3-D Finite-Element Modelling of Magnetotelluric Data With a Static Divergence Correction

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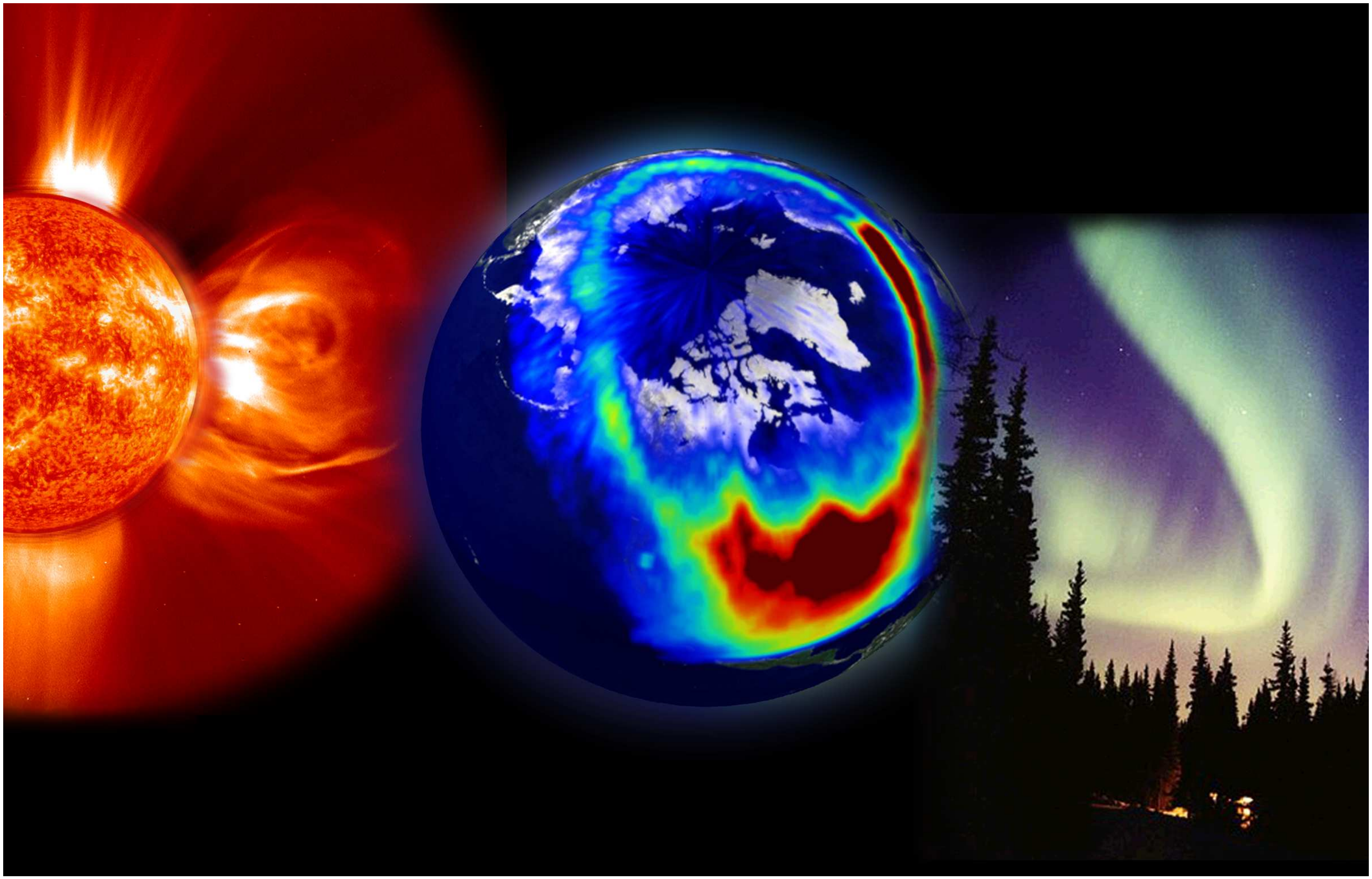
The Magnetotelluric Method



