3D stochastic geophysical inversion for contact surface geometry

Peter G. Lelièvre[1], Colin G. Farquharson[1] and Rodrigo Bijani[2]

plelievre@mun.ca

http://www.esd.mun.ca/~peter/Home.html

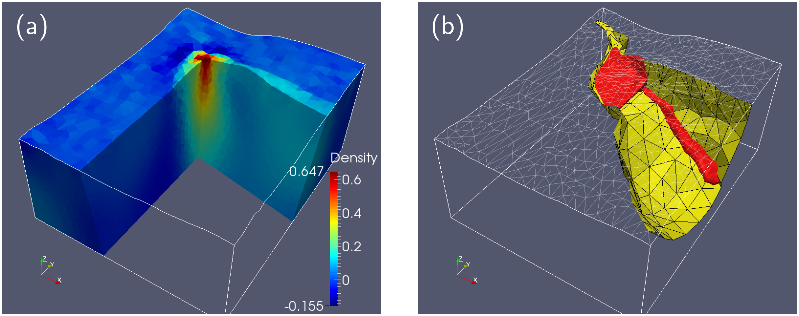
http://www.esd.mun.ca/~farq

[1]Memorial University, Department of Earth Sciences, St. John's, Canada

[2]Observatório Nacional, Rio de Janeiro, Brasil

EGU General Assembly 2015, PICO TS8.7

Motivation

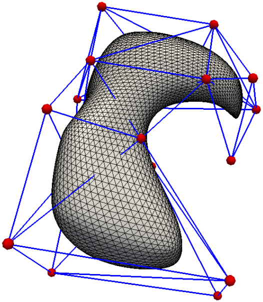


- Geophysical numerical methods typically work with mesh-based distributions of physical properties (a)
- Geologists' interpretations about the Earth typically involve wireframe contacts between distinct rock units (b)
- There are benefits to performing geophysical forward and inverse modelling on fundamentally different wireframe model discretizations

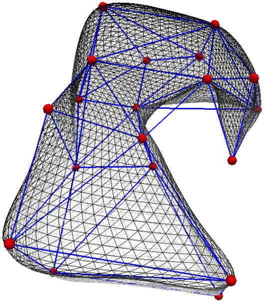
Methods: surface-based inversion for geometry

Use coarse control surfaces rather than the surfaces themselves!

Subdivision with cubic B-spline interpolation



Subdiv., Dyn-Levin-Gregory interp.



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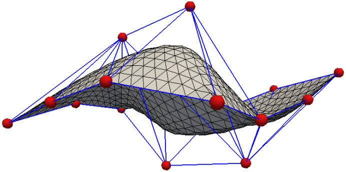
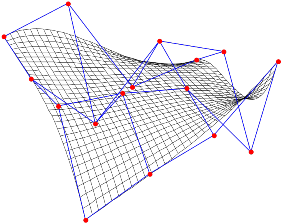


Bézier surface



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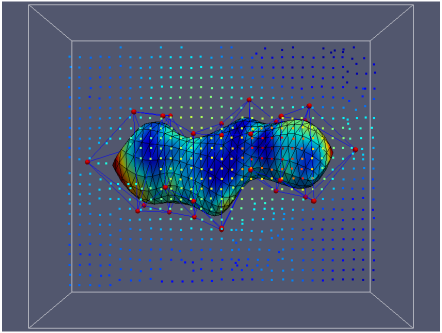
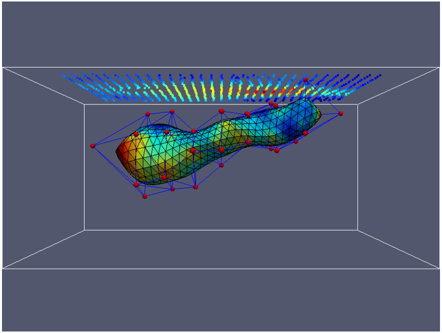
Selected result

IOCG deposit (real gravity data example)



Overhead view



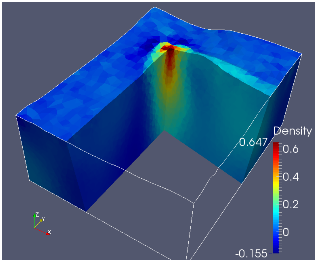
Side view

Model colours: standard deviation

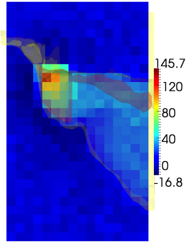
(low st. dev. = high confidence, high st. dev. = low confidence)

Geophysical inversion primer

Forward problem



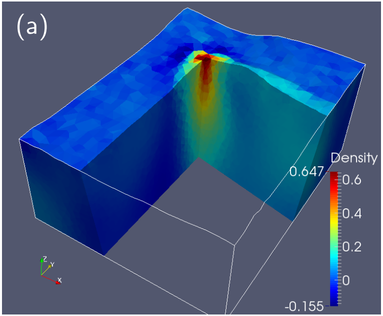
Earth model (e.g. density)

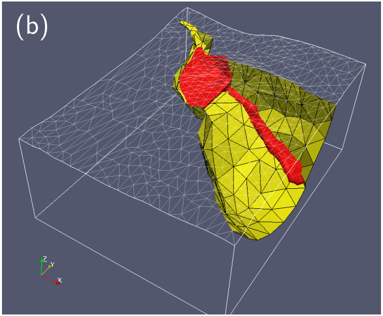


Survey data (e.g. gravity)

Inverse problem

Motivation





- Typical “minimum-structure” inversions discretize the Earth into many cells and seek smoothly varying models (a)
- In contrast, geologists’ interpretations about the Earth typically involve contacts between distinct rock units (b)
- There are benefits to performing fundamentally different inversions that seek the interfaces between proposed rock units

Types of geophysical inversion

1. Discrete body inversion
2. Mesh-based inversion
3. Surface-based inversion

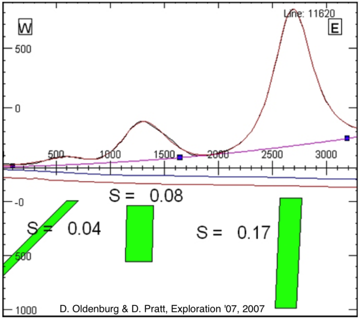
1. Discrete body inversion

Simplified representation of the Earth:

- Simple shapes for one or more causative target bodies
- Homogeneous background

Inversion:

- Very few parameters
- Data best-fit problem
- Low computational requirements
- Stochastic investigations feasible



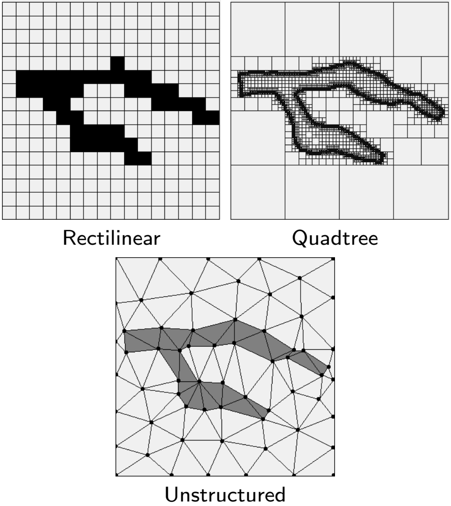
2. Mesh-based inversion

General representation of the Earth:

- Mesh of tightly packed cells
- Piecewise (pixellated) distribution of physical properties

Inversion:

- Very many parameters
- High computational requirements
- Stochastic investigations not very feasible



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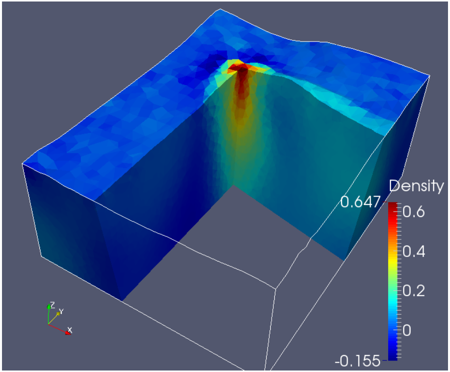


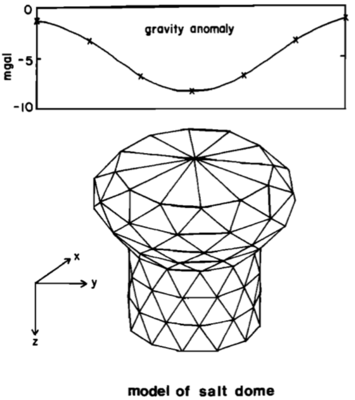
3. Surface-based inversion

Flexible representation of the Earth:

- Wireframe of nodes (vertices) and facets (e.g. triangles) representing contacts between rock units
- How geological models are built

Inversion:

- Physical properties remain fixed, geometry changes
- Moderate computational requirements
- Stochastic investigations somewhat feasible



Richardson & MacInnes, 1989, The inversion of gravity data into three-dimensional polyhedral models, JGR

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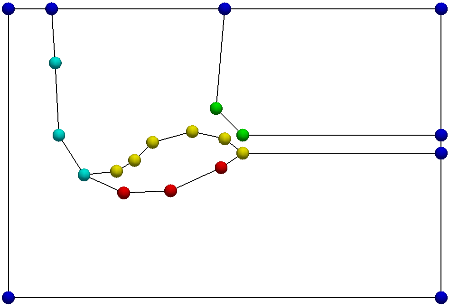

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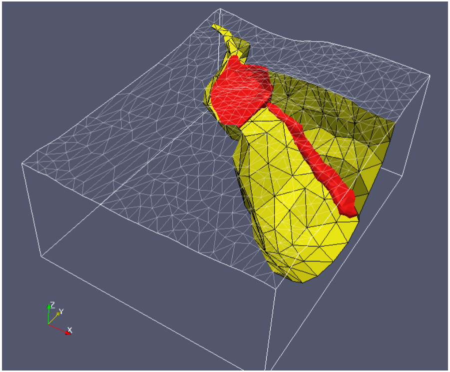

Inversion:

- Physical properties remain fixed, geometry changes
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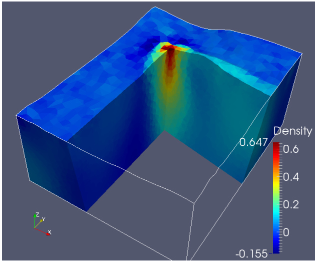
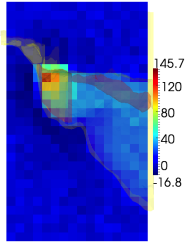
- Stochastic investigations somewhat feasible

To summarize, instead of doing this ...

Forward problem



Earth model (e.g. density)

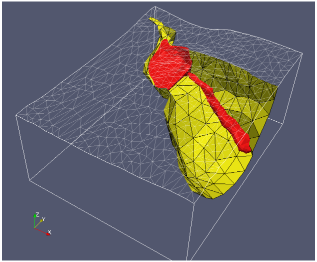


Survey data (e.g. gravity)

Inverse problem

... let's do this ...

Forward problem



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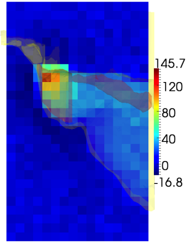


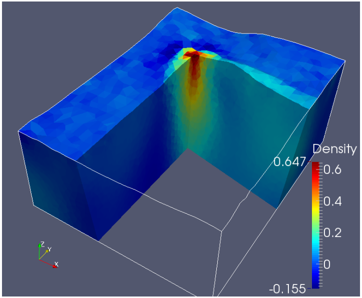
Survey data (e.g. gravity)

Inverse problem

Mesh-based inversion for smooth distributions

- Objective function

$$\Phi = \Phi_d + \beta \Phi_m$$



- Data misfit

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Model structure (regularization)

$$\Phi_m = \sum_j w_j (m_j - p_j)^2 + \sum_j \sum_k w_{j,k} (m_j - m_k)^2$$

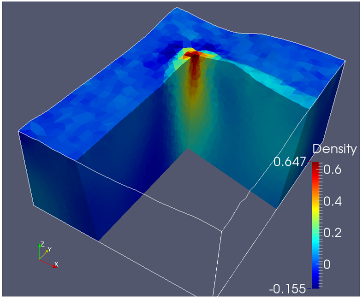
[smallness term] + [smoothness term]

