# Three-dimensional inversion of gravity data for blocky models using a minimum-structure algorithm and general measures

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## Acknowledgments

- Voisey's Bay Nickel Company, and Inco/CVRD, for access to the gravity data over the Ovoid.
- Brian Bengert of VBNC/Inco/CVRD, and Michael Ash of MUN, for their advice and assistance.
- MeshTools3D by Roman Shekhtman of UBC–Geophysical Inversion Facility.
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# Outline

- Motivation.
- Previous work.
- General minimum-structure inversion strategy.
  - General measures.
  - Iterative solution procedure.
  - Measure of model structure.
- Example: 3-D gravity inversion, Voisey's Bay Ovoid.
- Conclusions.

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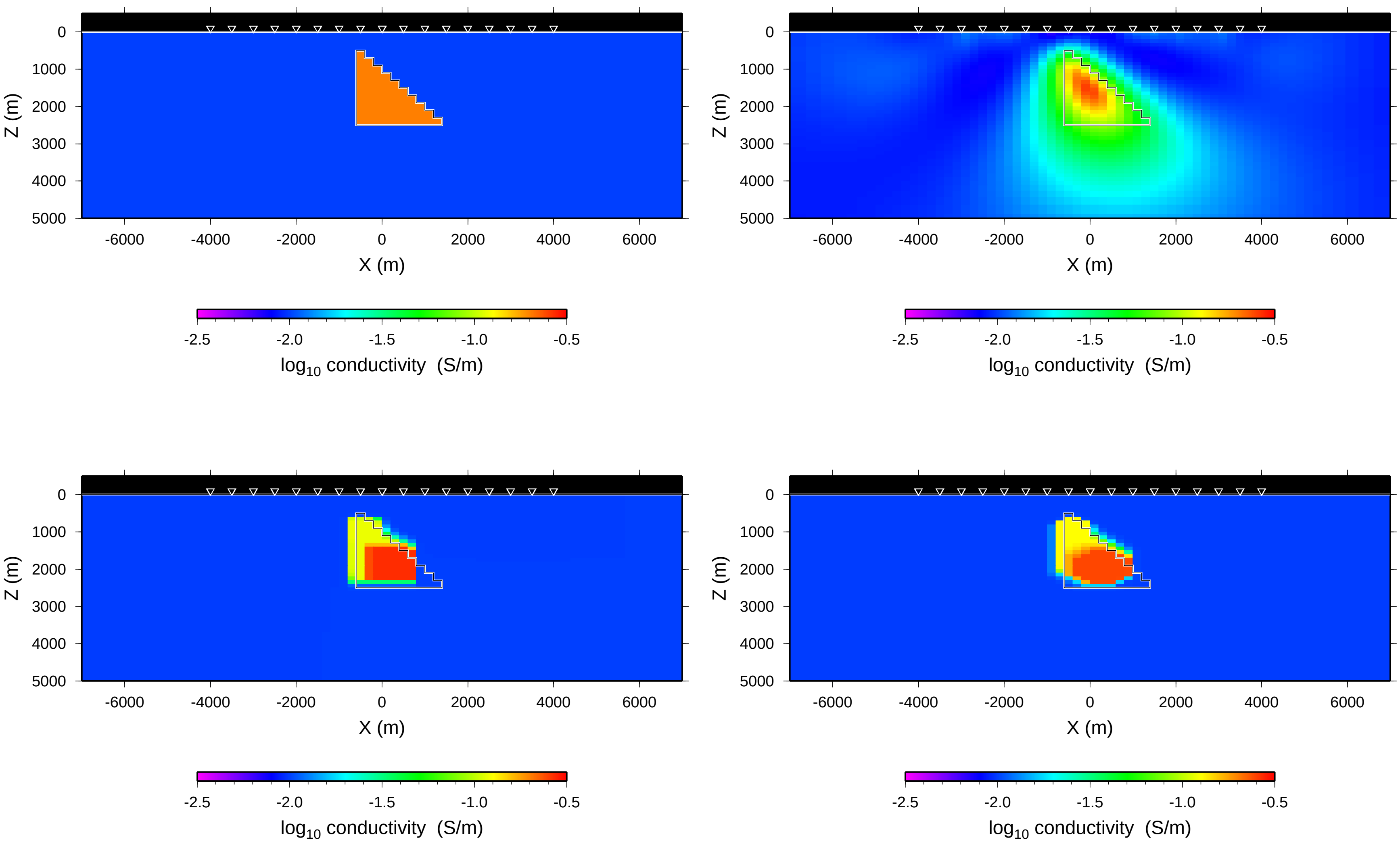
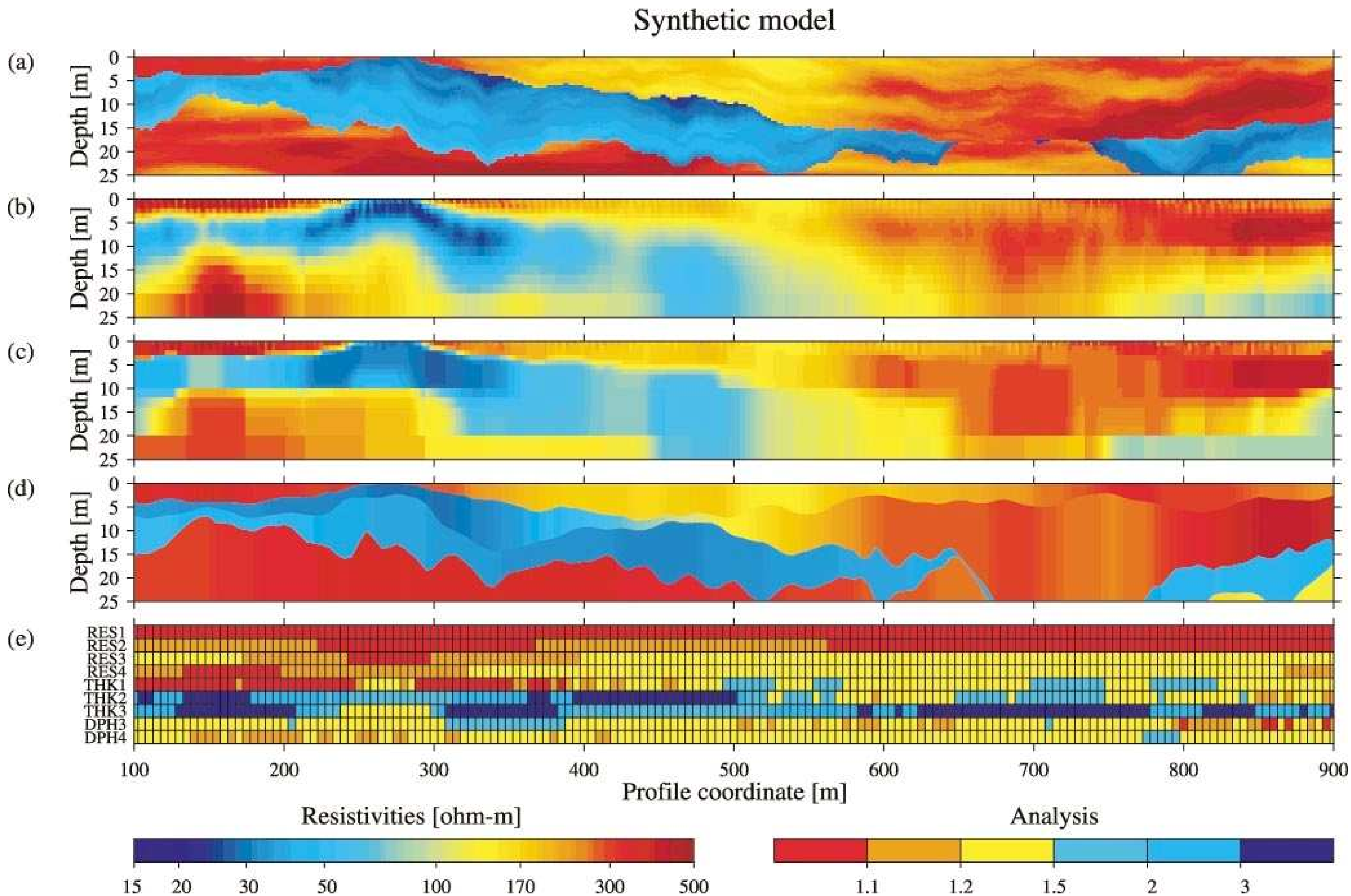
# Minimum-structure inversion for sharp interfaces



Auken & Christiansen (2004, Geophysics, 69, p752–761):



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# Previous work

"True" minimum-structure algorithms:

Farquharson & Oldenburg (1998, GJI), 1-D EM;
Portniaguine & Zhdanov (1999, Geophysics), 3-D focusing;
Loke, Acworth & Dahlin (2003, Expl. Geop.), 2-D resistivity;
Farquharson & Oldenburg (2003, SEGJ), 2-D resistivity.

Laterally constrained layered inversions:

Smith et al. (1999, Geophysics), 2-D MT;
Auken & Christiansen (2004, Geophysics), 2-D resistivity;
de Groot-Hedlin & Constable (2004, Geophysics), 2-D MT.

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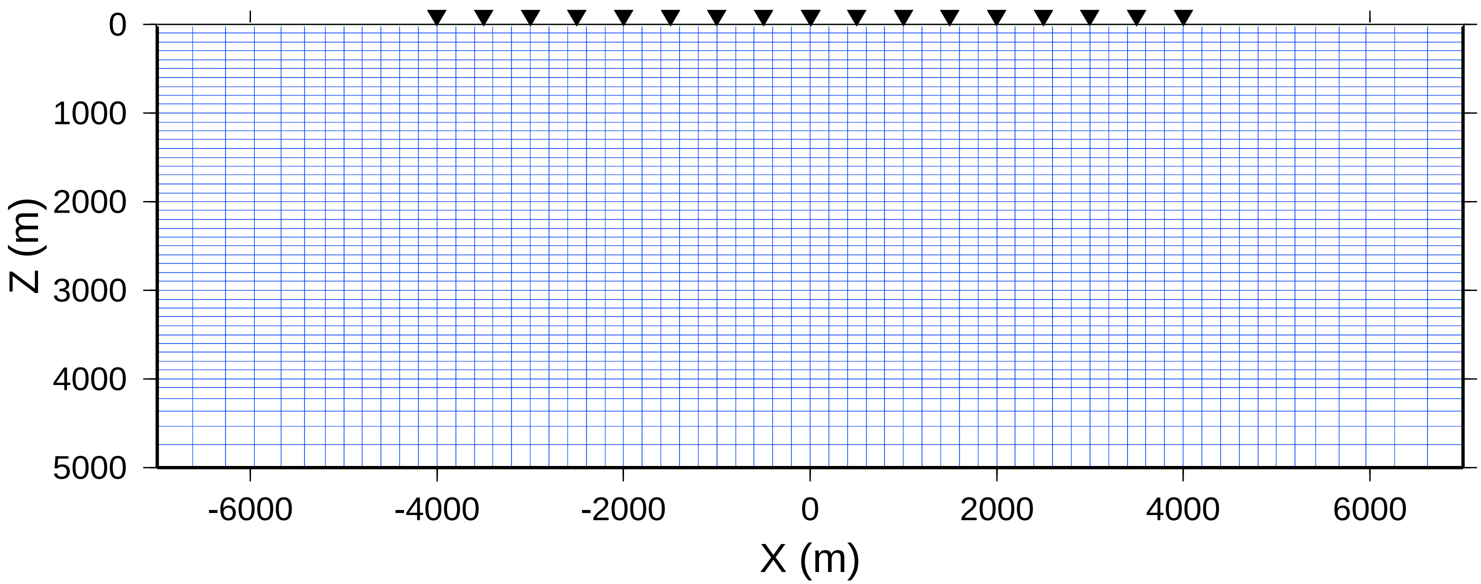
# General minimum-structure inversion strategy

- Mesh fixed during inversion; fine discretization.