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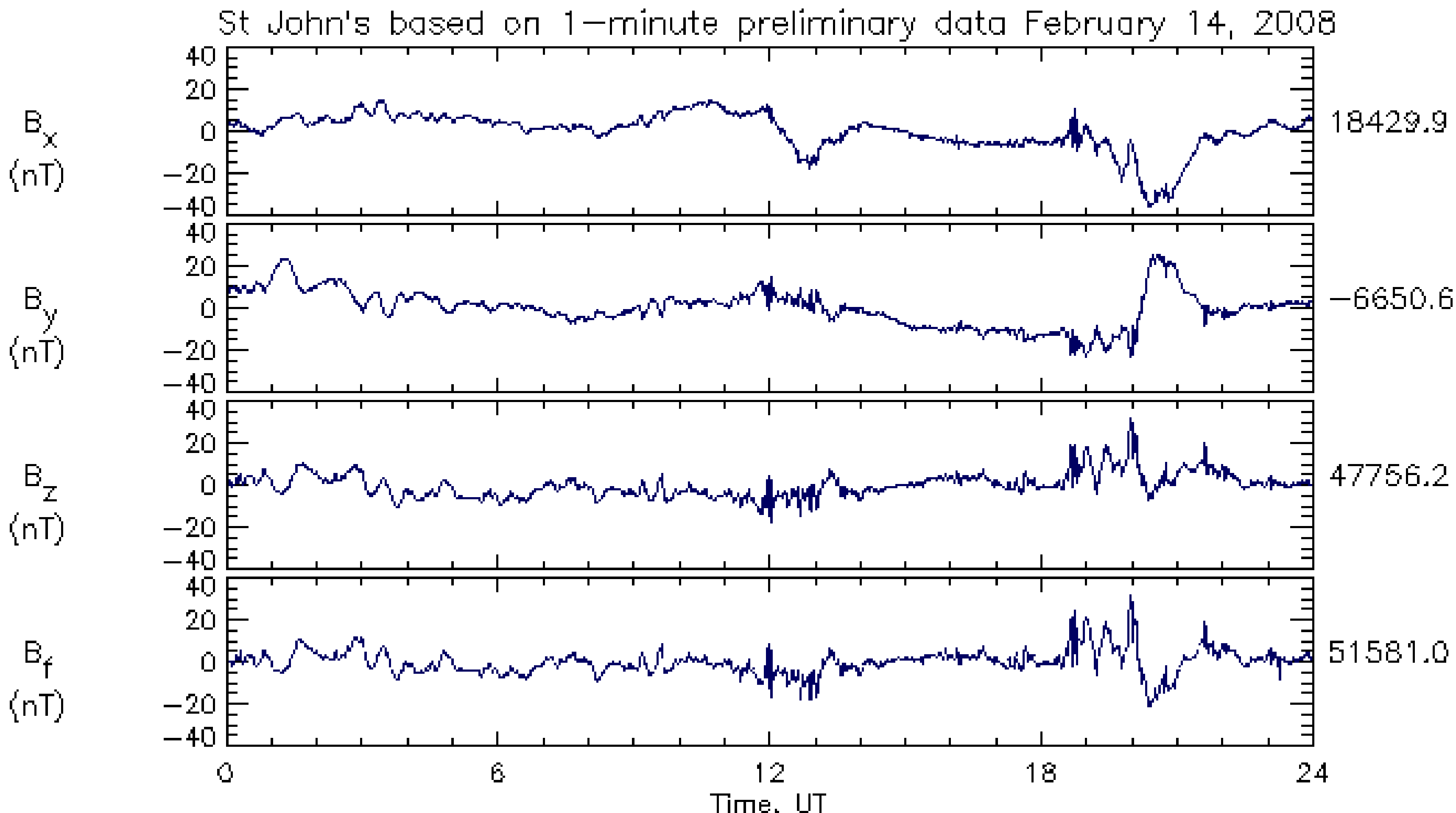
The Magnetotelluric Method