- Deterministic local optimization approach: one "best" solution

Mesh-based inversion for sharper features

- Objective function

$$\Phi = \Phi_d + \beta \Phi_m$$



- Data misfit

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

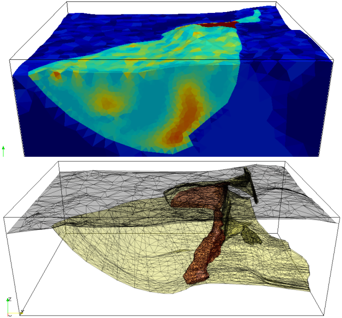
- Model structure (regularization)

$$\Phi_m = \sum_j w_j (m_j - p_j)^2 + \sum_j \sum_k w_{j,k} \left| m_j - m_k \right|^p + \Psi$$

[smallness term] + [smoothness term]

- Different norm, measures or re-weighted iterative procedure can help

Mesh-based inversion for sharper features



- Objective function

$$\Phi = \Phi_d + \beta \Phi_m$$

- Data misfit

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Model structure (regularization)

$$\Phi_m = \sum_j w_j (m_j - p_j)^2 + \sum_j \sum_k w_{j,k} (m_j - m_k)^2$$

[smallness term] + [smoothness term]

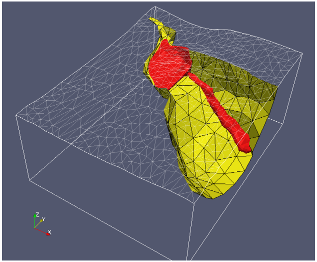
- The safest and most effective approach is to hardwire the surfaces

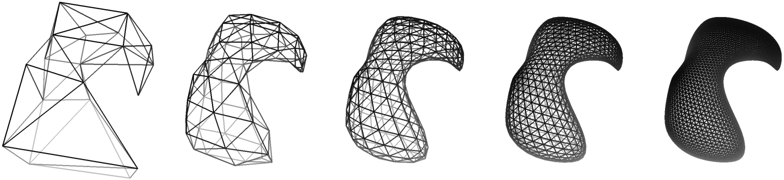
Surface-based inversion for geometry

- Inversion seeks positions of nodes in wireframe model
- Only data misfit is required

$$\Phi_d = \sum_i \left(\frac{F(m)_i - d_i}{\sigma_i} \right)^2$$

- Regularization not required: work on coarse representation, refine e.g. surface subdivision





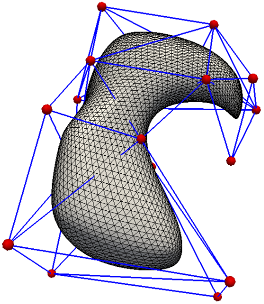
- Global optimization strategies (PSO, GA, MCMC) provide statistics:

$\Rightarrow$ many solution samples

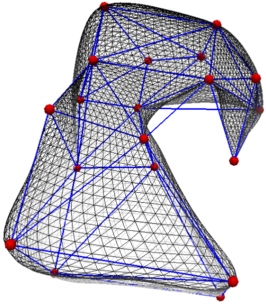
Surface-based inversion for geometry

Use coarse control surfaces rather than the surfaces themselves!

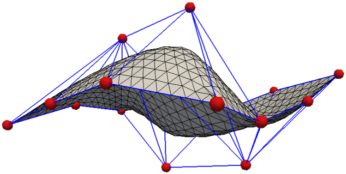
Subdivision with cubic B-spline interpolation



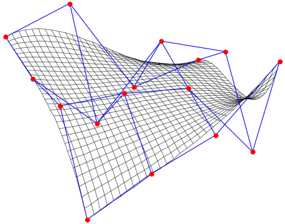
Subdiv., Dyn-Levin-Gregory interp.



Subdiv., cubic B-spline



Bézier surface



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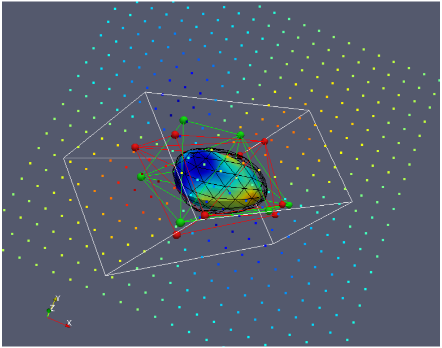
Examples

1. Isolated bodies (single surface)

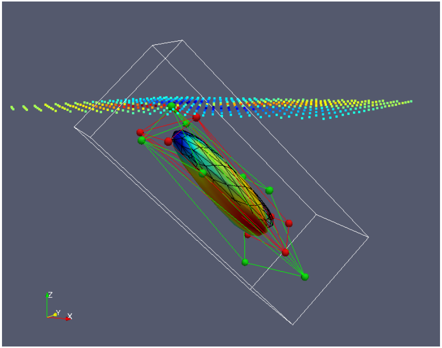
2. Complicated model (multiple surfaces)

Synthetic example #1: Isolated body

Black wireframe = true model
Red wireframe = control surface for true model
Green wireframe = control surface for recovered model
White box = bounds on control nodes during inversion
Coloured surface = recovered model (standard deviations, red high)
Coloured points = gravity data



