3D Potential Field Inversion for Wireframe Surface Geometry

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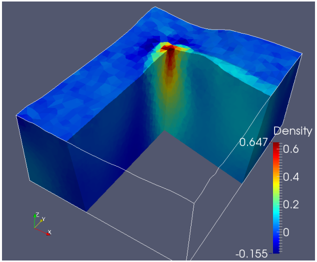
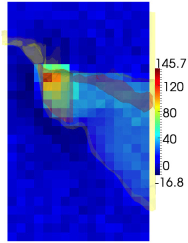
SEG 85th Annual Meeting, Oct. 2015, New Orleans, GM2

Geophysical inversion primer

Forward problem



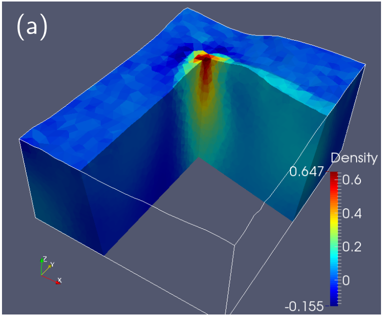
Earth model (e.g. density)

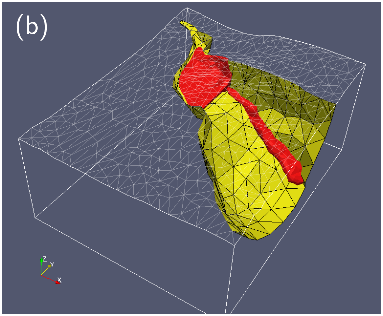


Survey data (e.g. gravity)

Inverse problem

Motivation





- Geophysical numerical methods typically work with mesh-based distributions of physical properties (a)
- Geologists' interpretations about the Earth typically involve wireframe contacts between distinct rock units (b)
- There is a disconnect here!

Types of geophysical inversion

1. Discrete body inversion
2. Mesh-based inversion
3. Surface-based inversion

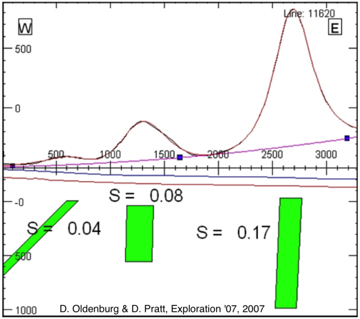
1. Discrete body inversion

Simplified representation of the Earth:

- Simple shapes for one or more causative target bodies
- Homogeneous background

Inversion:

- Few parameters (e.g. shape, location)
- Data best-fit problem
- Low computational requirements
- Stochastic investigations feasible



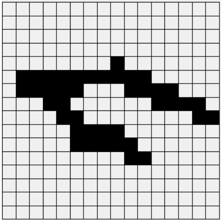
2. Mesh-based inversion

General representation of the Earth:

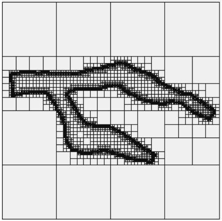
- Mesh of tightly packed cells
- Piecewise (pixellated) distribution of physical properties

Inversion:

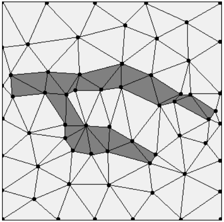
- Many parameters (many cells)
- High computational requirements
- Stochastic investigations not very feasible



Rectilinear



Quadtree



Unstructured

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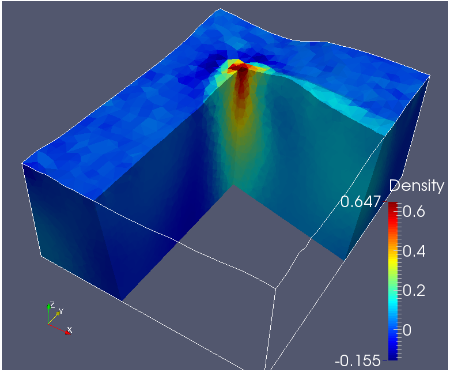