The Internet

The Magnetotelluric Method



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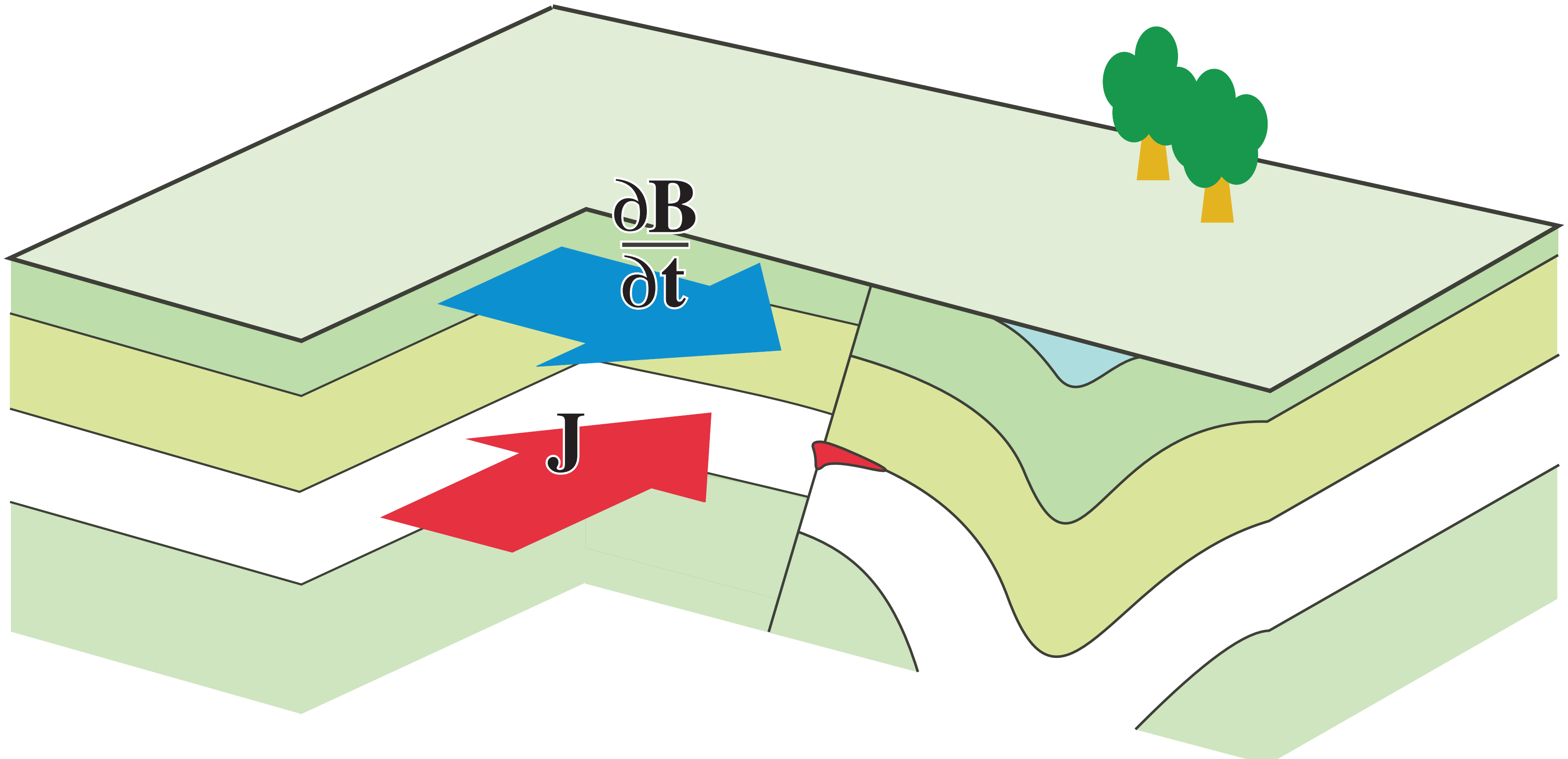
The Magnetotelluric Method



Measure **E** and **H**. Their ratio contains information about the electrical conductivity of the subsurface.