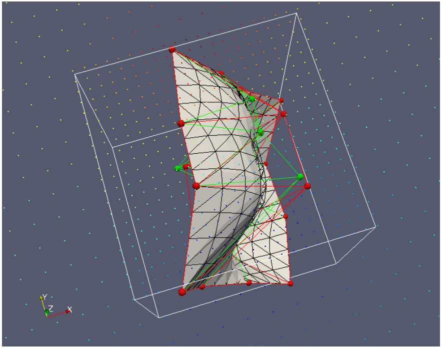
Overhead view



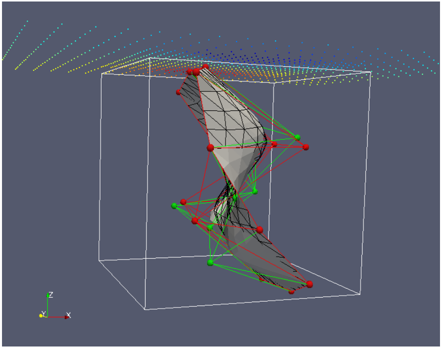
Side view

Synthetic example #2: Sheet-like contact surface

Black wireframe = true model
Red wireframe = control surface for true model
Green wireframe = control surface for recovered model
White box = bounds on control nodes during inversion
Light grey surface = recovered model (standard deviations not plotted here)
Coloured points = gravity data

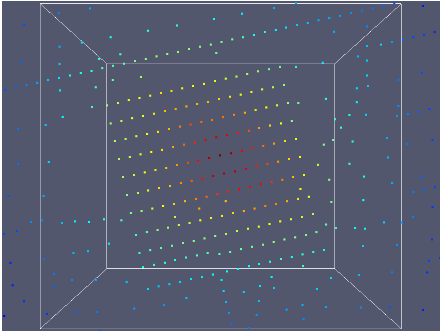


Overhead view

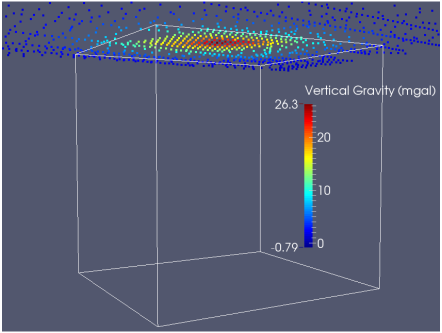


Side view

Real data example #1: Olympic Dam, gravity data



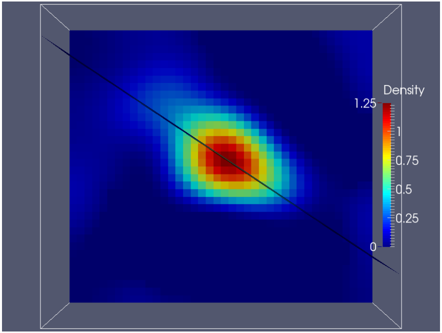
Overhead view



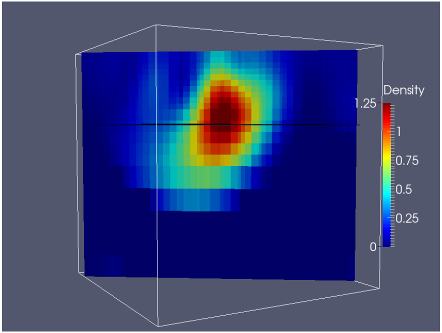
Side view

Colours: gravity

Olympic Dam: smooth mesh-based inversion



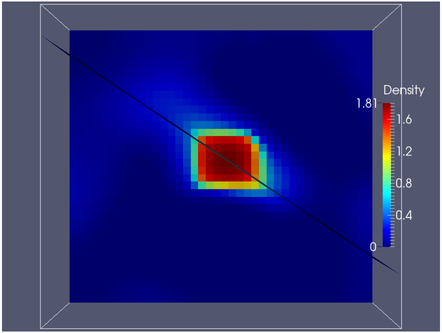
Overhead view



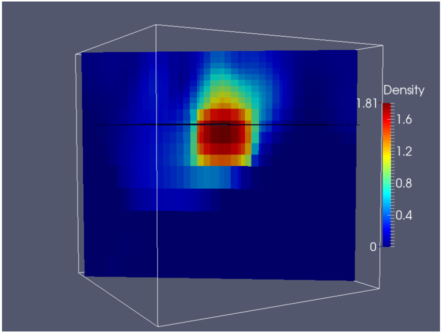
Side view

Colours: density

Olympic Dam: sharpened mesh-based inversion



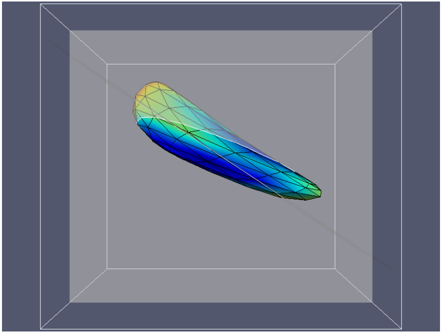
Overhead view



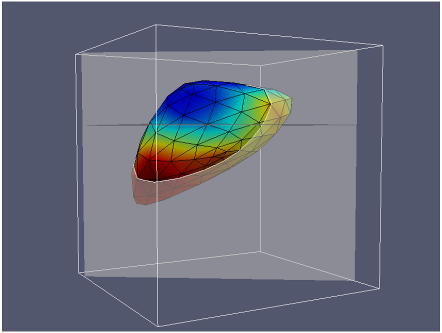
Side view

Colours: density

Olympic Dam: surface-based inversion



Overhead view

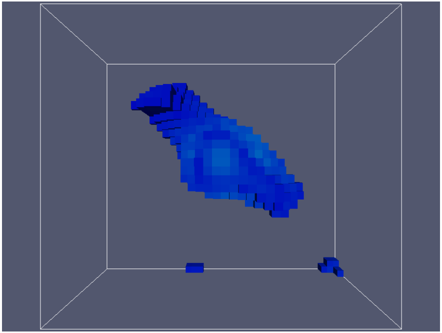


Side view

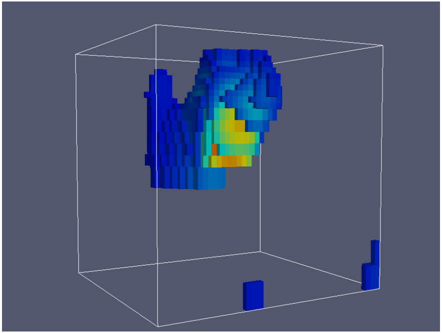
Colours: standard deviation

(low st. dev. = high confidence, high st. dev. = low confidence)