## General minimum-structure inversion strategy

- Minimize objective function:

$$\Phi = \phi_d \ + \ \beta \, \phi_m,$$

where $\phi_d$ is measure of data-misfit,

$$\phi_d = \phi_d(\mathbf{u}) \qquad \mathbf{u} = \mathbf{W}_d\big(\mathbf{d}^{\text{obs}} - \mathbf{d}^{\text{prd}}\big),$$

and $\phi_m$ is measure of structure in model,

$$\phi_m = \sum_k \alpha_k \phi_k(\mathbf{v}_k) \qquad \mathbf{v}_k = \mathbf{W}_k\big(\mathbf{m} - \mathbf{m}_k^{\text{ref}}\big).$$

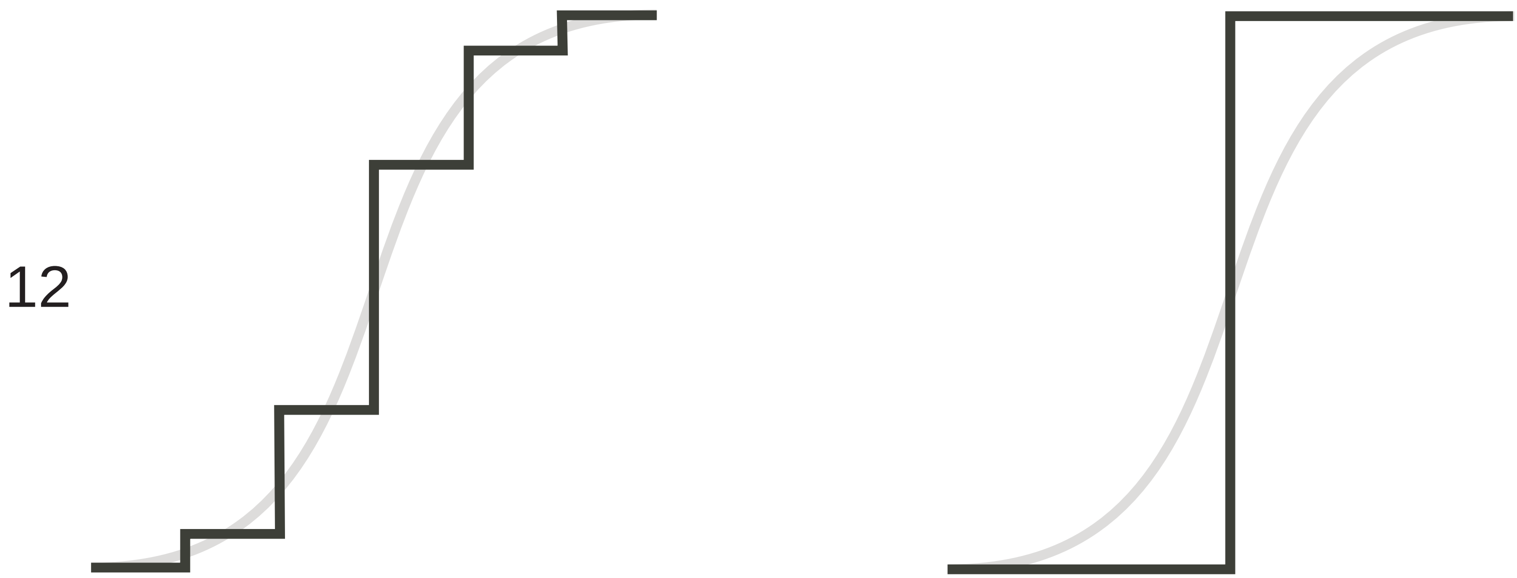
# General measures

- A general form for $\phi_d$ and $\phi_m$ is:

$$\phi(\mathbf{x}) = \sum_{j=1}^{N} \rho(x_j).$$

$$\begin{aligned}
&\text{For example, the } l_2\text{-norm:} & \rho(x) &= x^2; \\
&\text{the } l_p\text{-norm:} & \rho(x) &= |x|^p; \\
&\text{Ekblom's } l_p\text{-like measure:} & \rho(x) &= \left(x^2 + \epsilon^2\right)^{p/2}; \\
&\text{Huber's } M\text{-measure:} & \rho(x) &= \begin{cases} x^2 & |x| \le c, \\ 2c|x| - c^2 & |x| > c. \end{cases}
\end{aligned}$$

# General measures




| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 6 | 2 | 1 | = 12 | | 0 | 0 | 12 | 0 | 0 | = 12 |
| 1 | 4 | 36 | 4 | 1 | = 46 | | 0 | 0 | 144 | 0 | 0 | = 144 |

# Iterative solution procedure

- Differentiate $\Phi$ with respect to model parameters and equate to zero.

  Get normal system of equations:

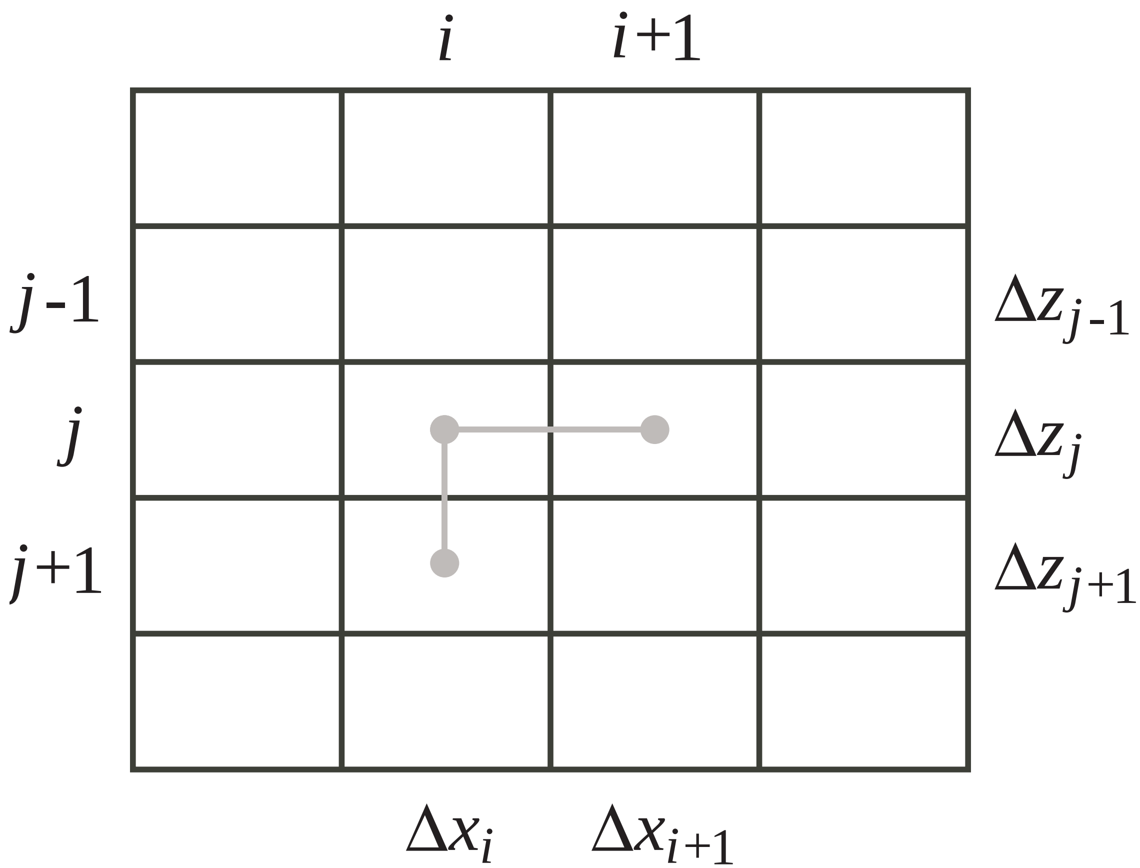
$$\begin{aligned}\Big[\mathbf{G}^T\mathbf{W}_d^T\mathbf{R}_d\mathbf{W}_d\mathbf{G} \;+\; \beta^n\sum_k \alpha_k\mathbf{W}_k^T\mathbf{R}_k\mathbf{W}_k\Big]\delta\mathbf{m} \\ = \;\mathbf{G}^T\mathbf{W}_d^T\mathbf{R}_d\mathbf{W}_d\big(\mathbf{d}^{\mathrm{obs}} - \mathbf{d}^{n-1}\big) + \\ \beta^n\sum_k \alpha_k\mathbf{W}_k^T\mathbf{R}_k\mathbf{W}_k\big(\mathbf{m}_k^{\mathrm{ref}} - \mathbf{m}^{n-1}\big).\end{aligned}$$

  Update $\mathbf{R}_d$ and $\mathbf{R}_k$.

## Measure of model structure

- Regularization via finite-difference matrices.

  Old way:



## Measure of model structure

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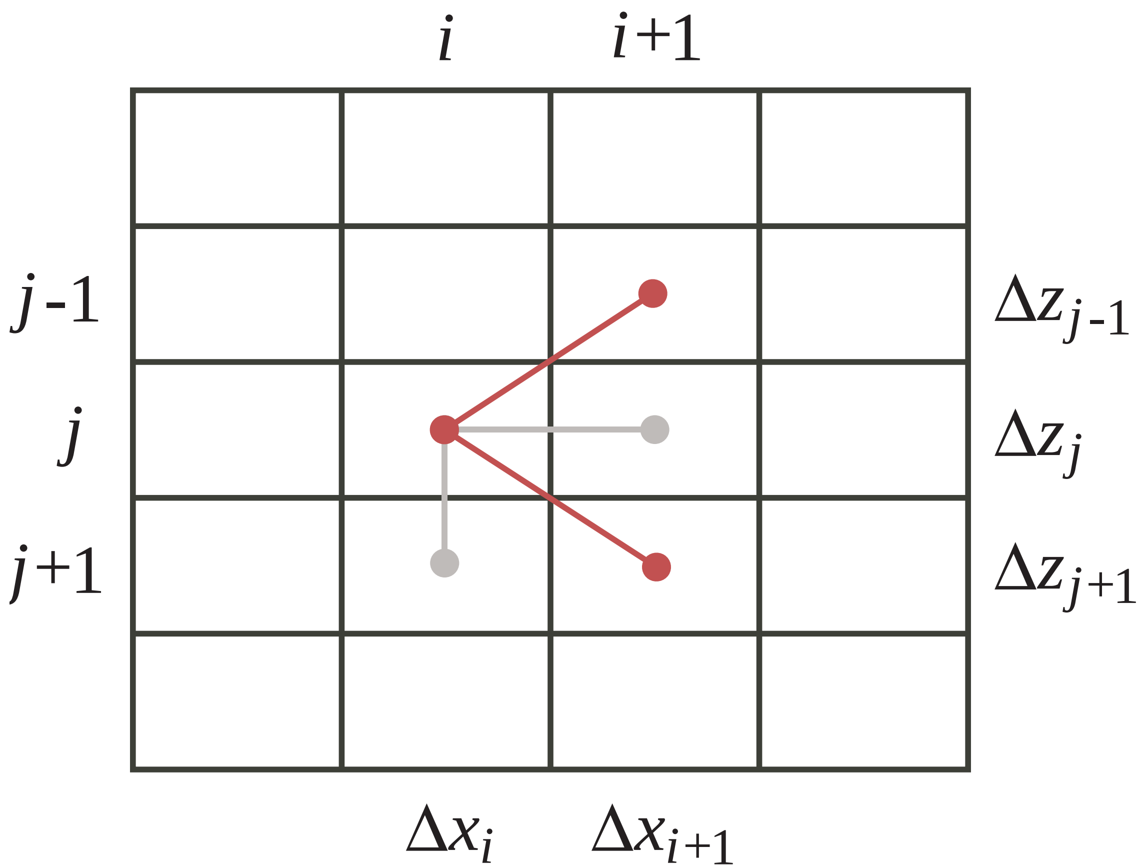
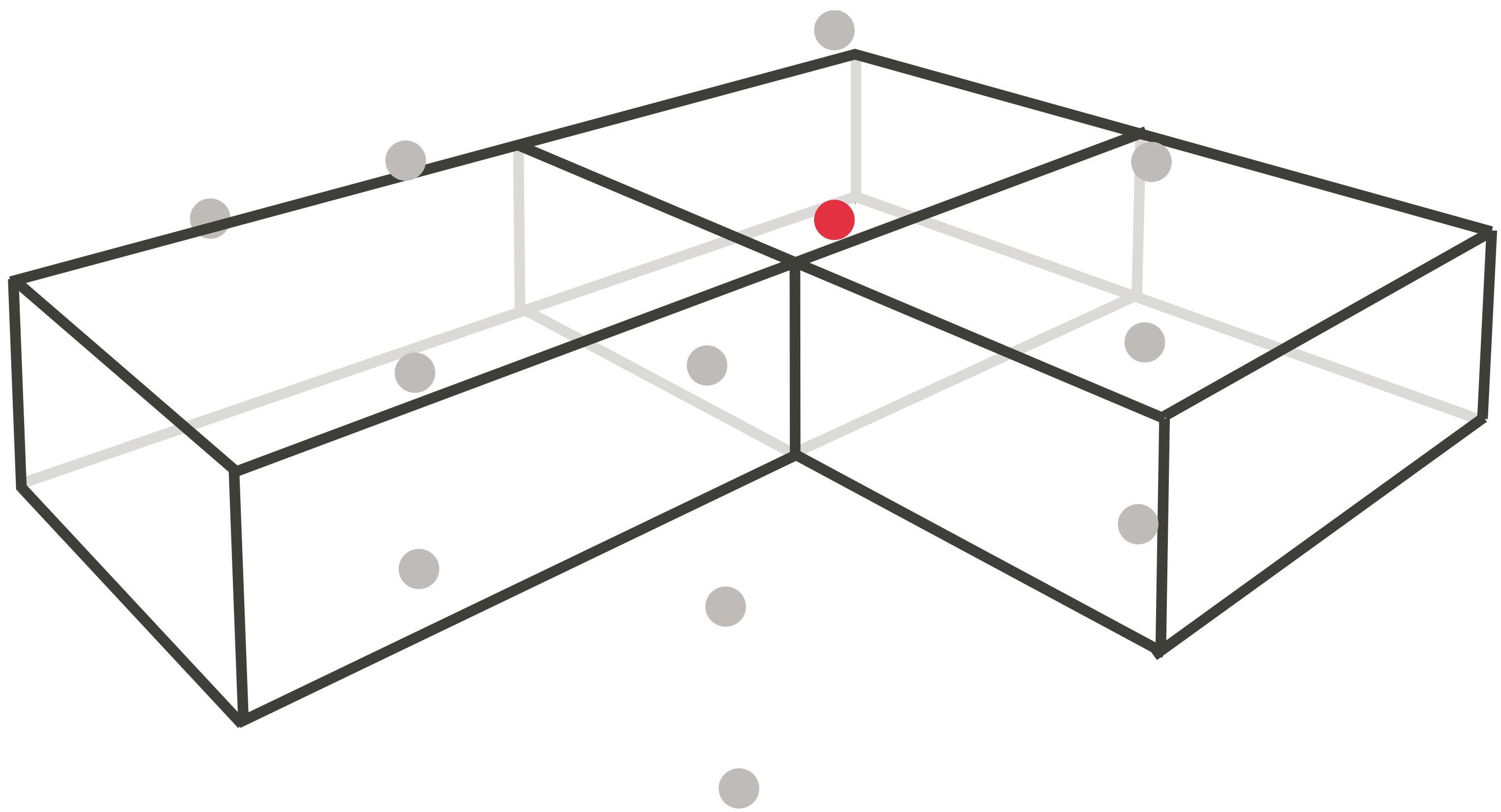
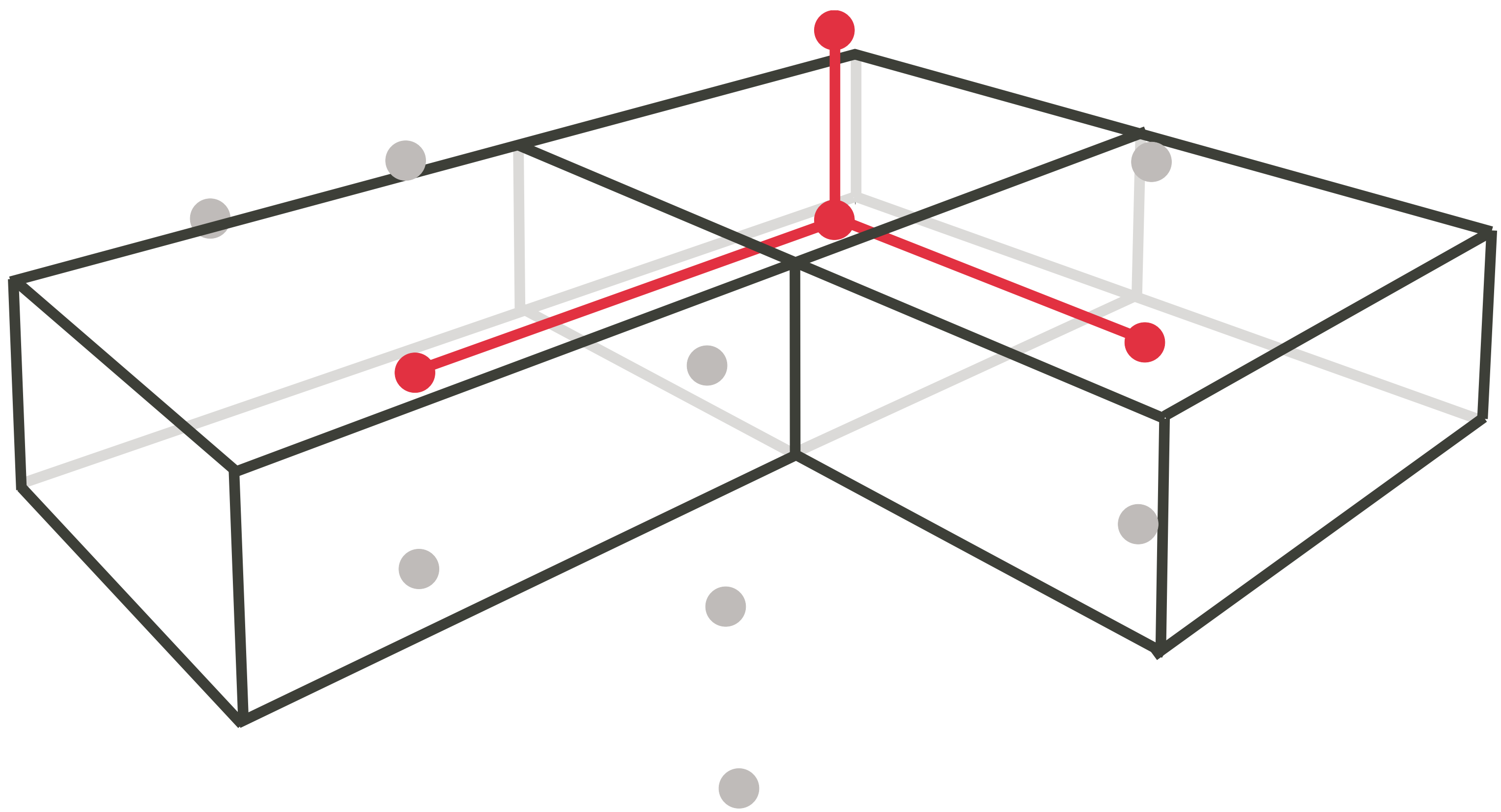
  New way:



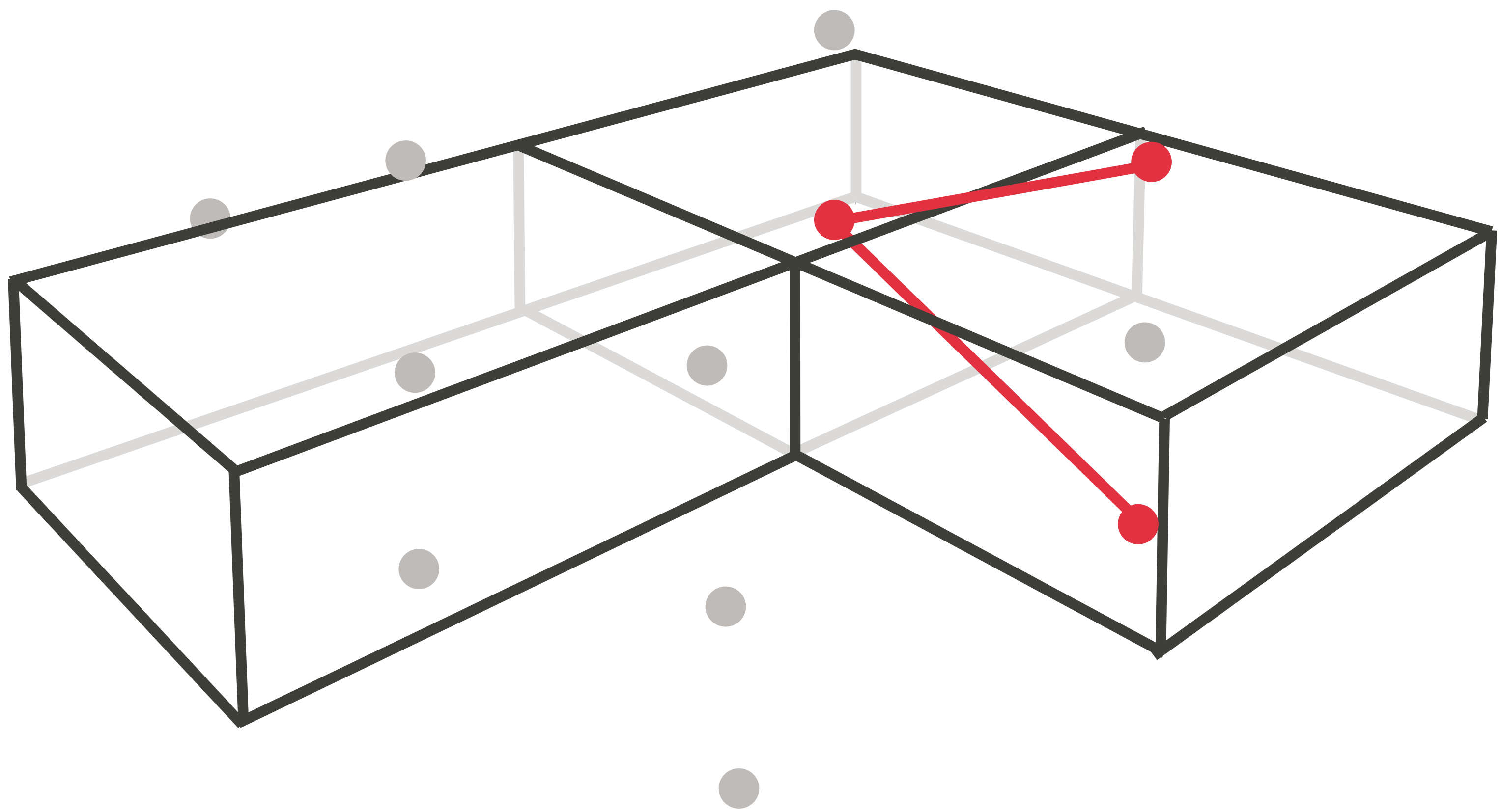
**Measure of model structure**



**Measure of model structure**



# Measure of model structure



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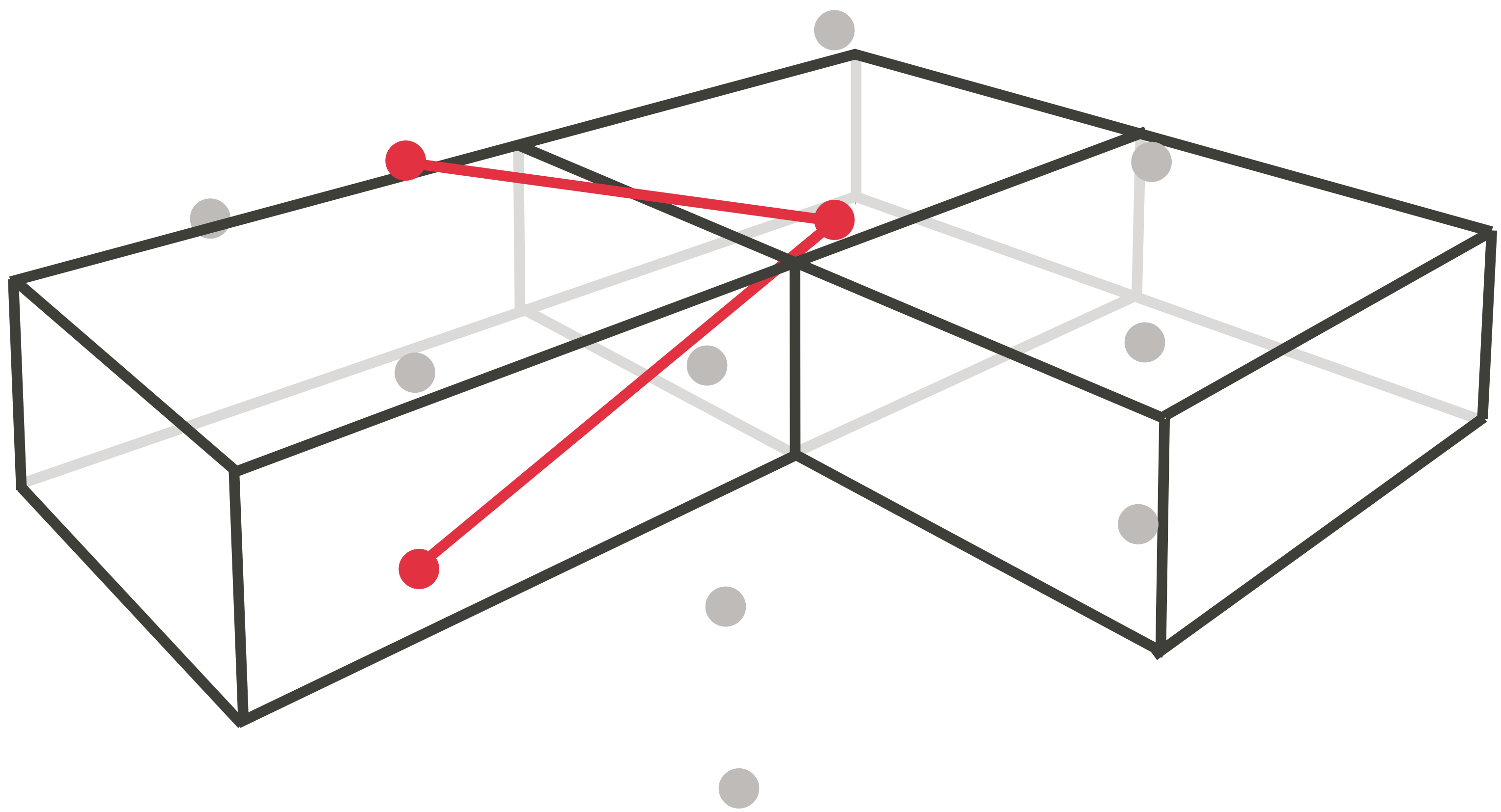

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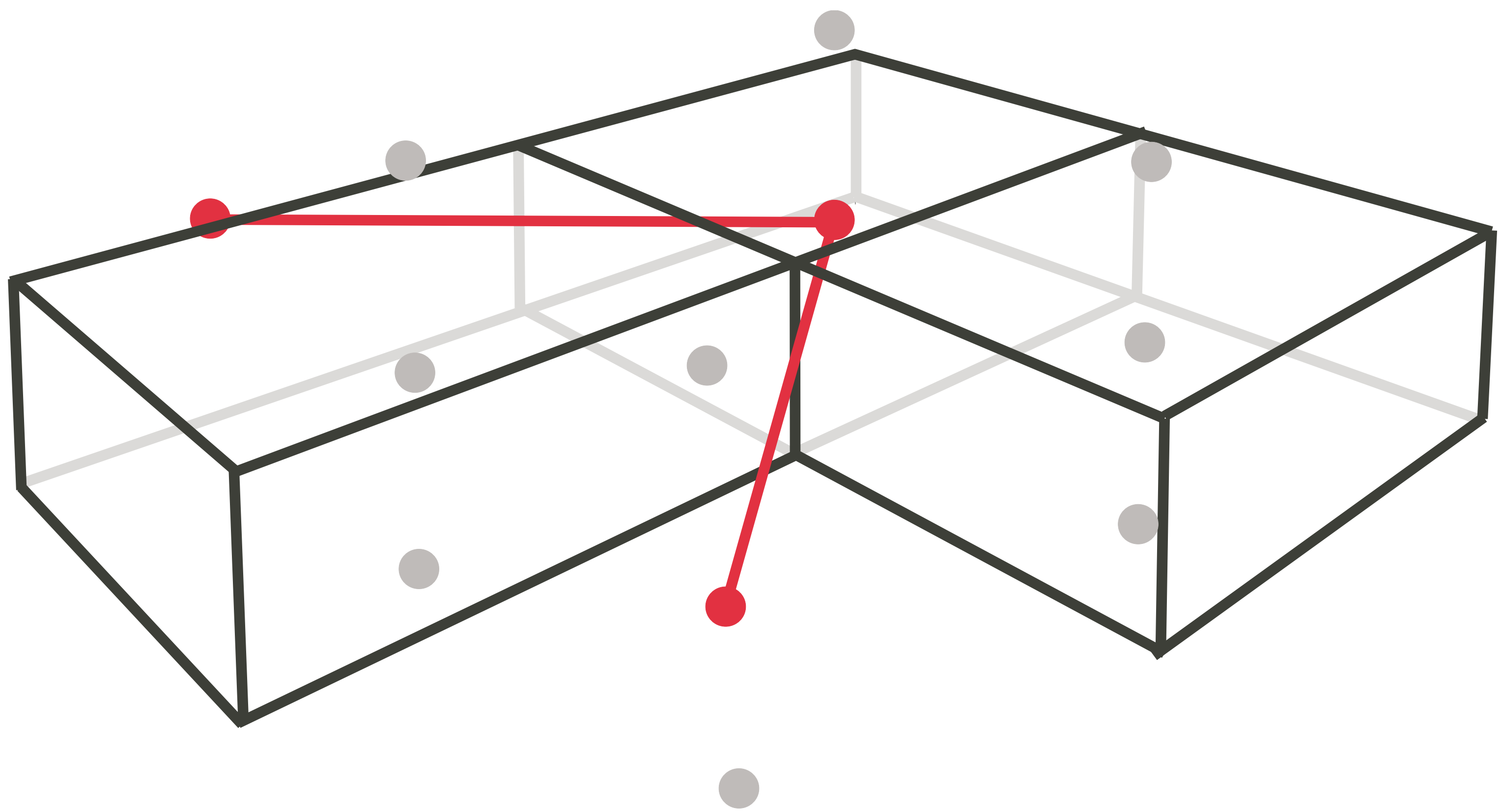

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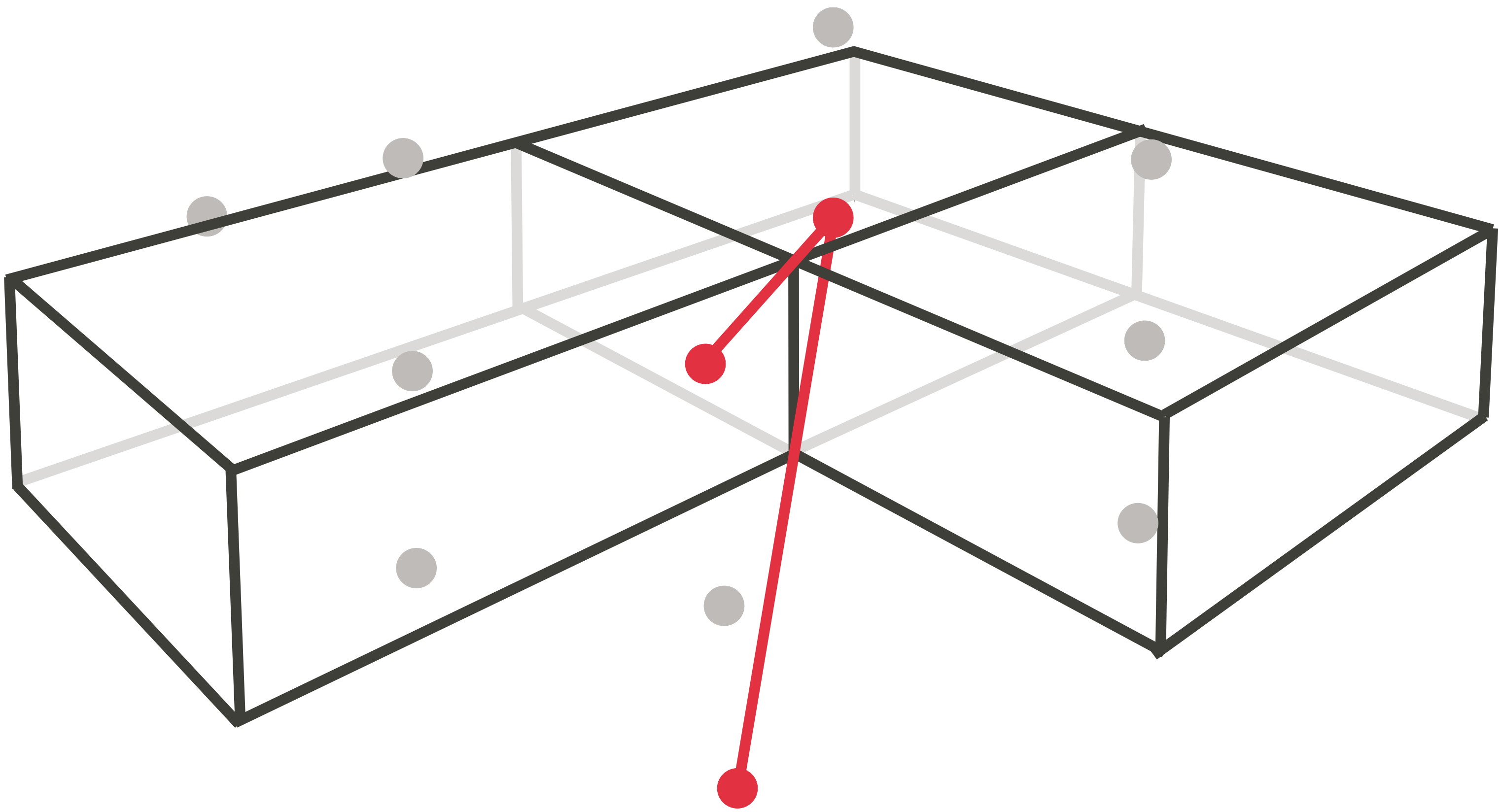

# Measure of model structure

- The measure of model structure becomes

$$\phi_m = \sum_k \alpha_k \phi_k(\mathbf{v}_k) \qquad \mathbf{v}_k = \mathbf{W}_k\left(\mathbf{m} - \mathbf{m}_k^{\text{ref}}\right),$$

where the summation is now over 14 terms, rather than 4.

