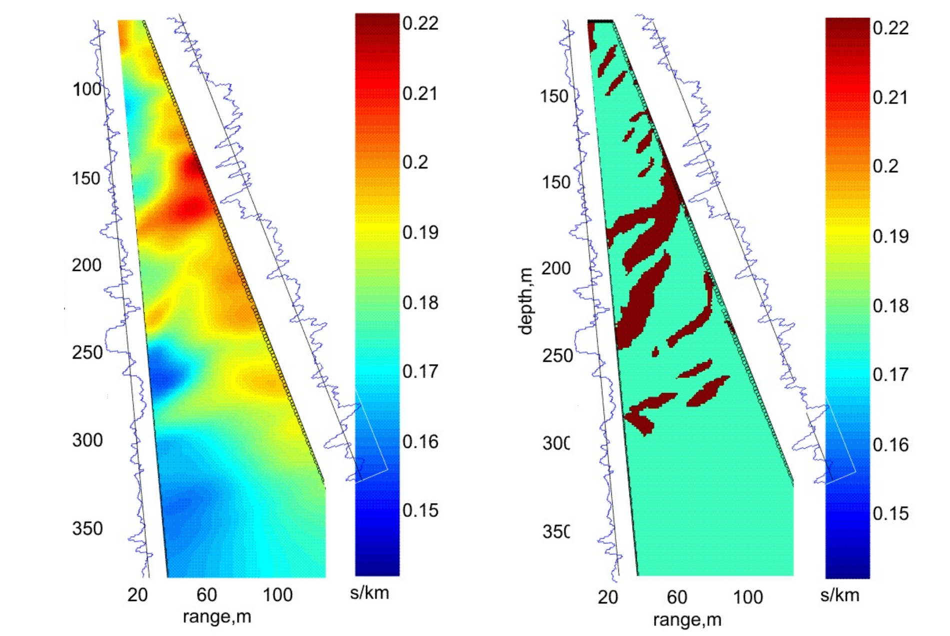


Slow inclusion example: (a) true model; (b) synthetic data with added 1% Gaussian noise; (c) forward solution from the final model; (d) difference between the noisy data and forward solution from the final model. In images (b)-(d) each pixel represents the travel time between one source-receiver pair. The color scale represents travel times in sec.



(a) Reconstruction of the 3 elongated slow inclusions with the level set inversion method. The color scale represents the P-wave speeds in m/sec. (b) Total objective function F, data misfit M and regularization βR during the inversion. (c) Model computed by a minimum structure inversion of the same data set.

Reconstruction from a real data set



(a) Reconstruction by minimum structure inversion using "smart arrivals"(courtesy of Brandon Reid) (b) Reconstruction by the level set method. Background and inclusion velocities are 5.7 and 4.5 km/s, corresponding to slowness 0.1754 and 0.2222 s/km, chosen based on the sonic log data. Number of data points: 16243, number of model points: 32076 in level set inversion, 10369 in minimum structure inversion.

Conclusions

- In this poster we considered an application of the level set method to the inversion of first arrival travel time data for a slowness model consisting of two phases, where the slowness values are assumed known.
- We tested our method on a synthetic example with the true medium consisting of three elongated fast and slow inclusions where the contrast between the slowness values is high. We obtained good reconstructions of the inclusions.
- Inversion of the real data set yields a tomogram that agrees well with the minimum structure inversion result and sonic log data.
- The method is rather insensitive with respect to the starting inclusion shape.
- Sensitivity studies with respect to the errors in the slowness values, and uncertainty analysis of the recovered inclusion shapes are necessary to determine our confidence in the performance of the level set method.

References and further reading

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