Olympic Dam: mesh-based inversion, threshold

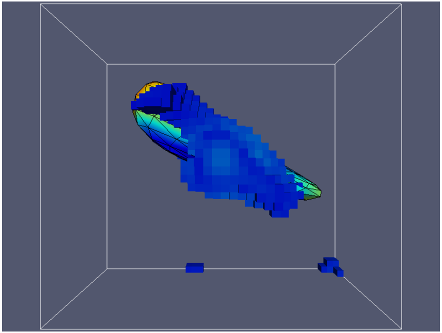


Overhead view

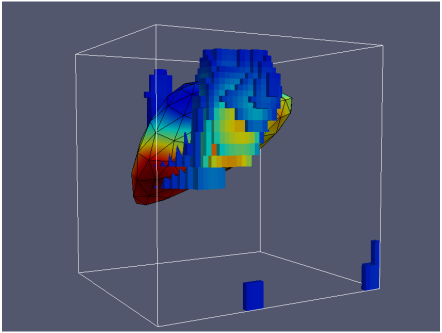


Side view

Olympic Dam: surface + threshold

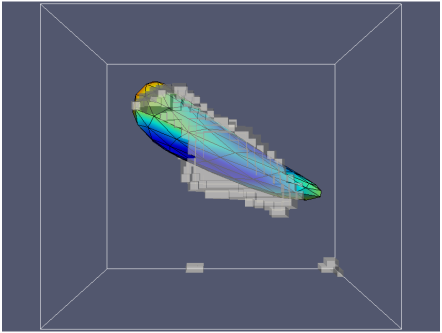


Overhead view

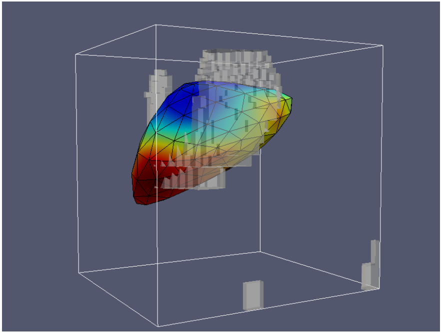


Side view

Olympic Dam: surface + opaque threshold



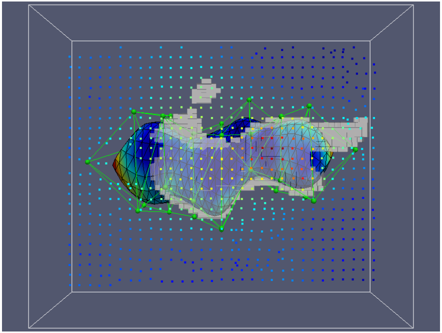
Overhead view



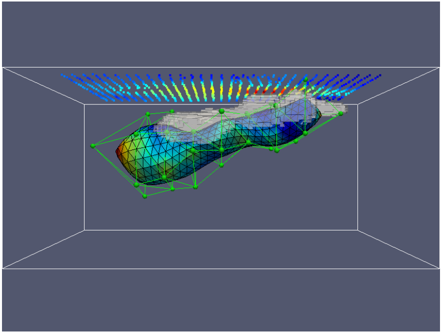
Side view

Surface-based inversion result consistent with understanding of geology, significant differences to mesh-based result require further study

Real data example #2: another IOCG deposit, gravity data



Overhead view



Side view

Surface-based inversion result consistent with understanding of geology, significant differences to mesh-based result require further study

Examples

1. Isolated bodies (single surface)

2. Complicated model (multiple surfaces)

Cocagne Subbasin, NB, Canada

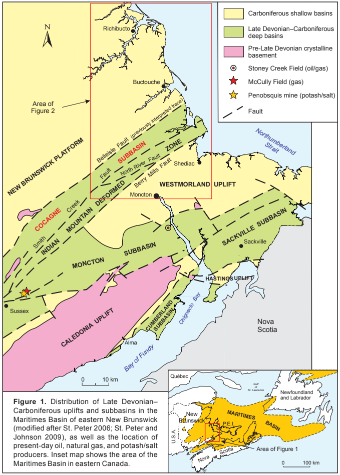


Figure 1. Distribution of Late Devonian–Carboniferous uplifts and subbasins in the Maritimes Basin of eastern New Brunswick (modified after St. Peter 2006; St. Peter and Johnson 2009), as well as the location of present-day oil, natural gas, and potash/salt producers. Inset map shows the area of the Maritimes Basin in eastern Canada.

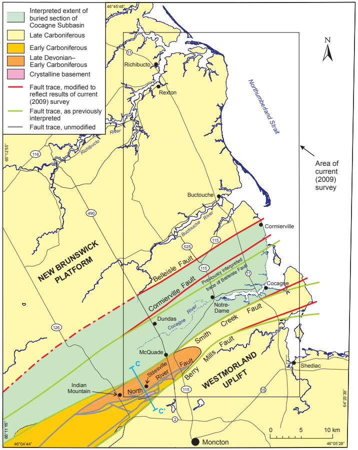


Figure 2. Simplified geology map of the report area (modified after Smith 2008), showing limits of the survey area. C–C' marks the line of section across the Indian Mountain Deformed Zone, illustrated in St. Peter and Johnson (2009, their Fig. 85). Figure 1 indicates the location of the report area in eastern New Brunswick.

J. Evangelatos & K. E. Butler, 2010. New gravity data in Eastern New Brunswick: implications for structural delineation of the Cocagne Subbasin. In Geological Investigations in New Brunswick for 2009. Edited by G. L. Martin. New Brunswick Department of Natural Resources; Lands, Minerals and Petroleum Division, Mineral Resource Report 2010–1, p. 98–122.

Cocagne Subbasin: gravity survey data

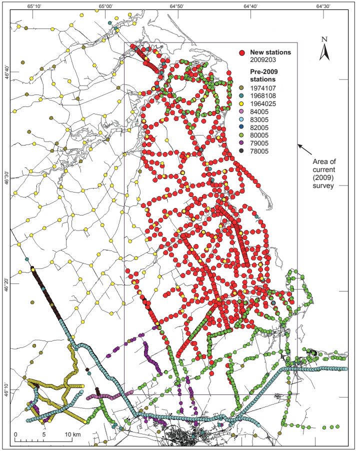


Figure 3. Map of the report area, showing distribution of the new and old gravity stations, numbered according to their Geodetic Survey Division database names. Large red dots indicate stations of the current (2009) survey. Figure 1 shows the location of the report area in eastern New Brunswick.