## Particulars of 3-D gravity inversion program used here

- Finite-difference forward solver.
- Preconditioned CG solver for Gauss-Newton equations.
- Preconditioner is ILU decomposition with approximate Jacobian.
- Sparse matrix-vector products, and solution of forward system.

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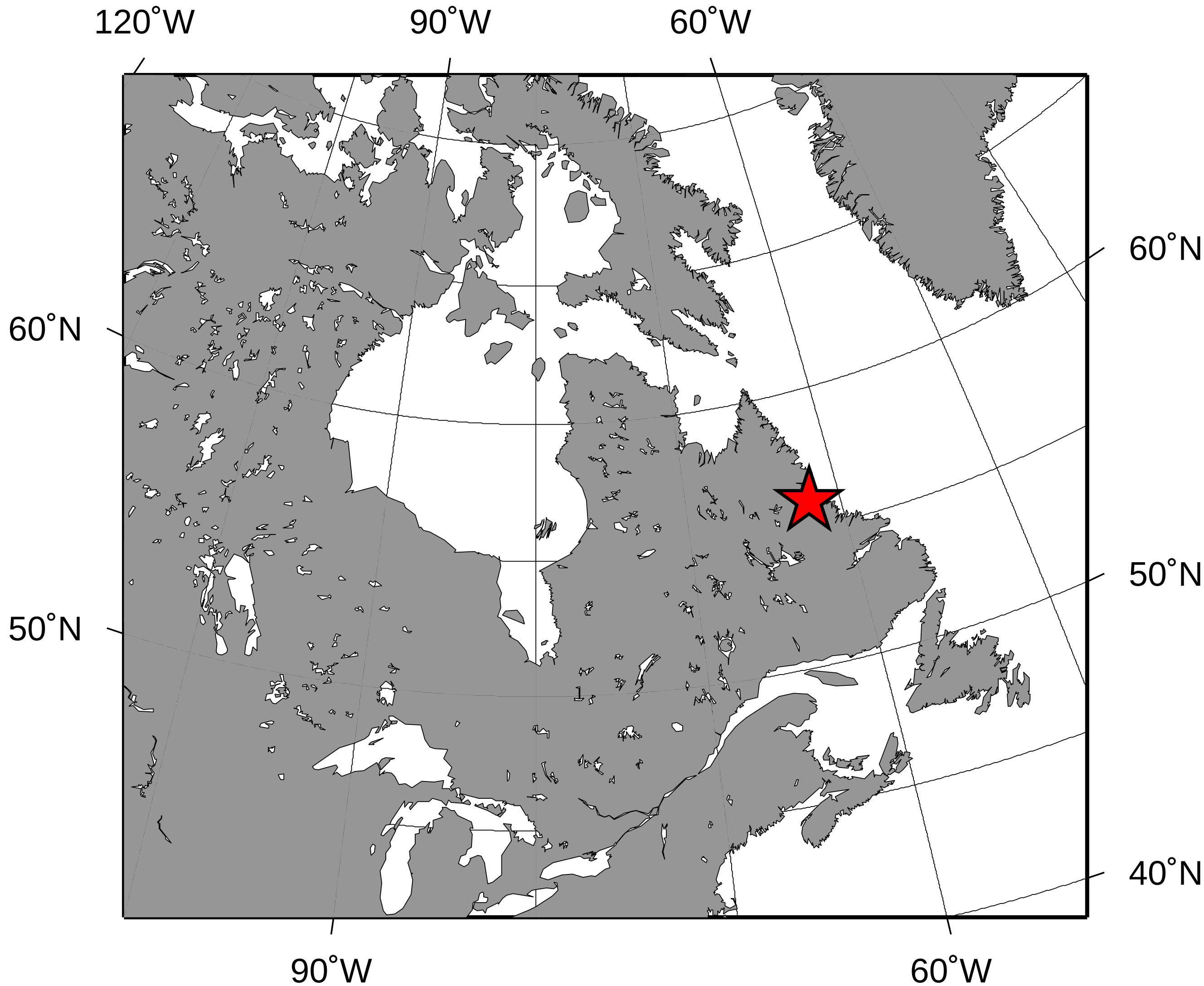
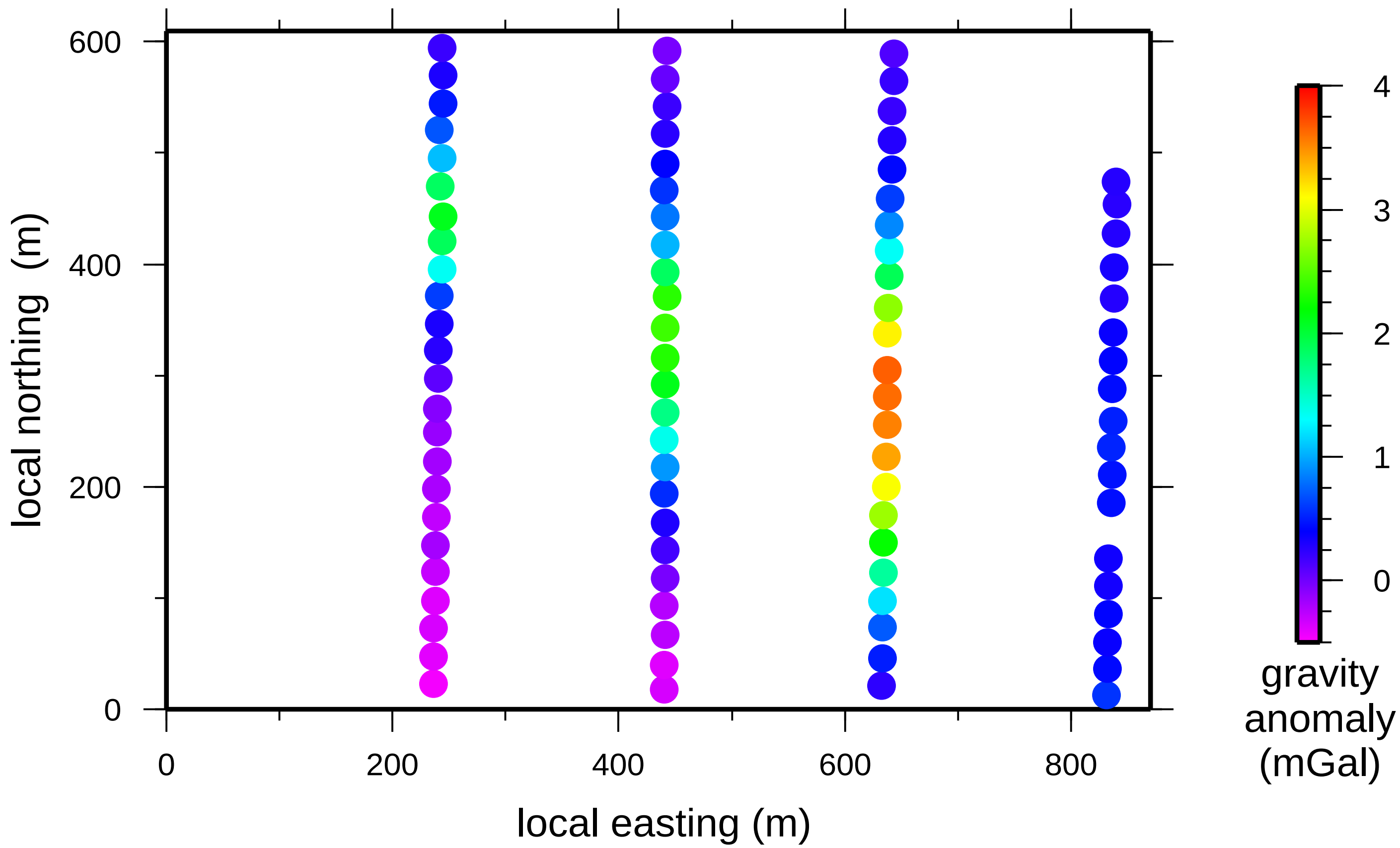
# Voisey's Bay



# Observed Bouguer anomaly over the Ovoid




- Bouguer anomaly relative to $2.67\,\mathrm{g/cm^3}$.
- Regional removal by upward continuation – Mike's talk.
- 89 data.
- Assumed measurement uncertainties of $0.05\,\mathrm{mGal}$.

# Inversions

- Results for three inversion coming up.
  For all inversions …

  Mesh: $87 \times 61 \times 54$ cells, each cell $10 \times 10 \times 5\,\mathrm{m}$.

  Topography incorporated.

  Overburden incorporated via the reference model.

  Same depth weighting as GRAV3D.

  More smoothing in easting direction (relative to northing);
  less smoothing in vertical direction (relative to northing).

# Inversions

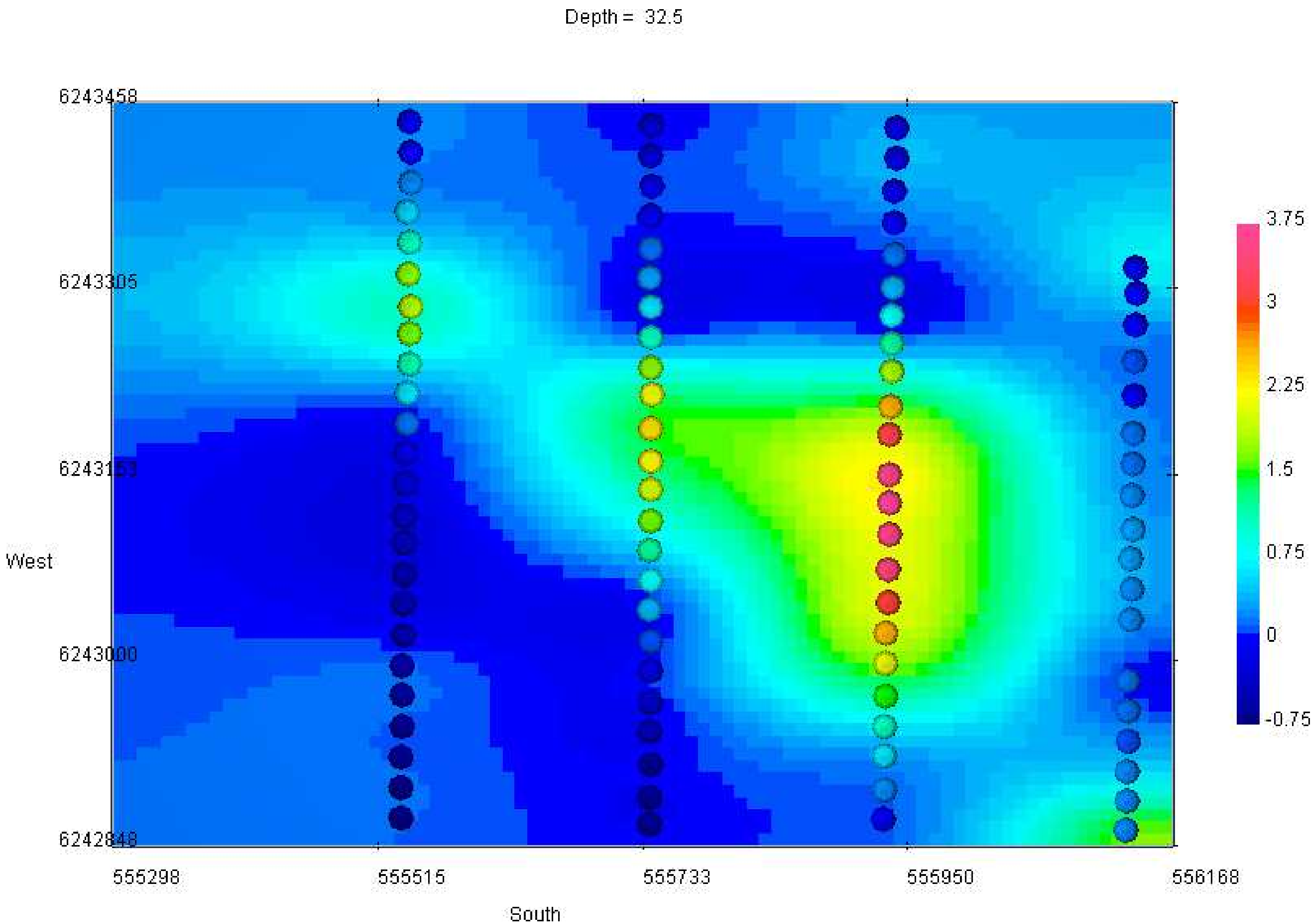
1. Traditional $l_2$ measure of model structure:
   - only the usual $x$, $y$, $z$ finite differences in $\phi_m$.