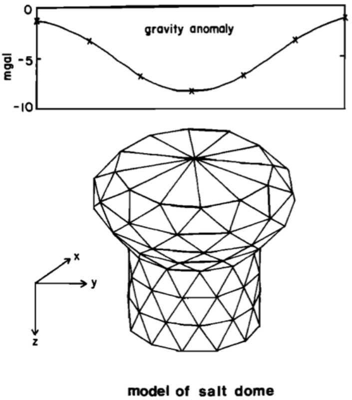
3. Surface-based inversion

Flexible representation of the Earth:

- Wireframe of nodes and connecting facets representing contacts between rock units
- How geological models are built

Inversion:

- Surface geometry defined by moderate number of parameters
- Moderate computational requirements
- Stochastic investigations somewhat feasible



Richardson & MacInnes, 1989, The inversion of gravity data into three-dimensional polyhedral models, JGR

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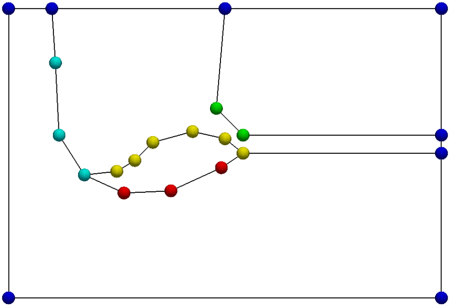

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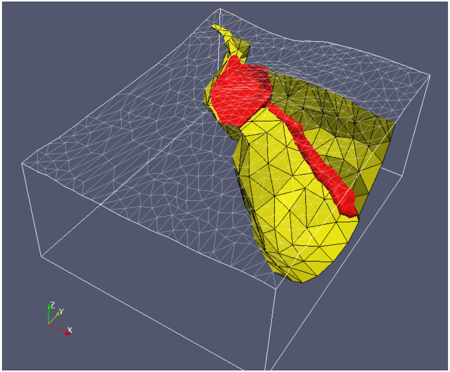


Types of geophysical inversion

1. Discrete body inversion
2. Mesh-based inversion
3. Surface-based inversion

Mesh-based inversion for smooth distributions

- Objective function

$$\Phi = \Phi_d + \beta \Phi_m$$

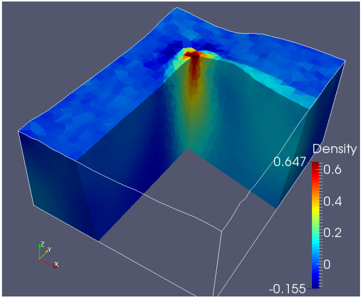
- Data misfit

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Model structure (regularization)

$$\Phi_m = \sum_j w_j (m_j - p_j)^2 + \sum_j \sum_k w_{j,k} (m_j - m_k)^2$$

[smallness term] + [smoothness term]



- Deterministic local optimization approach: one "best" solution

Mesh-based inversion for sharper features

- Objective function

$$\Phi = \Phi_d + \beta \Phi_m$$

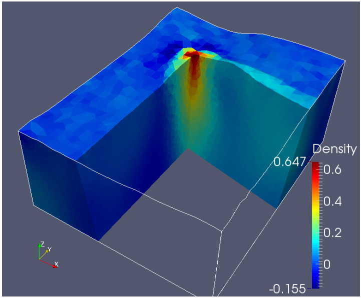
- Data misfit

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Model structure (regularization)

$$\Phi_m = \sum_j w_j (m_j - p_j)^2 + \sum_j \sum_k w_{j,k} \left| m_j - m_k \right|^{p} + \Psi$$

[smallness term] + [smoothness term]

