Lithological and surface geometry joint inversions using multi-objective global optimization methods

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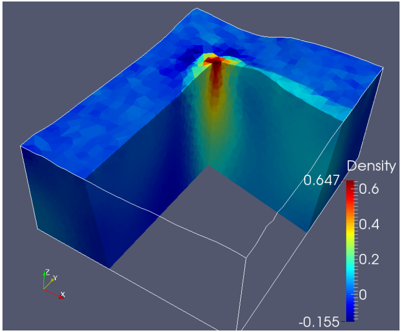
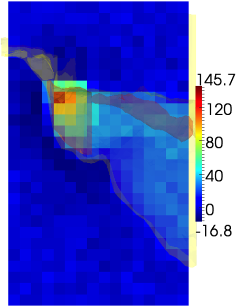
Geophysical inversion primer

Forward problem



Earth model (e.g. density)



Survey data (e.g. gravity)

Inverse problem

Typical numerical approach for mesh-based inversion (underdetermined)

- Local descent-based minimization
- Weighted aggregate objective function

$$\min_{\mathbf{m}} \quad f_a(\mathbf{m}) = \sum_i w_i f_i(\mathbf{m}) \qquad (1)$$

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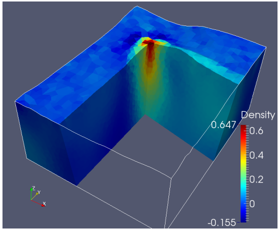
$$\min_{\mathbf{m}} \quad f_a(\mathbf{m}) = \lambda f_d(\mathbf{m}) + f_m(\mathbf{m})$$



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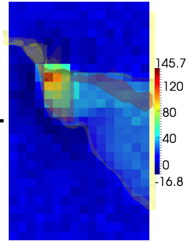

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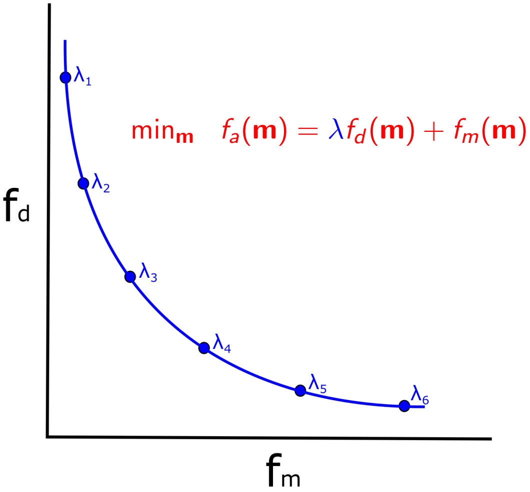

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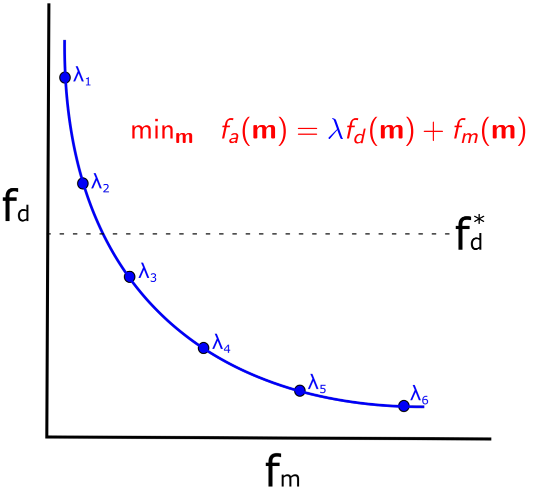

Disadvantages of the typical inversion approach

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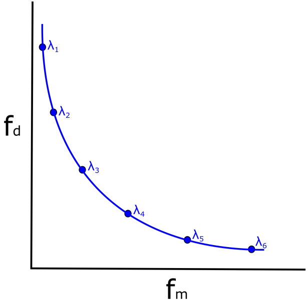
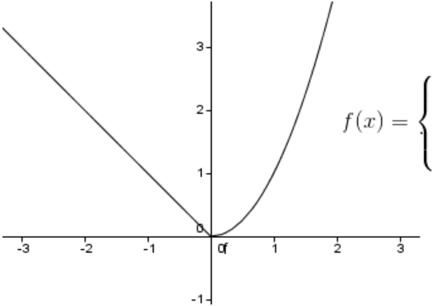


Disadvantages of the typical inversion approach

1. Appropriate weights must be determined for the aggregate objective function
2. All objective functions must be differentiable



$$f(x) = \begin{cases} x^2 & \text{if } x > 0 \\ -x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Disadvantages of the typical inversion approach

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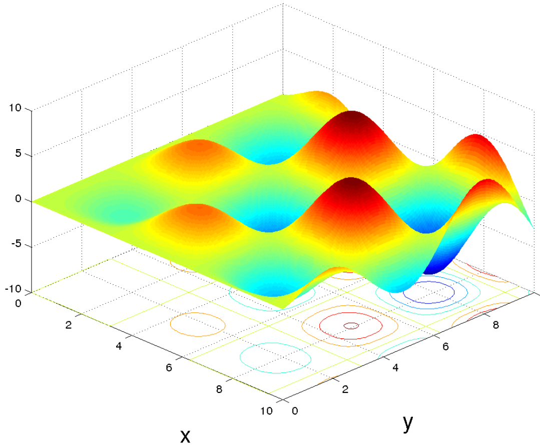
3. Local minima entrapment may occur, providing suboptimal solutions