Standard Finite-Element Modelling

Fundamental equations:

$$\nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H} \qquad \nabla \times \mathbf{H} = \mathbf{J}$$

$$\mathbf{J} = \sigma\mathbf{E}, \quad \sigma = \sigma(\mathbf{r})$$

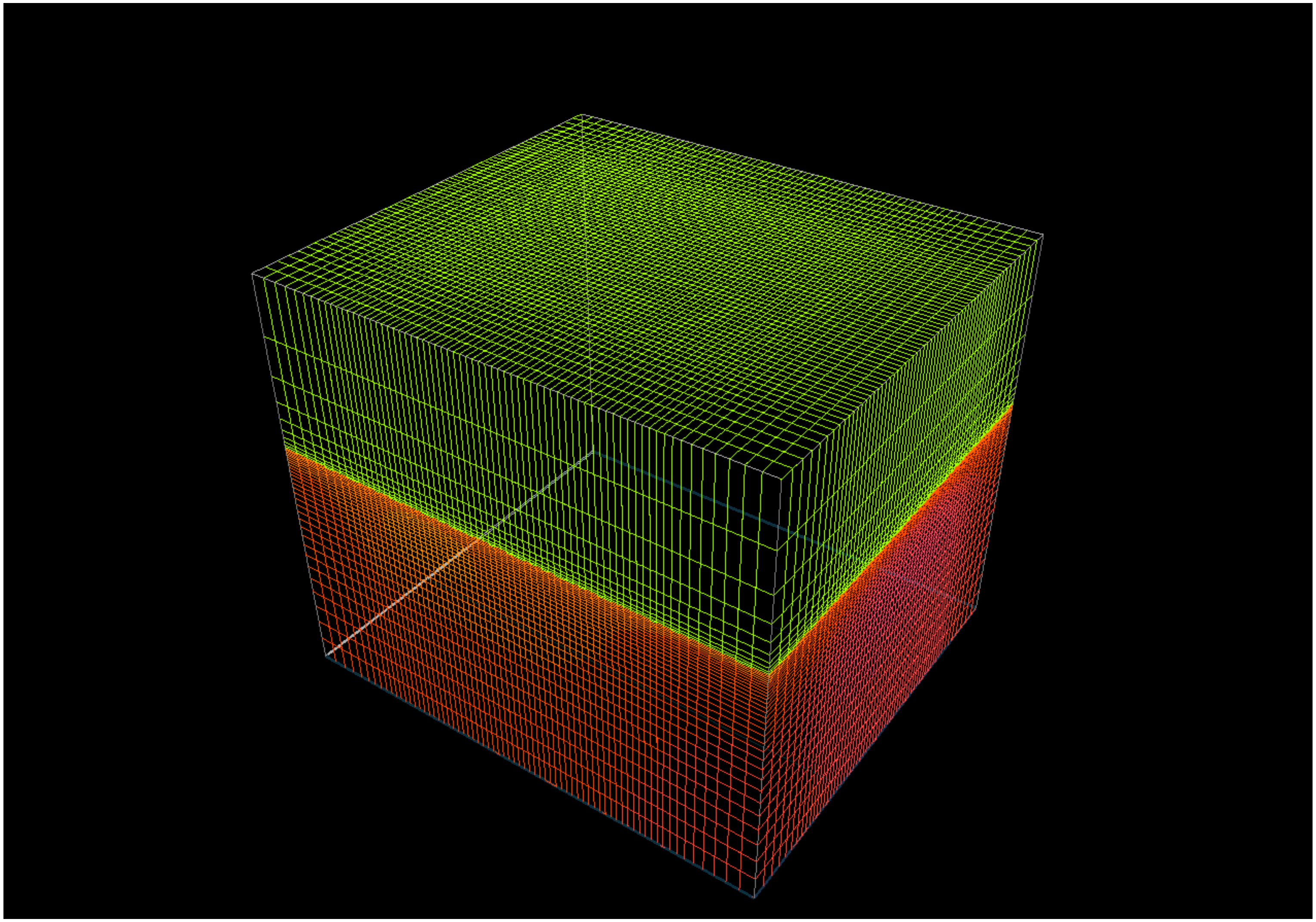
(Quasi-static)