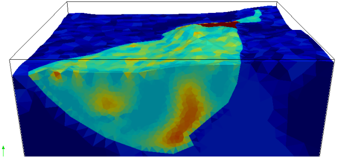


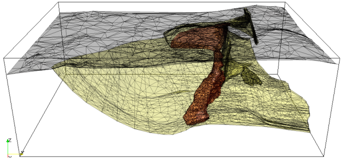
- Different norm, measures or re-weighted iterative procedure can help

Mesh-based inversion for sharper features

- Objective function

$$\Phi = \Phi_d + \beta \Phi_m$$





- Data misfit

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Model structure (regularization)

$$\Phi_m = \sum_j w_j (m_j - p_j)^2 + \sum_j \sum_k {\color{red} w_{j,k}} (m_j - m_k)^2$$

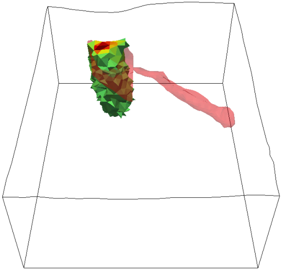
[smallness term] + [smoothness term]

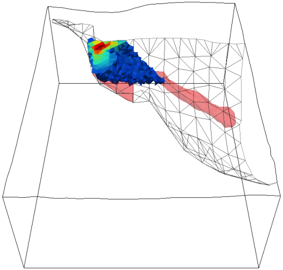
- The safest and most effective approach is to hardwire the surfaces

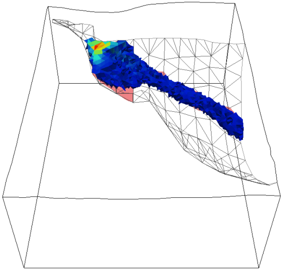
Mesh-based inversion for sharper features

Hardwiring surfaces ⇒ unstructured meshes become important

Inversion of gravity gradiometry data:


Unconstrained


Hardwired surface


Additional regularization

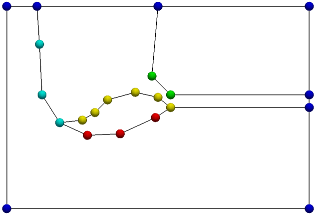
Types of geophysical inversion

1. Discrete body inversion
2. Mesh-based inversion
3. Surface-based inversion (2D)

2D Surface-based inversion for geometry

- Inversion seeks positions of nodes in wireframe model
- Only data misfit is required

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$



- Regularization not required: work on coarse representation, refine e.g. splines

- Global optimization strategies (PSO, GA, MCMC) provide statistics:

$\Rightarrow$ many solution samples

Cocagne Subbasin: gravity survey data

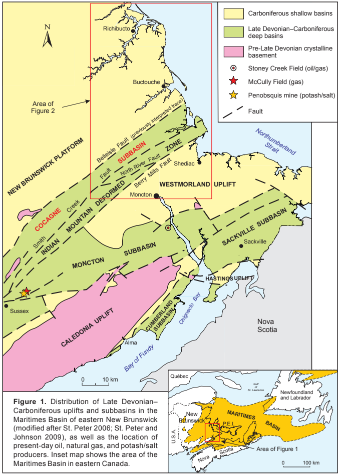


Figure 1. Distribution of Late Devonian–Carboniferous uplifts and subbasins in the Maritimes Basin of eastern New Brunswick (modified after St. Peter 2006; St. Peter and Johnson 2009), as well as the location of present-day oil, natural gas, and potash/salt producers. Inset map shows the area of the Maritimes Basin in eastern Canada.

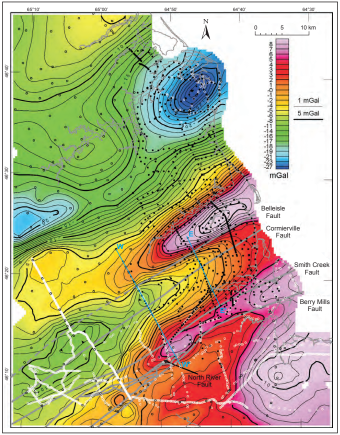


Figure 8. New terrain-corrected Bouguer anomaly map of the report area, with fault boundaries (modified after Smith 2008) shown as thick grey lines. Blue lines E–E' and W–W' mark the location of section profiles in Figures 11 and 12, respectively. Gravity stations are as identified in Figure 6. Figure 1 shows the location of this report area in eastern New Brunswick.