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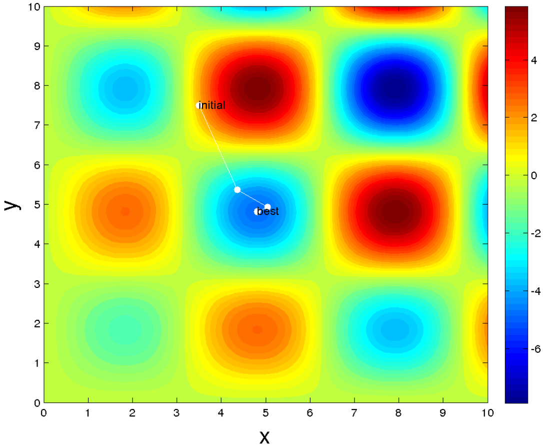

Pareto Multi-Objective Global Optimization (PMOGO)

- Multi-Objective Problem (MOP):

$$\min_{\mathbf{m}} \quad \mathbf{f}(\mathbf{m}) = \left[f_1(\mathbf{m}), f_2(\mathbf{m}), \ldots, f_L(\mathbf{m}) \right]$$

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$$\begin{aligned} f_i(\mathbf{m}_a) &\leq f_i(\mathbf{m}_b) \qquad \textit{for } i = 1, \ldots, L \\ f_i(\mathbf{m}_a) &< f_i(\mathbf{m}_b) \qquad \textit{for at least one } i \end{aligned}$$

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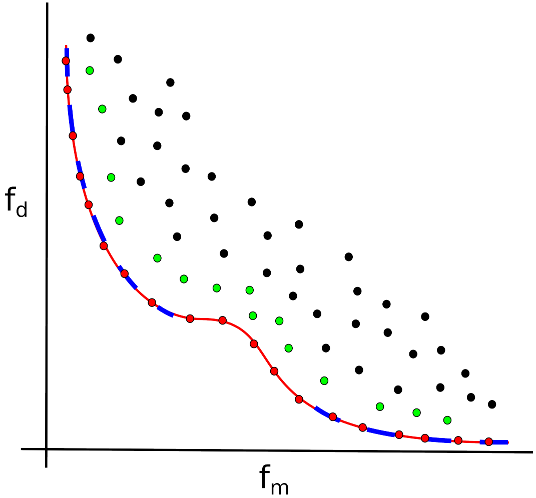

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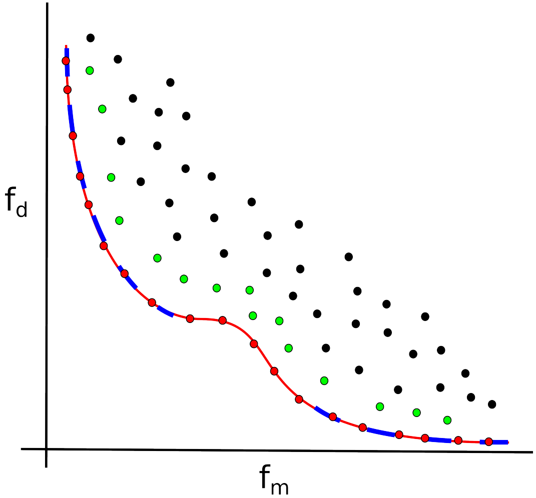
- We want to converge to the Pareto-optimal curve/surface (related to Tikhonov curve)



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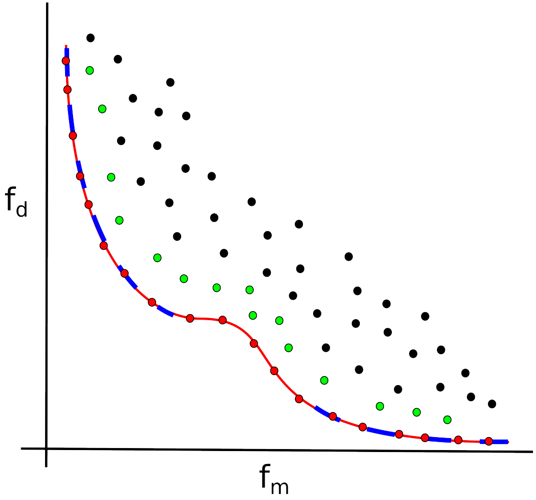
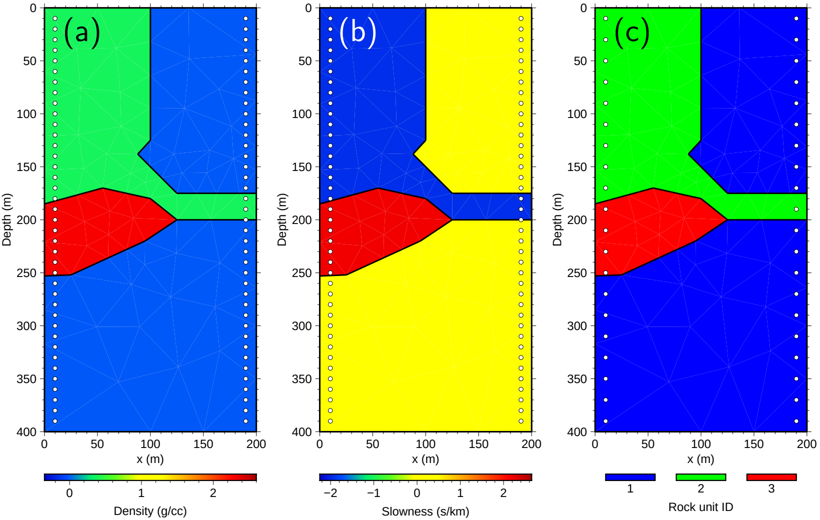
- We want to converge to the Pareto-optimal curve/surface (related to Tikhonov curve)
- Non-dominated Sorting Genetic Algorithm (NSGA-II) of Deb et al. (2002)