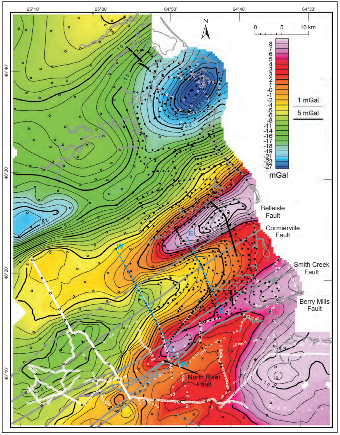


Figure 8. New terrain-corrected Bouguer anomaly map of the report area, with fault boundaries (modified after Smith 2008) shown as thick grey lines. Blue lines E–E' and W–W' mark the location of section profiles in Figures 11 and 12, respectively. Gravity stations are as identified in Figure 6. Figure 1 shows the location of this report area in eastern New Brunswick.

J. Evangelatos & K. E. Butler, 2010. New gravity data in Eastern New Brunswick: implications for structural delineation of the Cocagne Subbasin. In Geological Investigations in New Brunswick for 2009. Edited by G. L. Martin. New Brunswick Department of Natural Resources; Lands, Minerals and Petroleum Division, Mineral Resource Report 2010–1, p. 98–122.

Cocagne Subbasin: rudimentary geological model

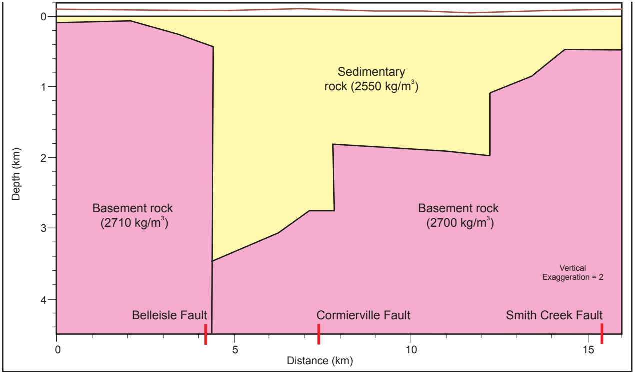
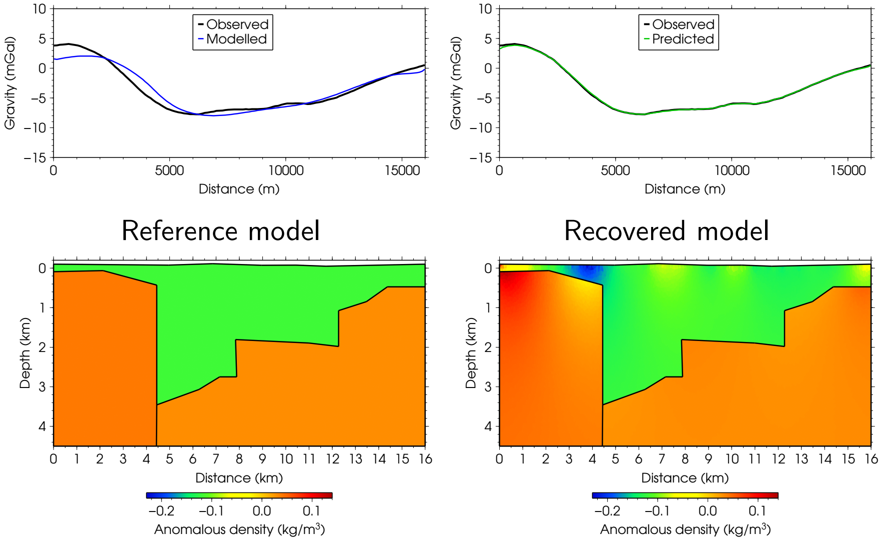


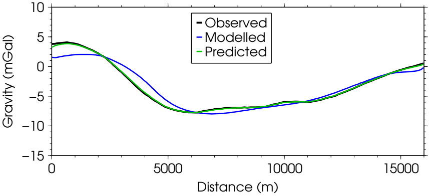
Figure 11. A two-dimensional geological model, simulating the observed gravity response across the eastern part of the Cocagne Subbasin along profile E–E' in Figure 8. The magnetic profile is extracted from the regional aeromagnetic dataset (Canadian Aeromagnetic Data Base 2010). The Belleisle, Cormierville, and Smith Creek fault traces in Figure 8 are shown here as red ticks on the horizontal axis.

J. Evangelatos & K. E. Butler, 2010. New gravity data in Eastern New Brunswick: implications for structural delineation of the Cocagne Subbasin. In Geological Investigations in New Brunswick for 2009. Edited by G. L. Martin. New Brunswick Department of Natural Resources; Lands, Minerals and Petroleum Division, Mineral Resource Report 2010–1, p. 98–122.