J. Evangelatos & K. E. Butler, 2010. New gravity data in Eastern New Brunswick: implications for structural delineation of the Cocagne Subbasin. In Geological Investigations in New Brunswick for 2009. Edited by G. L. Martin. New Brunswick Department of Natural Resources; Lands, Minerals and Petroleum Division, Mineral Resource Report 2010–1, p. 98–122.

Cocagne Subbasin: rudimentary geological model

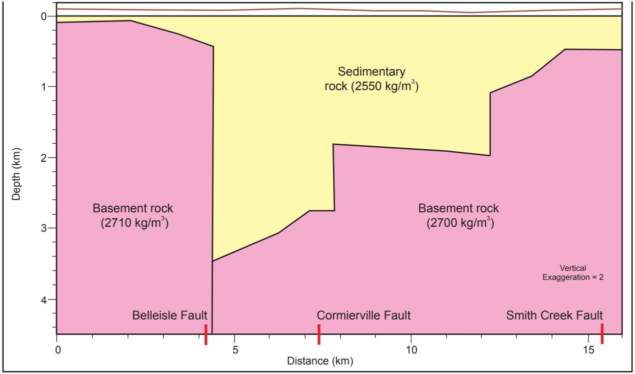
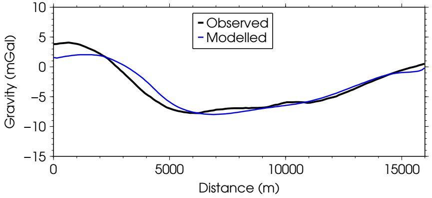
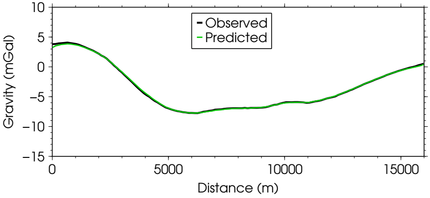


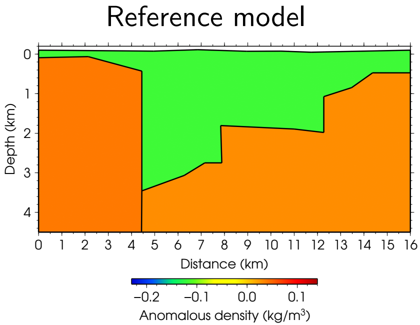
Figure 11. A two-dimensional geological model, simulating the observed gravity response across the eastern part of the Cocagne Subbasin along profile E–E' in Figure 8. The magnetic profile is extracted from the regional aeromagnetic dataset (Canadian Aeromagnetic Data Base 2010). The Belleisle, Cormierville, and Smith Creek fault traces in Figure 8 are shown here as red ticks on the horizontal axis.

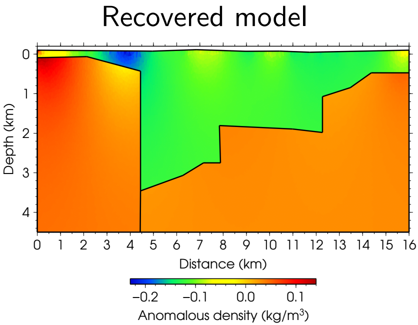
J. Evangelatos & K. E. Butler, 2010. New gravity data in Eastern New Brunswick: implications for structural delineation of the Cocagne Subbasin. In Geological Investigations in New Brunswick for 2009. Edited by G. L. Martin. New Brunswick Department of Natural Resources; Lands, Minerals and Petroleum Division, Mineral Resource Report 2010–1, p. 98–122.