Massive sulphide deposit scenario from Carter-McAuslan et al. (2015)



Rock types:

- background (gneiss)
- intrusive (troctolite)
- lens (sulphide)

Joint inversion of gravity data and traveltimes

Aggregate objective function:

$$
\begin{array}{cccccc}
 & & \text{Data misfits} & & \text{Model roughness} & & \text{Coupling} \\
f_a(\mathbf{m}_1, \mathbf{m}_2) & = & \left[\lambda_1 f_{d1}(\mathbf{m}_1) + \lambda_2 f_{d2}(\mathbf{m}_2)\right] & + & \left[f_{m1}(\mathbf{m}_1) + f_{m2}(\mathbf{m}_2)\right] & + & \rho f_c(\mathbf{m}_1, \mathbf{m}_2)
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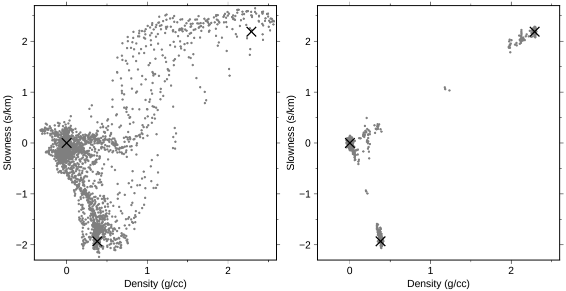
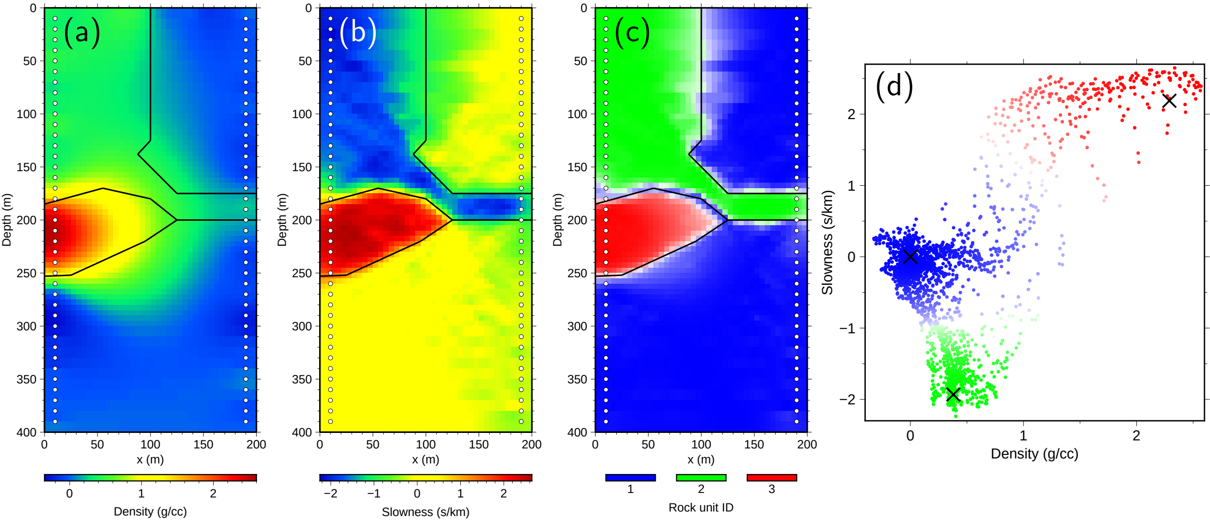
Joint coupling term (fuzzy c-means):

$$f_c(\mathbf{m}_1,\mathbf{m}_2) = \sum_{i=1}^{C}\sum_{k=1}^{M} w_{ik}^2 z_{ik}^2$$