2. $l_1$-type measure of model structure:
   - only the usual $x$, $y$, $z$ finite differences in $\phi_m$;
   - 20 iterations.

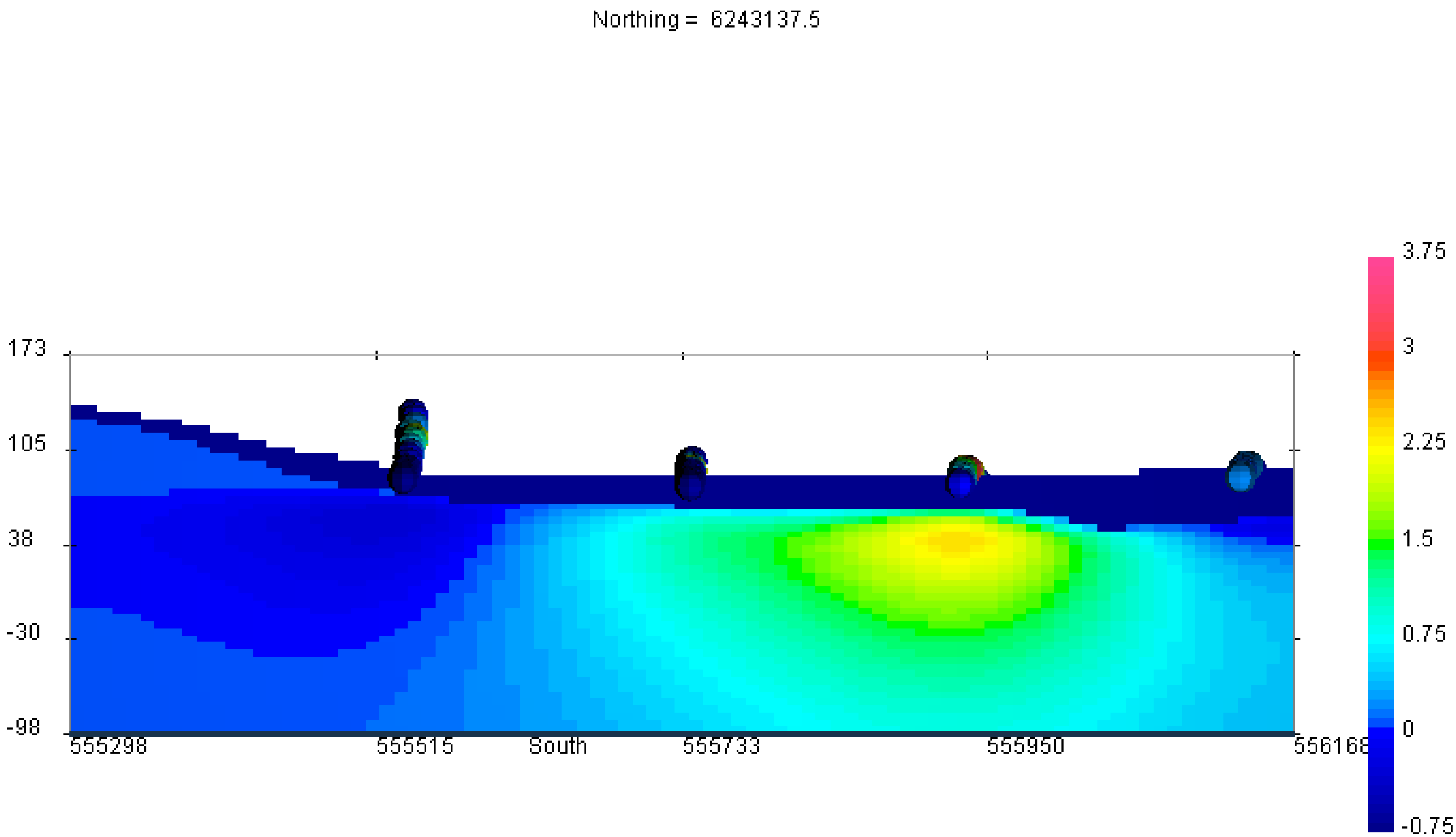
3. $l_1$-type measure of model structure:
   - all diagonal finite differences included in $\phi_m$;
   - 20 iterations.

- Sum-of-squares, $l_2$ data misfit used in all inversions. (Final misfits for the three inversions: 108, 103, 100.)

# Inversion 1: $l_2$



# Inversion 1: $l_2$



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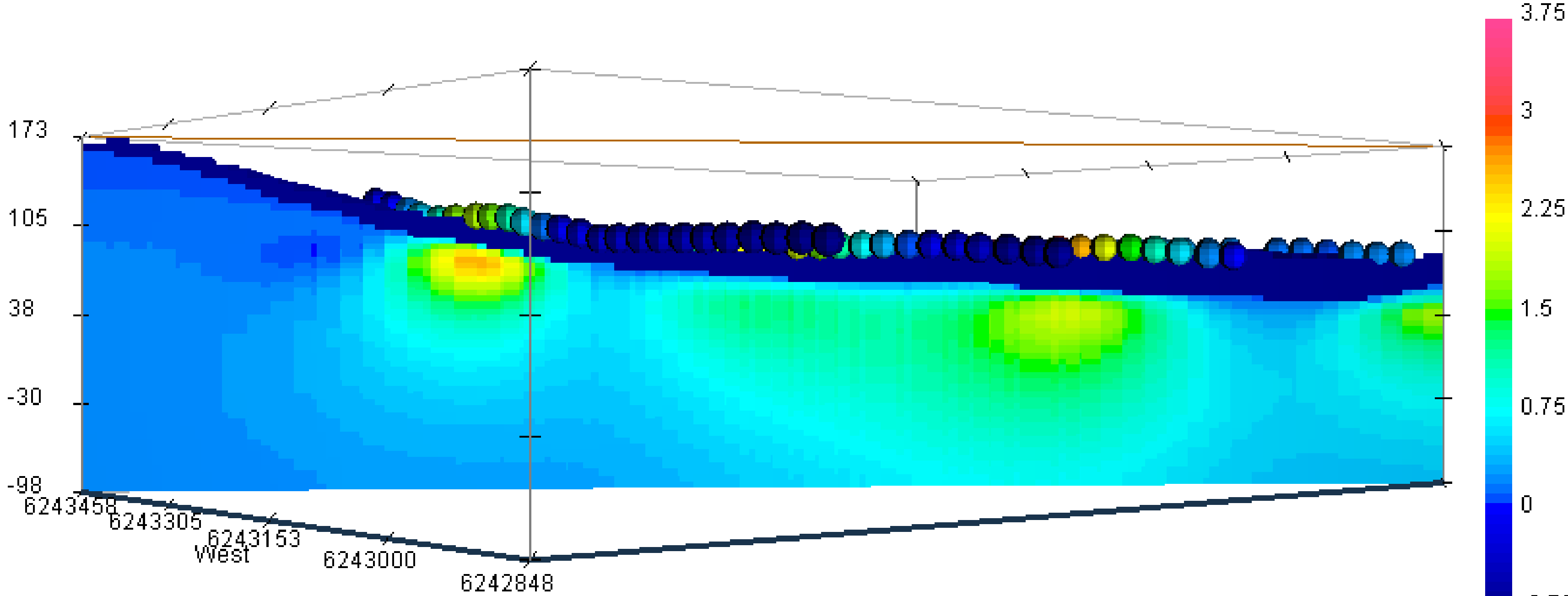
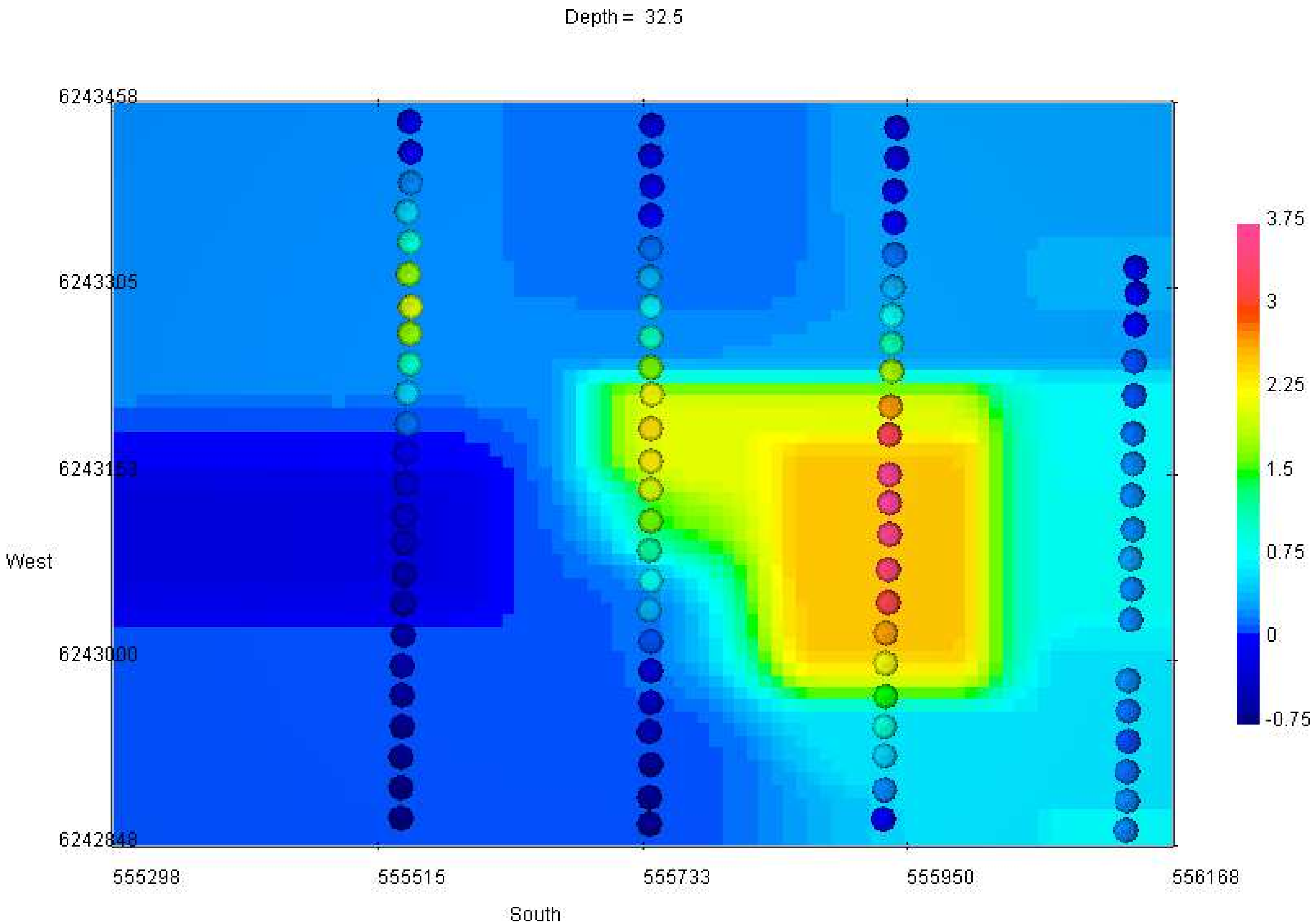