Cocagne Subbasin: mesh-based inversion

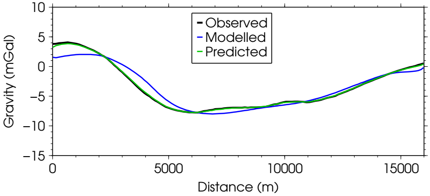


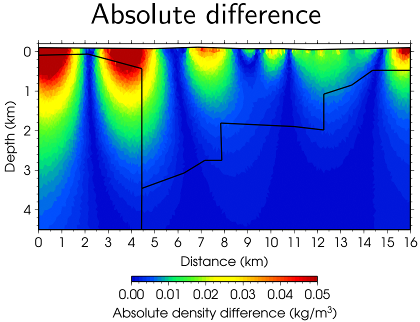


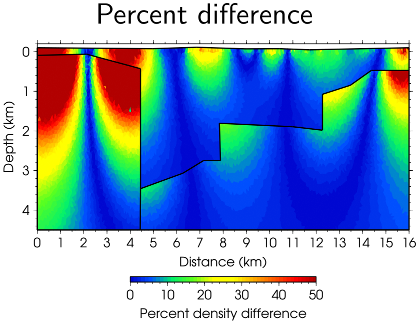




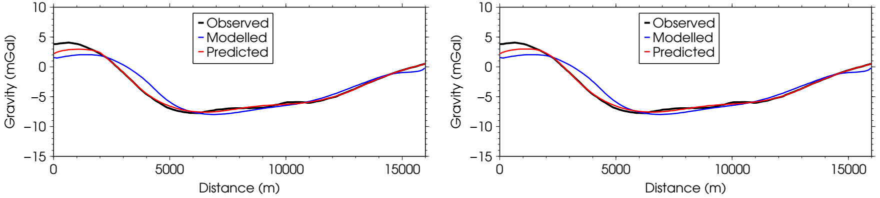
Cocagne Subbasin: mesh-based inversion

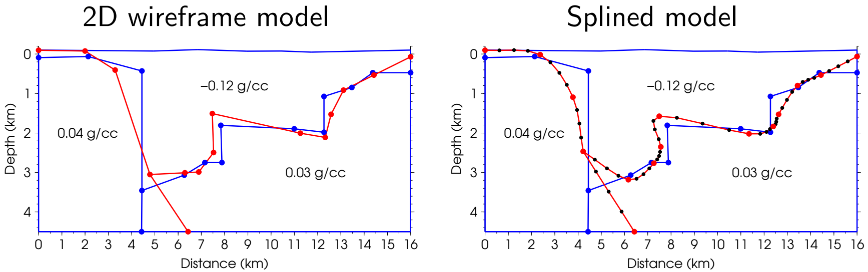




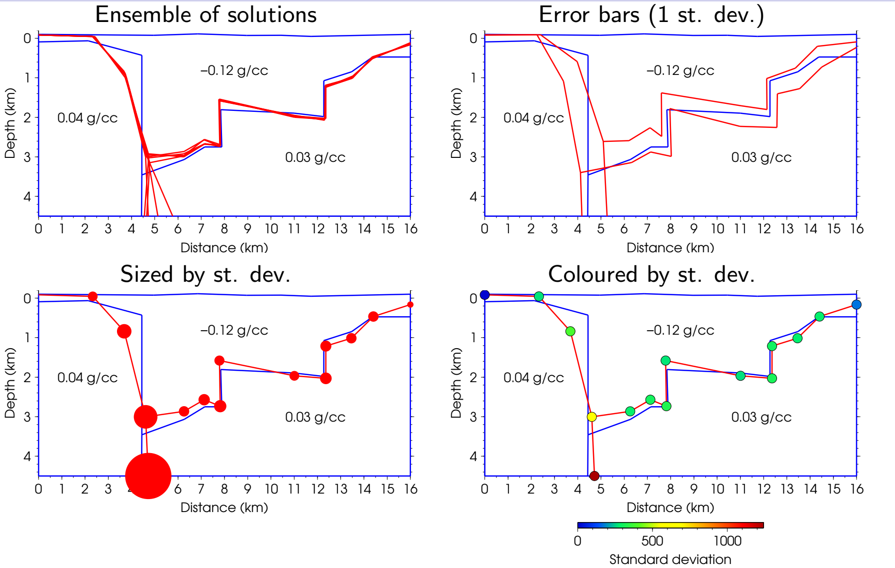


Cocagne Subbasin: surface-based inversion





Options for visualizing uncertainty



Types of geophysical inversion

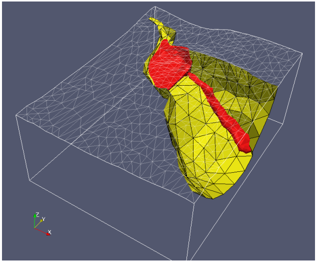
1. Discrete body inversion
2. Mesh-based inversion
3. Surface-based inversion (3D)

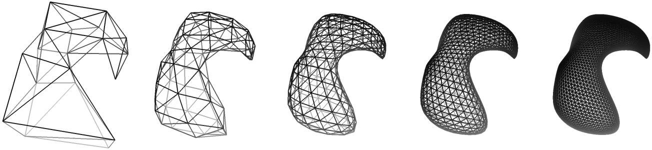
Surface-based inversion for geometry

- Inversion seeks positions of nodes in wireframe model
- Only data misfit is required