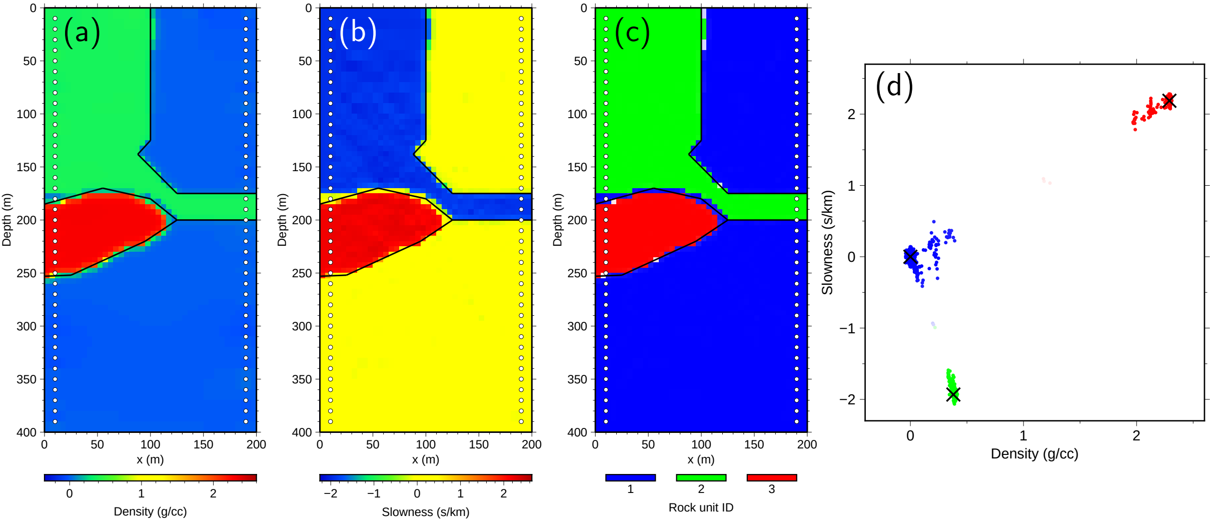
$$z_{ik}^2 = (\mathbf{m}_{1,k} - u_{1,i})^2 + (\mathbf{m}_{2,k} - u_{2,i})^2$$



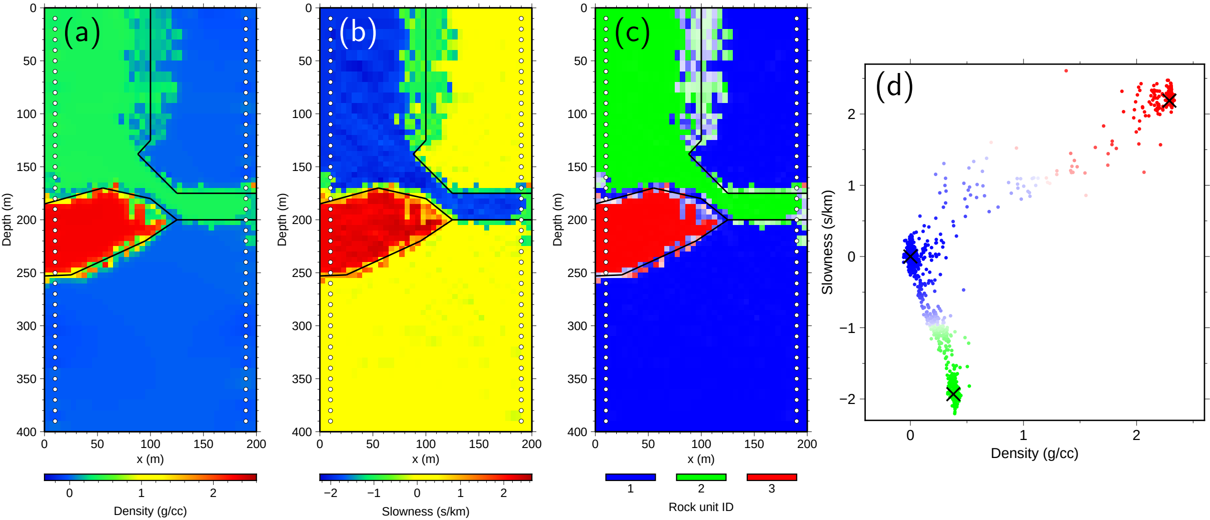
Independent inversion results (aggregate & local minimization)



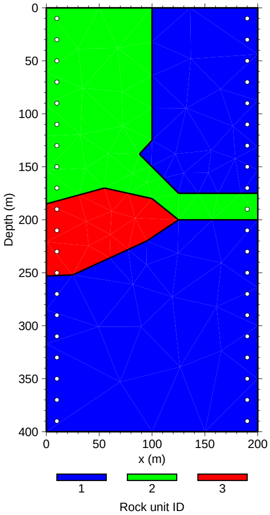
Joint inversion with careful local minimization (hand holding)



Joint inversion with careless local minimization



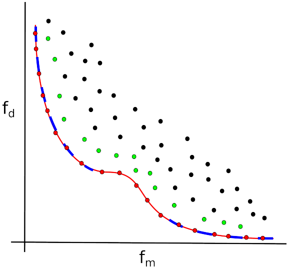
Lithological inversion

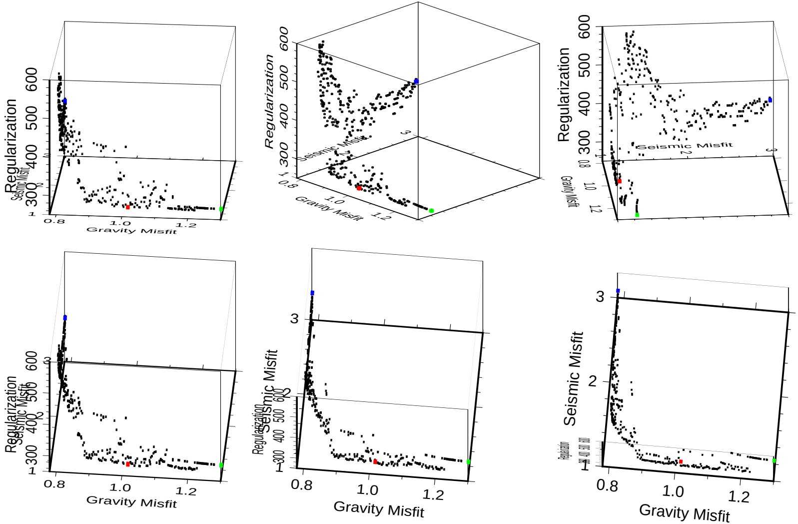


- Define several a priori rock types (3 here)
- Define physical property distributions for each rock type (homogeneous here)
- PMOGO inversion assigns a rock type to each mesh cell
- Simple to add numerically complicated topological constraints, e.g. the model must contain:
    - 4 contiguous regions
    - all 3 rock types
    - 2 regions corresponding to the background