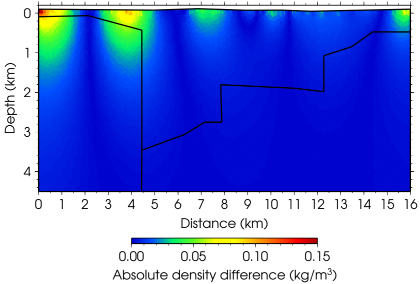
Cocagne Subbasin: mesh-based inversion



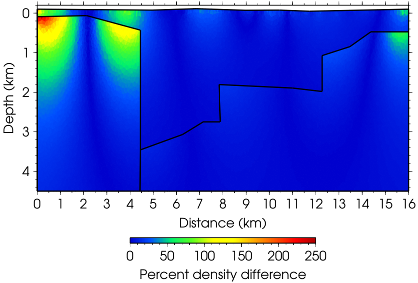
Cocagne Subbasin: mesh-based inversion



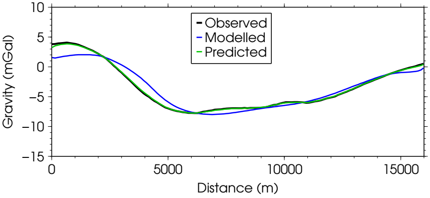
Absolute difference

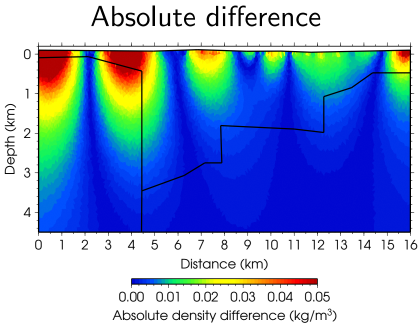


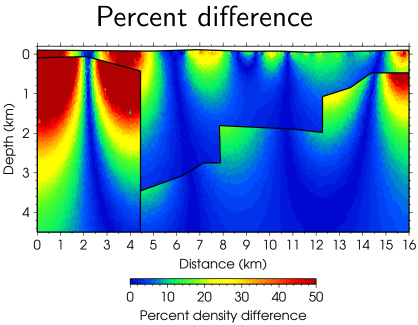
Percent difference



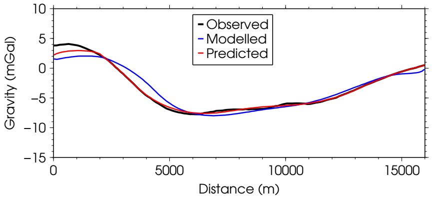
Cocagne Subbasin: mesh-based inversion

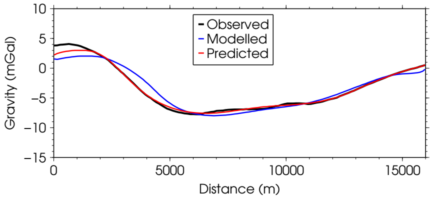




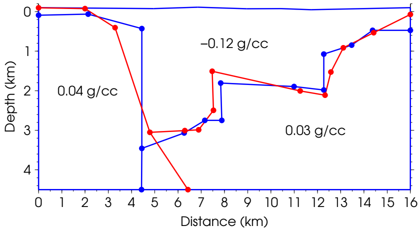


Cocagne Subbasin: surface-based inversion

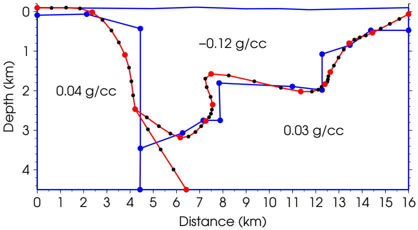




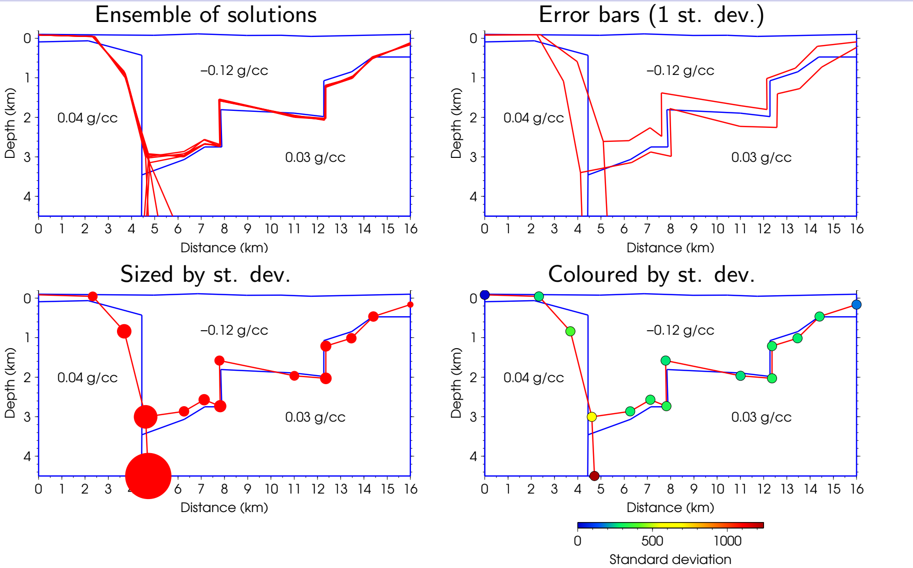
2D wireframe model



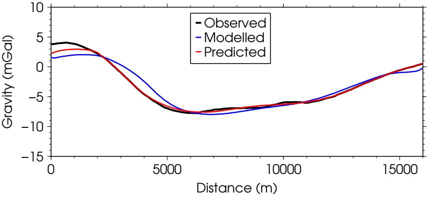
Splined model

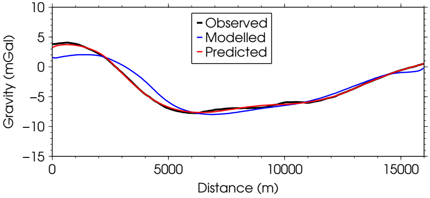


Options for visualizing uncertainty

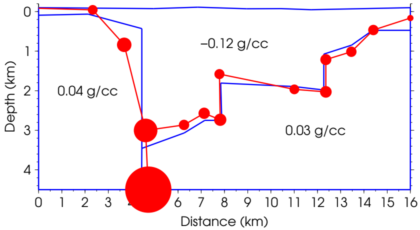


Investigating different physical properties

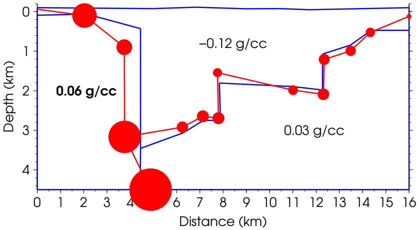




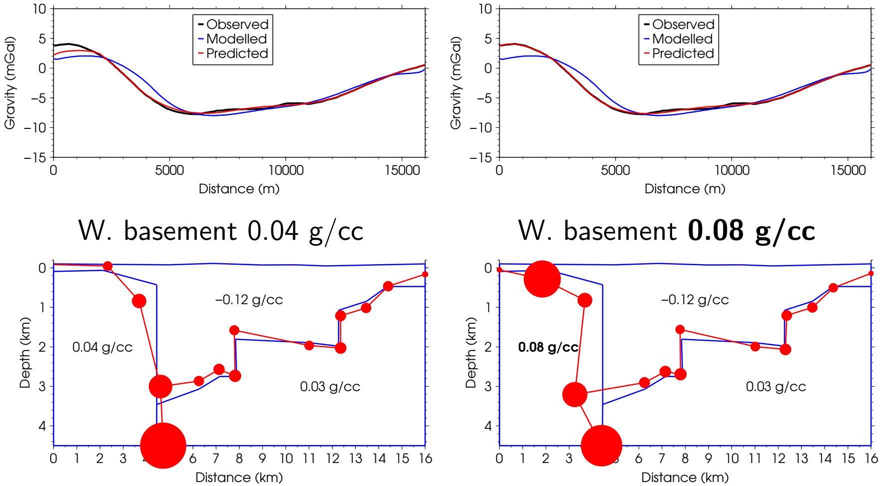
W. basement 0.04 g/cc



W. basement **0.06 g/cc**



Investigating different physical properties



Summary

- Mesh-based "distribution inversion" is standard and numerically well behaved until you try to recover sharp features
- Surface-based "geometry inversion" is challenging but can model sharp contacts and provide likelihood information

Summary

There are benefits to performing fundamentally different inversions that seek the interfaces between proposed rock units:

- Geological and geophysical models defined using the same underlying representation can be, in essence, the same Earth model