# Inversion 2: $l_1$, no diagonal differences



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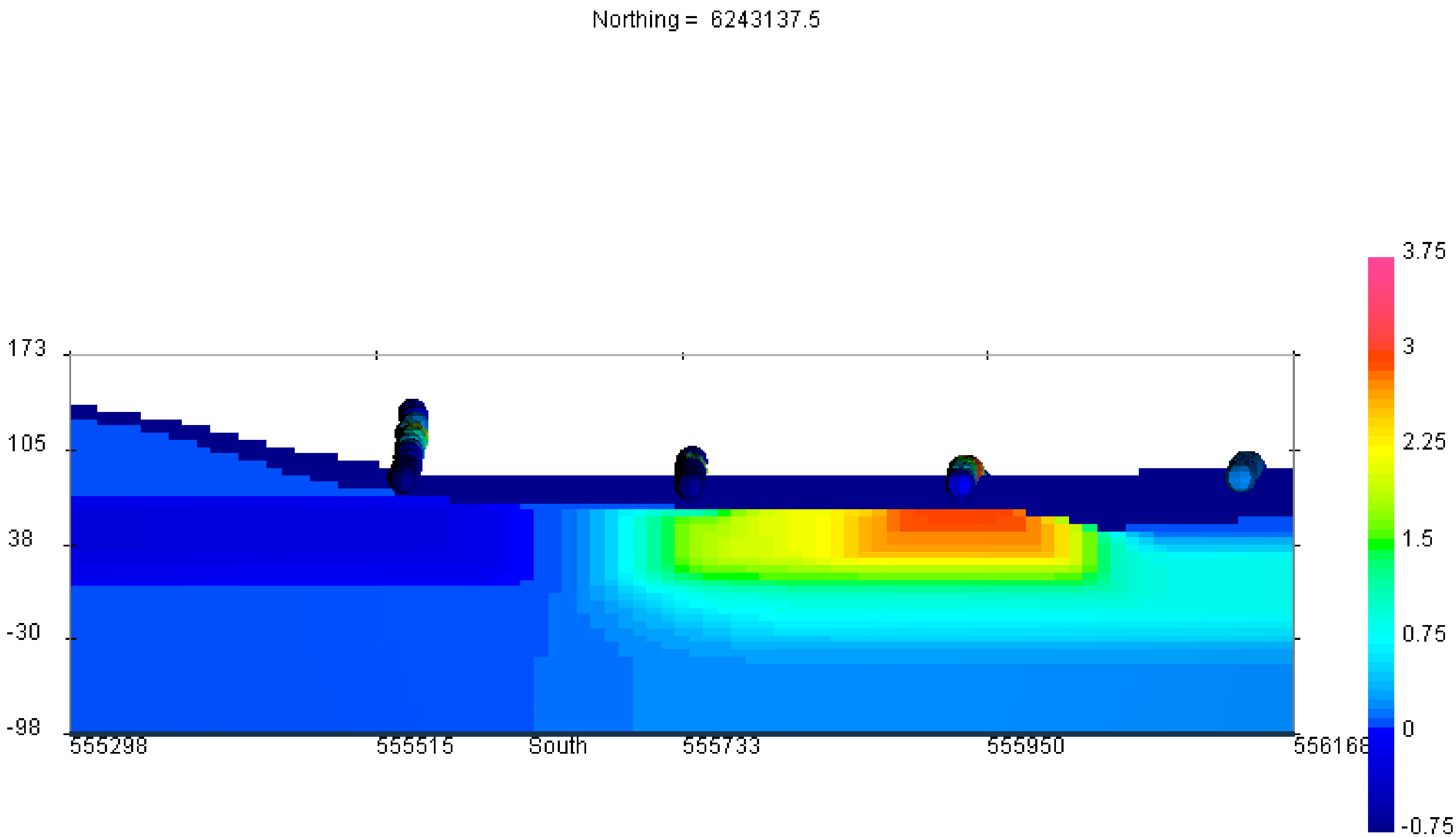

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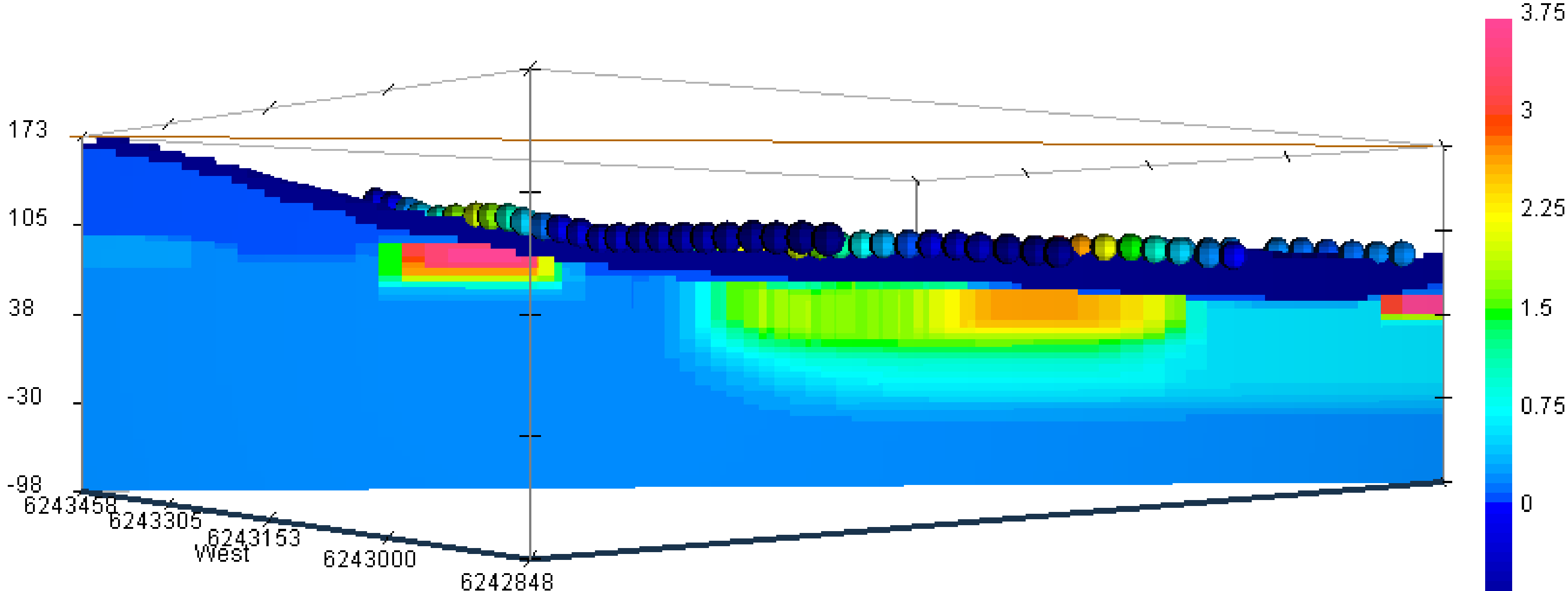
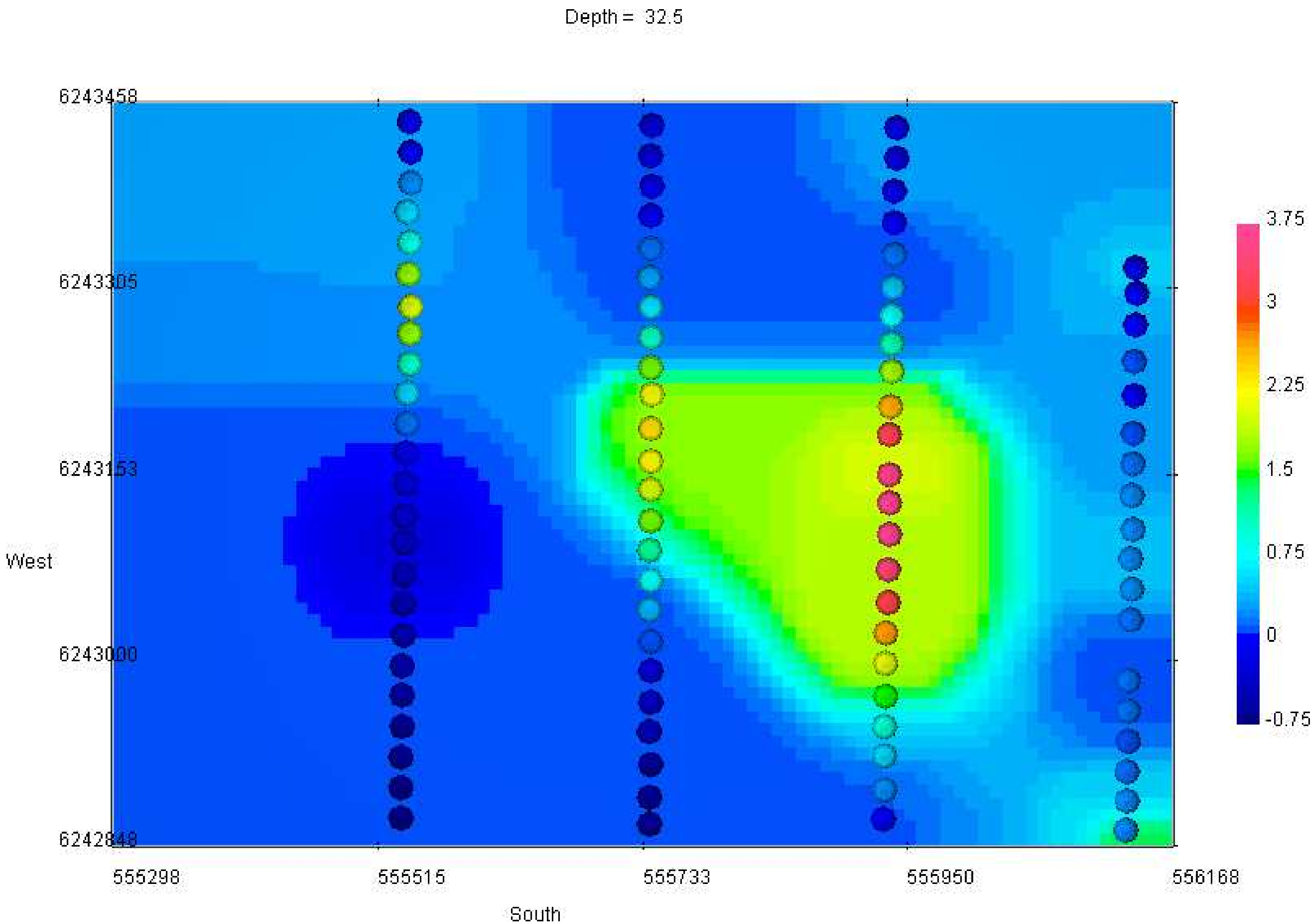