Lithological inversion: 3D Pareto front

Pareto surface vs curve





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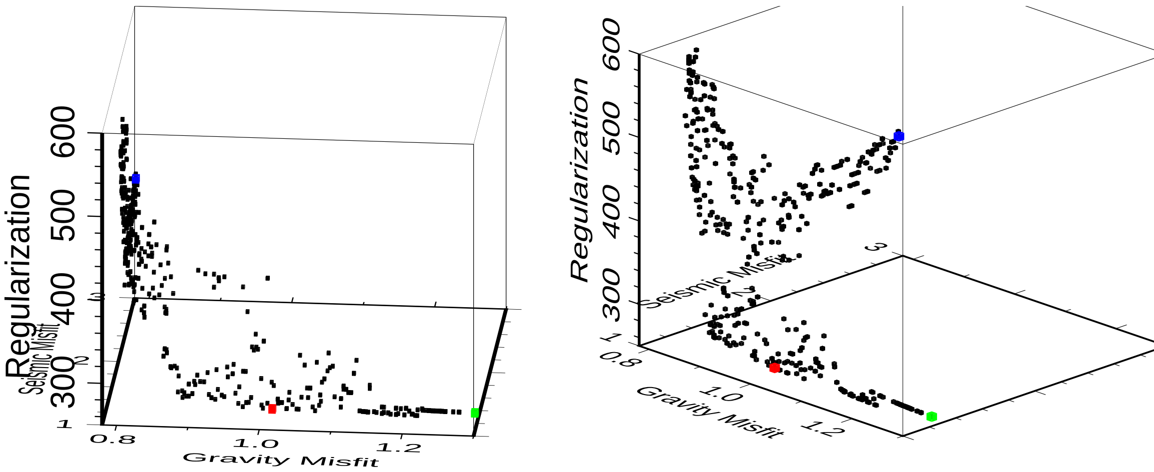
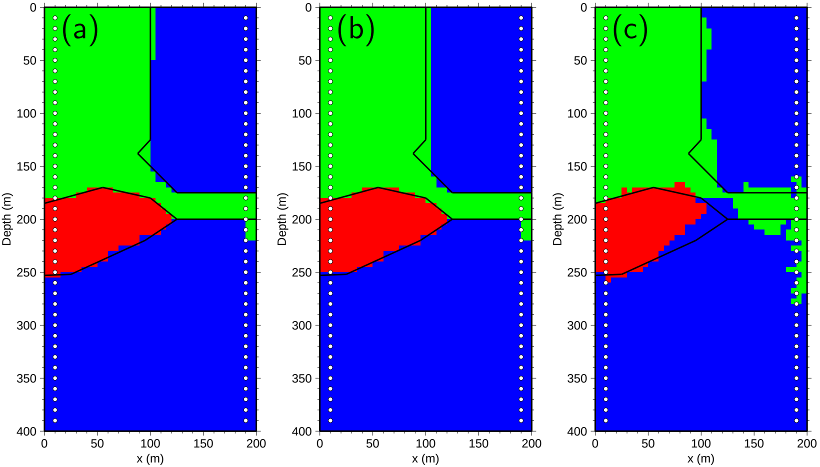


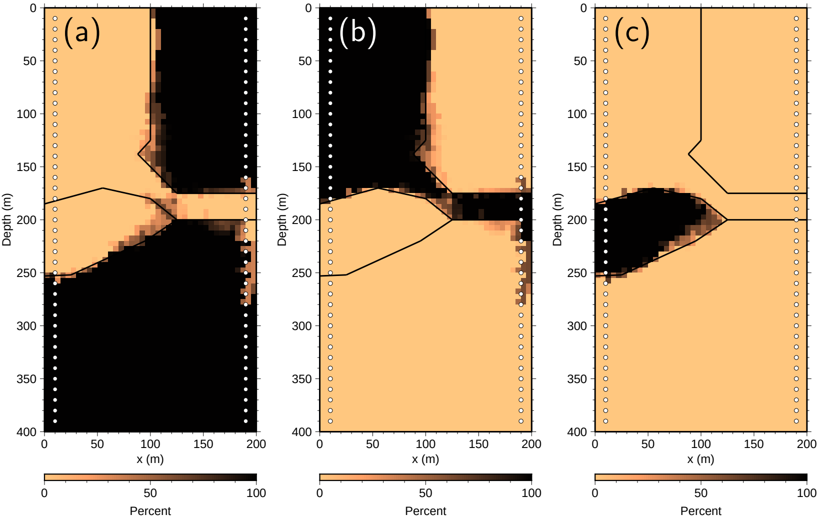
Lithological inversion: three models in the Pareto front