- Aids in the creation of Common Earth models, consistent with all geological and geophysical information available
- Wireframe discretization can flexibly and efficiently generate complicated geological features
- Global optimization strategies can provide statistics (many solution "samples")
- Allows you to ask different exploration questions

Future work

- Joint inversion (gravity + magnetics + ?)
  - Multi-objective optimization methods for joint inverse problems ...
- Alternate global optimization approaches
  - Genetic algorithms ...
- Work directly with complicated 3D common Earth model ...
- Hybrid approach ...

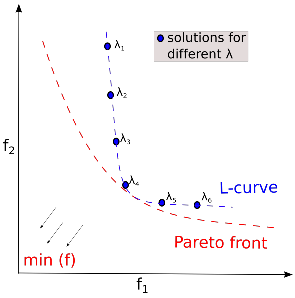
Multi-objective genetic algorithms for joint inversion

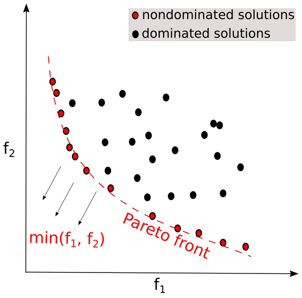
Single-objective GA:

- Aggregate of objectives:
  $$\min(f) \quad f = f_1 + \lambda f_2 + \ldots$$
- One single best solution
- Difficult to find best λ value(s)

Multi-objective GA:

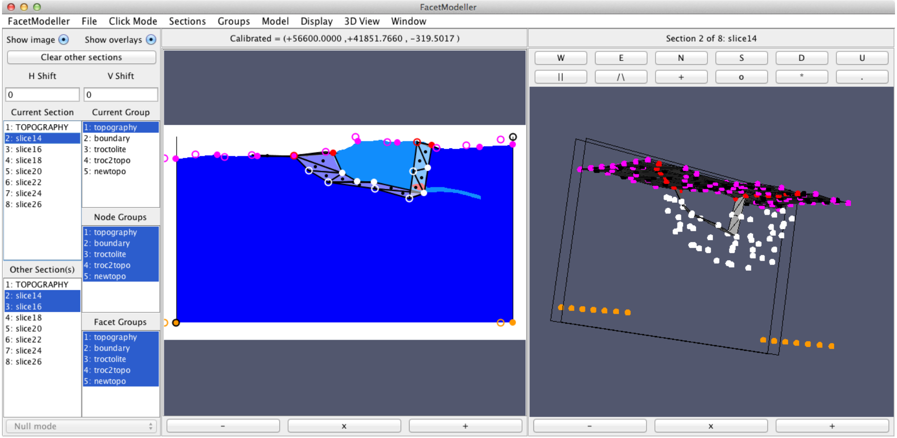
- Objectives treated separately:
  $$\min(f_1, f_2)$$
- Several solutions along the Pareto front (nondominated)





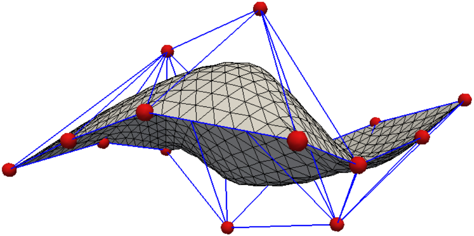
Building and manipulating complicated 3D models

FacetModeller

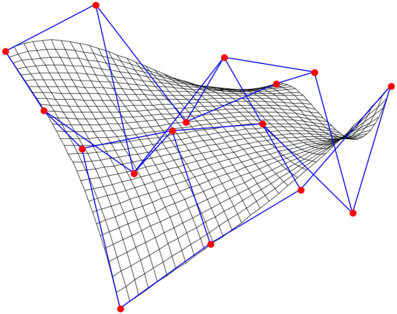


Can we extract the control surfaces, rather than the surfaces themselves, from the modelling software?

Subdiv., cubic B-spline



Bézier surface



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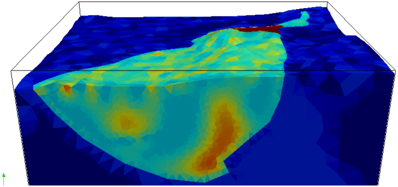
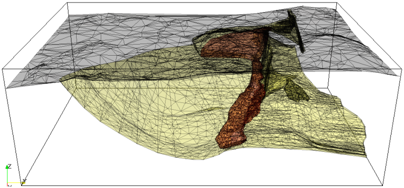
A third approach?

1. Mesh-based inversion with sharp features



2. Surface-based inversion



3. Hybrid approach?