# Inversion 3: $l_1$, diagonal differences



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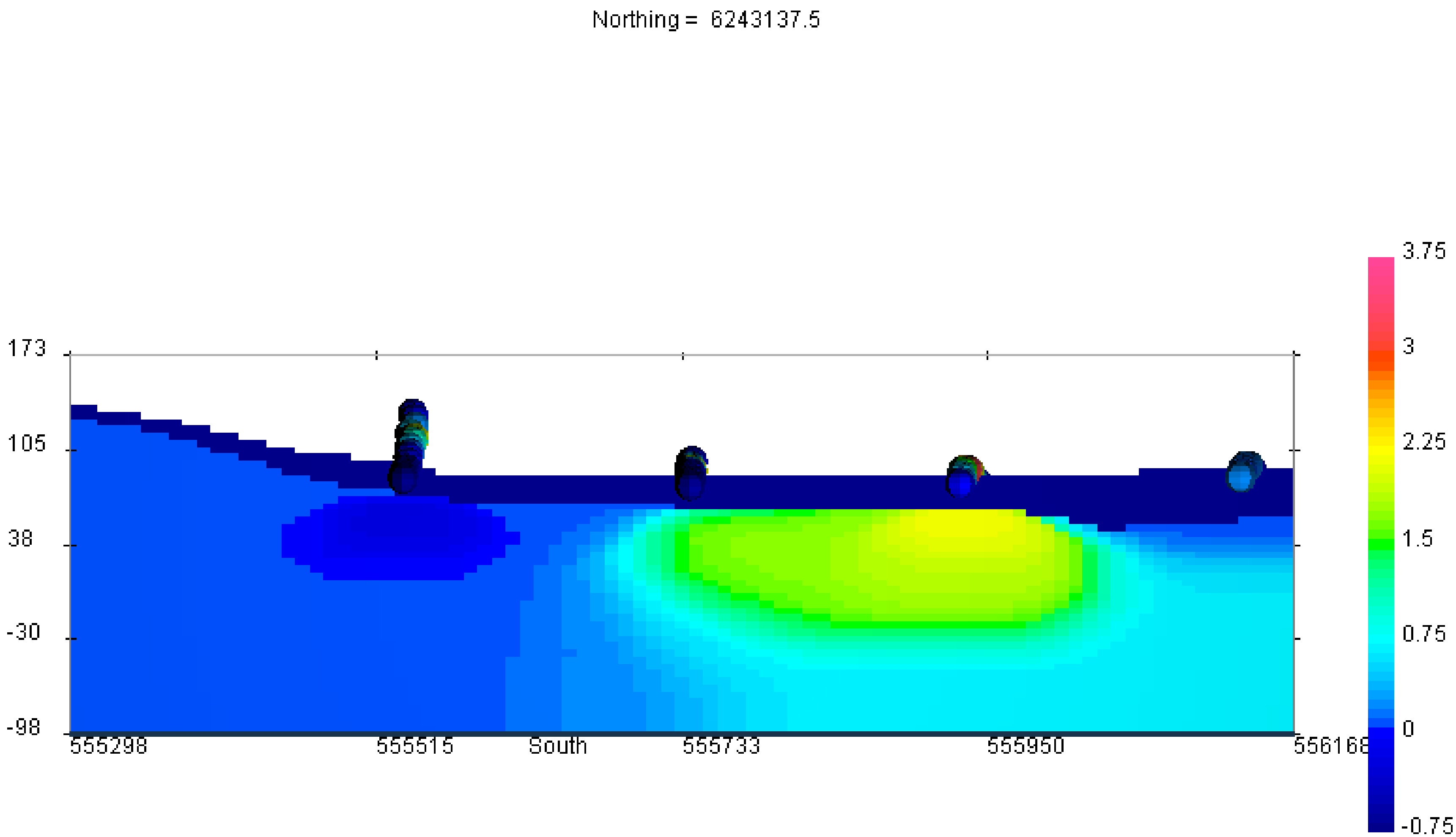

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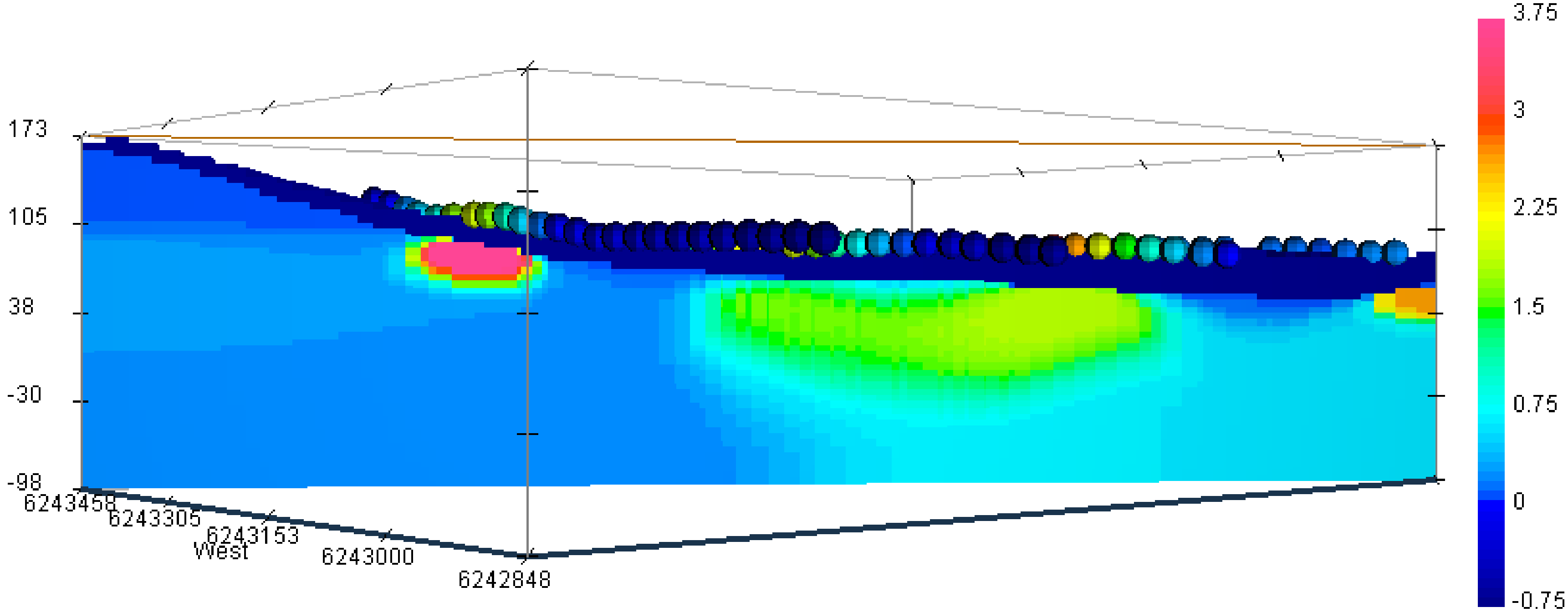

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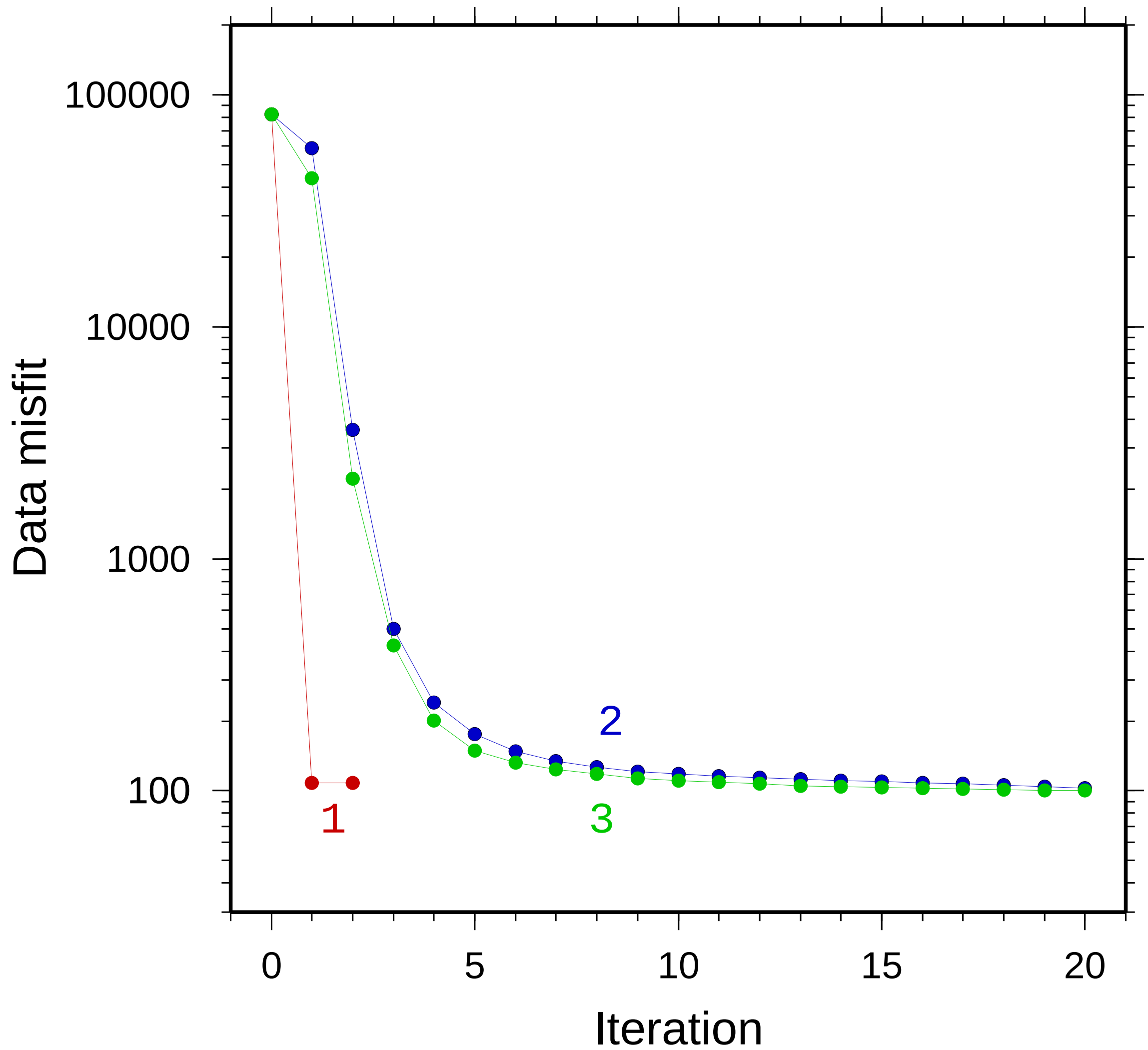

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# Conclusions

- *Minimum-structure inversions* can be made to produce *blocky models* by using *non-$l_2$ measures* and *iterative* solution procedures.

- Explicit inclusion of *diagonal differences* in the measure of model structure allows *dipping interfaces* to be produced.

◦ Computation time is significantly increased for linear inverse problems: not such an onerous increase for an already non-linear problem.

◦ Interfaces not quite as sharp as I had hoped – because of CG solver of Gauss-Newton system?