Misfits:

- (a) expected targets
- (b) high gravity
- (c) high seismic

Lithological inversion: some rudimentary statistics

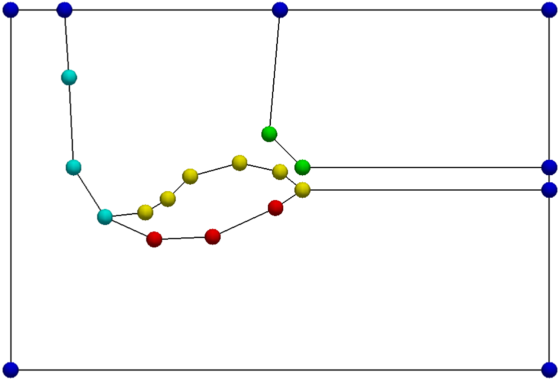


Rock types:

(a) background
(b) intrusive
(c) sulphide

Surface-based inversion

- Parameterization defines interfaces between rock units
- No mesh required (very fast forward problem)
- PMOGO inversion moves vertices



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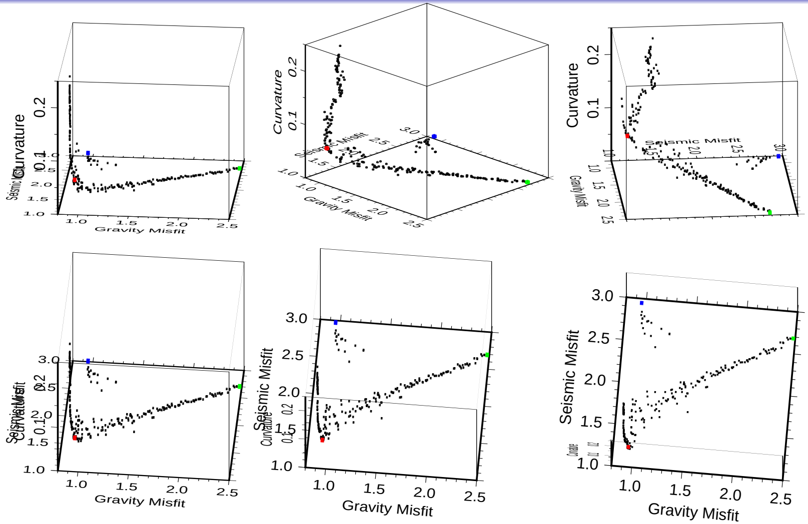

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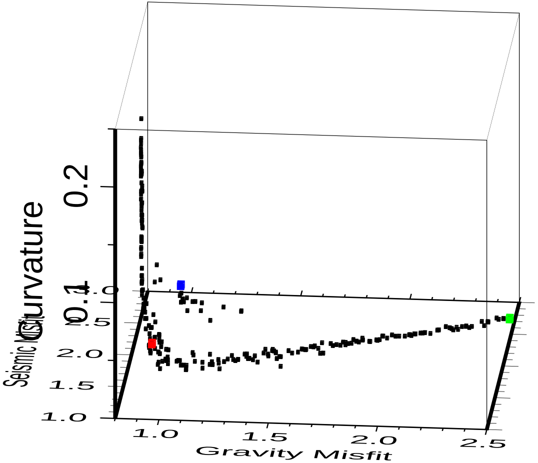

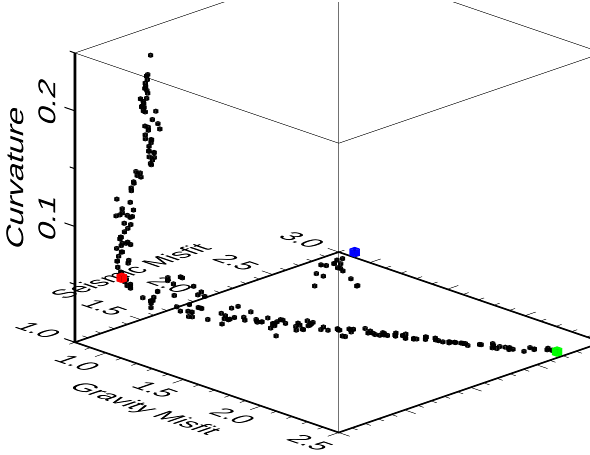

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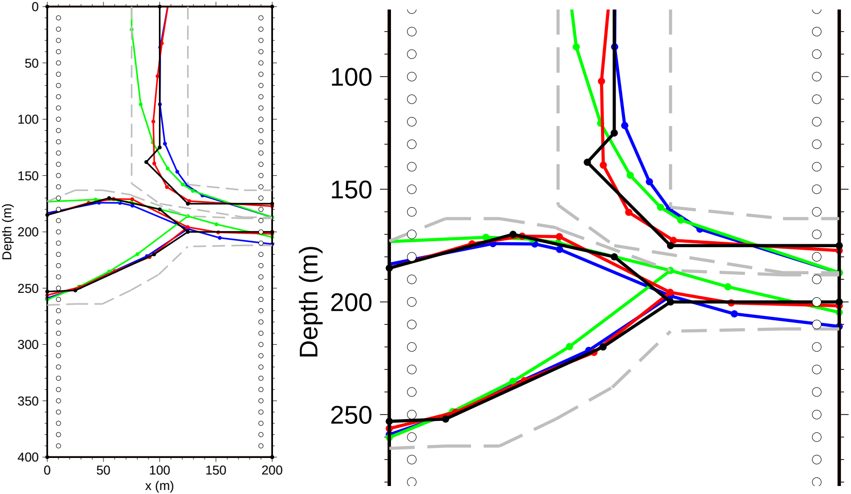