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Regularization not required: work on coarse representation, refine e.g. surface subdivision





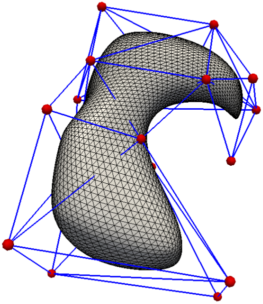
- Global optimization strategies (PSO, GA, MCMC) provide statistics:

$\Rightarrow$ many solution samples

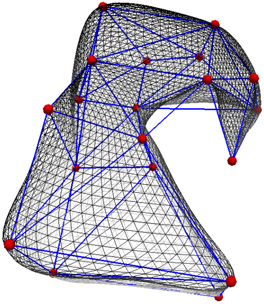
Surface-based inversion for geometry

Use coarse control surfaces rather than the surfaces themselves!

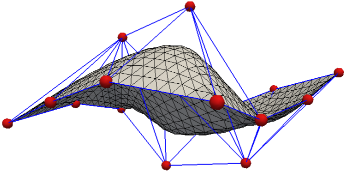
Subdivision with cubic B-spline interpolation



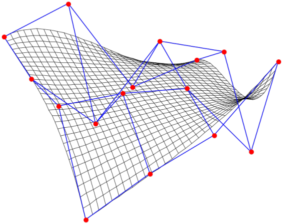
Subdiv., Dyn-Levin-Gregory interp.



Subdiv., cubic B-spline



Bézier surface



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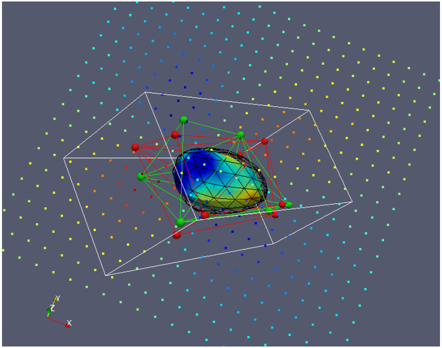
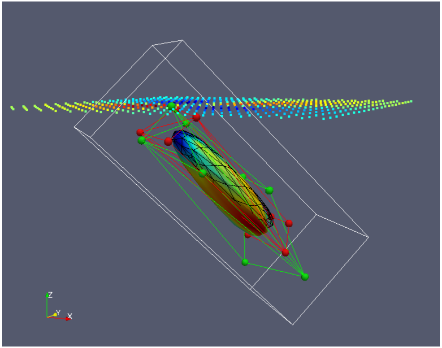
Synthetic example #1: Isolated body