Legend:

- - - feasible region
- — true interfaces
- — expected target misfits
- — both misfits high
- — high seismic misfit

Advantages and disadvantages of PMOGO

Advantages:

- Obviate the requirement of having to deal with trade-off parameters
  $\rightarrow$ Joint inversion greatly simplified

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- Avoid local minima entrapment
  → Fundamentally different model parameterizations

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Disadvantages:

- Increased computing time (100-300x for mesh-based inversions, WITH CAVEATS)

Geophysical Inversion With Multi-Objective Global Optimization Methods

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[1]Memorial University, Department of Earth Sciences, St. John's, NL, Canada
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Summary

We are investigating the use of Pareto multi-objective global optimization (PMOGO) methods to solve numerically complicated geophysical inverse problems. PMOGO methods can be applied to highly nonlinear inverse problems, to those where derivatives are discontinuous or simply not obtainable, and to those were multiple minima exist in the problem space. PMOGO methods generate a suite of solutions that minimize multiple objectives (e.g. data misfits, regularization and joint coupling measures) in a Pareto-optimal sense. This allows a more complete assessment of the possibilities and provides opportunities to calculate statistics regarding the likelihood of particular model features. We are applying PMOGO methods to several types of inverse problems.

Disadvantages of Typical Mesh-Based Inversion

The typical numerical approach for underdetermined mesh-based inversion is to perform a local descent minimization of a weighted aggregate objective function, subject to some constraints:

$$\min_{\mathbf{m}} \; f_a(\mathbf{m}) = \sum_i w_i f_i(\mathbf{m}) \qquad \text{s.t. } f_c(m) \geq 0$$

Joint inverse problems are generally posed as:

$$f_a(\mathbf{m}) = \sum_i \lambda_i f_{d,i}(\mathbf{m}_i) + \sum_j \alpha_j f_{m,j}(\mathbf{m}_j) + \sum_k \rho_k f_{c,k}(\mathbf{m}_{k,1}, \mathbf{m}_{k,2})$$

This approach has several disadvantages:

- appropriate weights must be determined
- all objective functions must be differentiable
- local minima entrapment may yield poor solutions

Pareto Multi-Objective Global Optimization

Multi-Objective Problem (MOP):

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Concept of dominance:

$$f_i(\mathbf{m}_a) \leq f_i(\mathbf{m}_b) \quad \text{for } i = 1, \ldots, L$$
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We want to converge to the Pareto curve/surface. We use the Non-dominated Sorting Genetic Algorithm of [3].

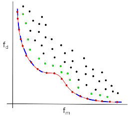

Figure 1 : Tikhonov curve, Pareto-optimal curve, and PMOGO solution population including Pareto front and secondary front.

Mesh-Based Lithological Inversion

In standard mesh-based inverse problems, the physical property values in each cell are treated as continuous variables. In lithological mesh-based inversions, the cells can only take discrete physical property values corresponding to known or assumed rock units. Here we apply such an inversion to the massive sulphide deposit scenario from [2] (joint inversion of gravity and seismic first arrival times).

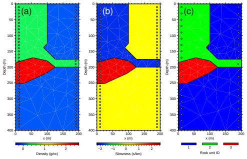

Figure 2 : True model. Rock units in (c): background, intrusive, sulphide lens.

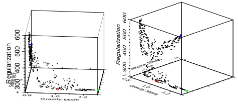

Figure 3 : 3D Pareto front from the lithological inversion.

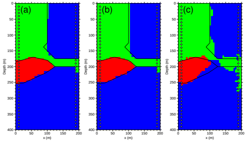

Figure 4 : Three representative models in the Pareto front corresponding to the (a) red, (b) green and (c) blue points in Fig. 3.

Mesh-Based Lithological Inversion (continued)

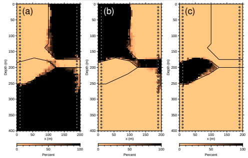

Figure 5 : Some rudimentary statistics: likelihoods for the (a) background, (b) intrusive and (c) sulphide lens rock units.

Surface Geometry Inversion

In surface geometry inversions, we consider a fundamentally different type of problem in which a model comprises wireframe surfaces representing contacts between rock units. The physical properties of each rock unit can remain fixed while the inversion controls the position of the contact surfaces via control nodes. Surface geometry inversion can be used to recover the unknown geometry of a target body or to investigate the viability of a proposed Earth model.

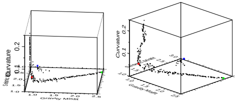

Figure 6 : 3D Pareto front from the surface geometry inversion.

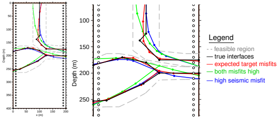

Figure 7 : Three representative models in the Pareto front corresponding to the (a) red, (b) green and (c) blue points in Fig. 6.