Black wireframe = true model
Red wireframe = control surface for true model
Green wireframe = control surface for recovered model
White box = bounds on control nodes during inversion
Coloured surface = recovered model (standard deviations, red high)
Coloured points = gravity data



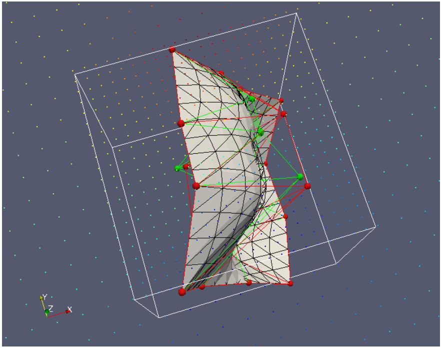
Overhead view



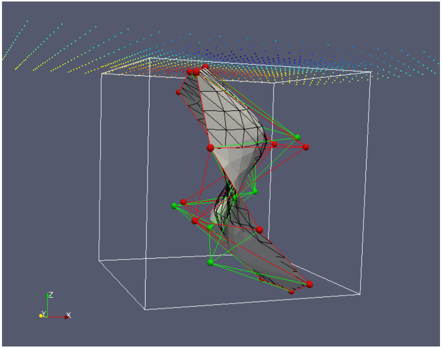
Side view

Synthetic example #2: Sheet-like contact surface

Black wireframe = true model
Red wireframe = control surface for true model
Green wireframe = control surface for recovered model
White box = bounds on control nodes during inversion
Light grey surface = recovered model (standard deviations not plotted here)
Coloured points = gravity data

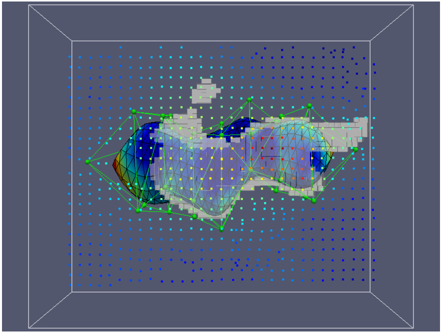


Overhead view

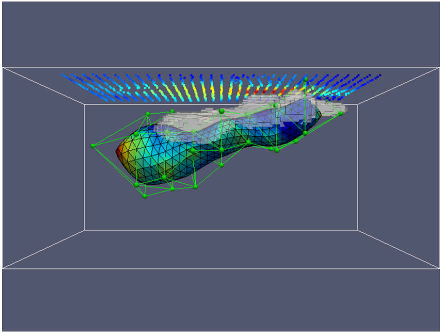


Side view

Real data example: IOCG deposit, gravity data



Overhead view



Side view

Surface-based inversion result consistent with understanding of geology, significant differences to mesh-based result require further study

Summary

- Mesh-based "distribution inversion" is standard and numerically well-behaved until you try to recover sharp features (best way is to hardwire them)
- Surface-based "geometry inversion" is challenging but:
  - can model sharp contacts and provide statistical information
  - geological and geophysical models can be, in essence, the same Earth model: there is no longer a disconnect!

Future work

- Joint surface-based inversion
  - Multi-objective optimization methods for joint inverse problems ...
- Alternate global optimization approaches
  - Particle swarm, genetic algorithms, ant colony, ...
- Work directly with complicated 3D common Earth models ...
- Hybrid approach ...