Discrete Body Inversion

In discrete body inverse problems, the inversion determines values of several parameters that define the location, orientation, size and physical properties of an anomalous body represented by a simple shape, for example a sphere, ellipsoid, cylinder or cuboid. A PMOGO inversion can simultaneously determine the optimal number of bodies, the optimal shapes and the optimal shape parameters.

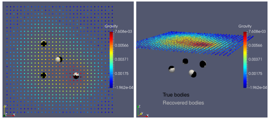

Figure 8 : Discrete body inversion for multiple source bodies (e.g. applicable to UXO problems).

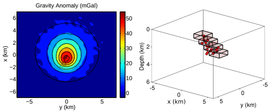

Figure 9 : Discrete body inversion using multiple point mass elements with regularization based on the minimum spanning tree concept. The true model is a stepped prism. Modified from [1].

Conclusion

PMOGO methods can solve numerically complicated problems that can not be solved with standard descent-based local minimization methods. This includes three of the classes of inverse problem we have described. There are significant increases in the computational requirements when PMOGO methods are used but these can be ameliorated using strategies such as parallelization and problem dimension reduction. We think it likely that with future work it will not be long before PMOGO methods can be applied to 3D inverse problems of useful size, particularly potential field inversions, and PMOGO methods could become an accepted standard approach for 2D inverse problems.

References

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[2] A. Carter-McAuslan, P. G. Lelièvre, and C. G. Farquharson. A study of fuzzy c-means coupling for joint inversion, using seismic tomography and gravity data test scenarios. *Geophysics*, 80(1):W1–W15, doi:10.1190/geo2014–0056.1, 2015.

[3] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, doi:10.1109/4235.996017, 2002.

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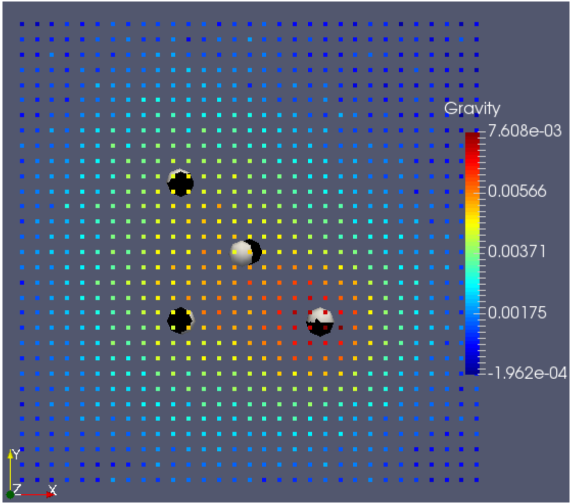
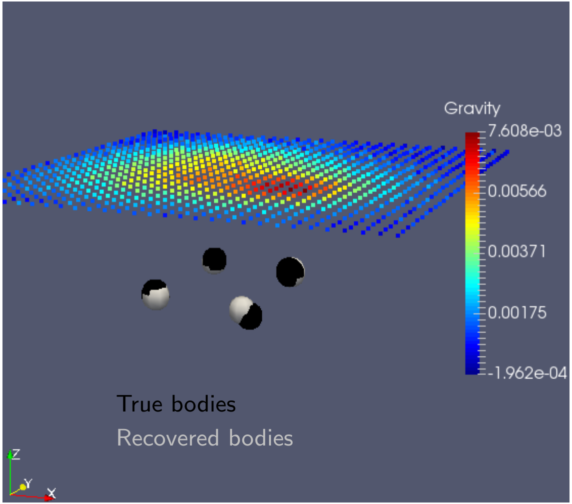
Discrete body inversion

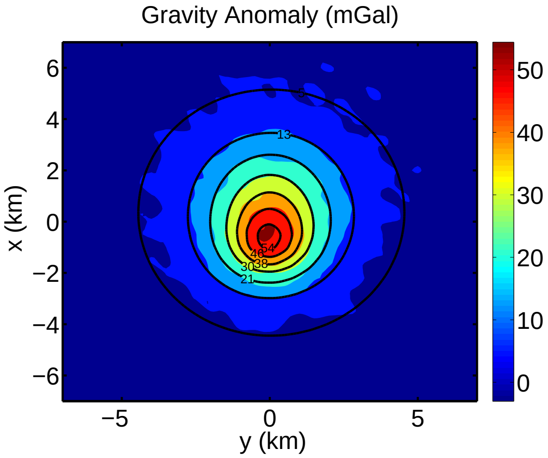
- Anomalous body represented by a simple shape: e.g. sphere, ellipsoid, cylinder, cuboid
- Inversion determines values of several parameters: e.g. location, orientation, phys. props.

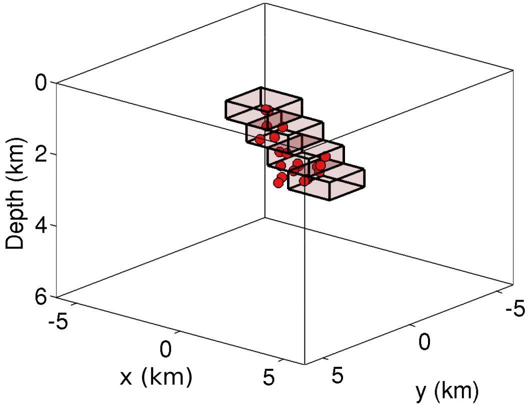




Discrete body inversion

- Regularization measures based on graph theory to control spacing of bodies (Bijani et al., 2015)





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