(additional slides follow)

Multi-objective genetic algorithms for joint inversion

Single-objective GA:

- Aggregate of objectives:

$$\min(f) \quad f = f_1 + \lambda f_2 + \ldots$$

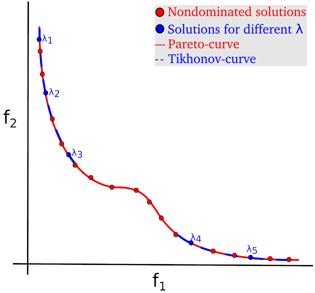
- One single best solution
- Difficult to find best λ value(s)

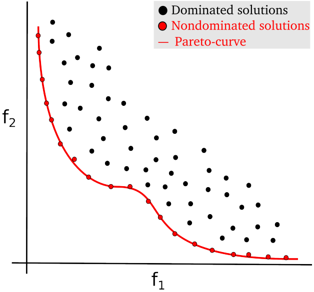
Multi-objective GA:

- Objectives treated separately:

$$\min(f_1, f_2)$$

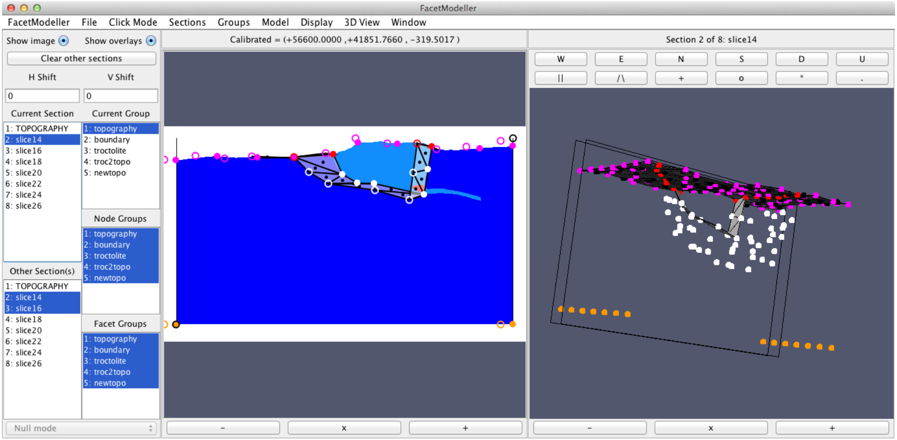
- Several solutions along the Pareto front (nondominated)





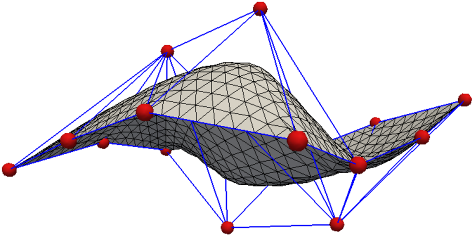
Building and manipulating complicated 3D models

FacetModeller

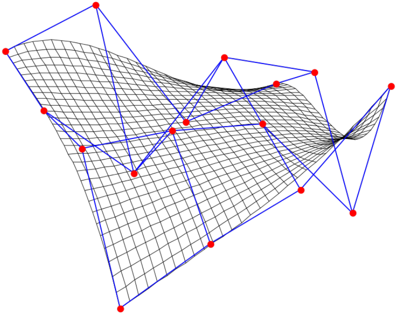


Can we extract the control surfaces, rather than the surfaces themselves, from the modelling software?

Subdiv., cubic B-spline



Bézier surface



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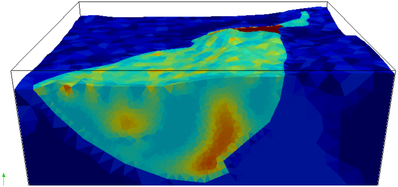
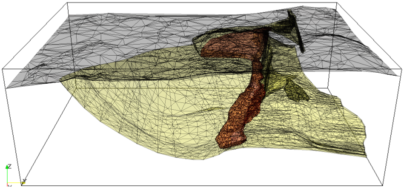
A third approach?

1. Mesh-based inversion with sharp features



2. Surface-based inversion



3. Hybrid approach?