PDE for electric field:

$$\nabla \times \nabla \times \mathbf{E} - i\omega\mu_0\sigma\mathbf{E} = 0$$

Standard Finite-Element Modelling

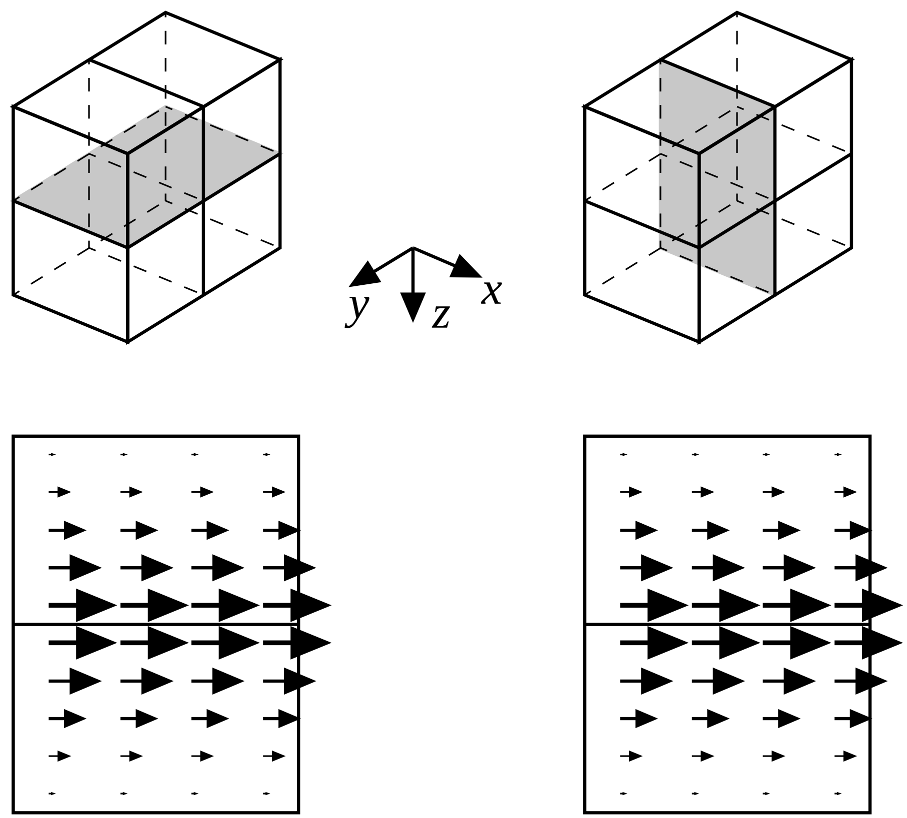
Discretization:



Standard Finite-Element Modelling

Edge-element basis functions:

$$\tilde{\mathbf{E}} = \sum_{j=1}^{N} \tilde{E}_j \, \mathbf{v}_j$$



Standard Finite-Element Modelling

Constructing system of equations:

$$\nabla \times \nabla \times \mathbf{E} - i\omega\mu_0\sigma\mathbf{E} = 0$$

$$\sum_{j=1}^{N} \tilde{E}_j \left\{ \nabla \times \nabla \times \mathbf{v}_j \; - \; i\omega\mu_0\sigma\,\mathbf{v}_j \right\} \; - \; \mathbf{R} \; = \; 0$$

$$\sum_{j=1}^{N} \tilde{E}_j \left\{ \int_V \mathbf{v}_i \cdot \nabla \times \nabla \times \mathbf{v}_j \, dv \; - \; i\omega\mu_0 \int_V \sigma\,\mathbf{v}_i \cdot \mathbf{v}_j \, dv \right\} \; = \; 0$$

$$\sum_{j=1}^{N} \tilde{E}_j \left\{ \int_V (\nabla \times \mathbf{v}_i) \cdot (\nabla \times \mathbf{v}_j) \, dv \; - \; i\omega\mu_0 \int_V \sigma\,\mathbf{v}_i \cdot \mathbf{v}_j \, dv \right\} \; = \; 0$$

$$(i = 1, \ldots, N)$$

Standard Finite-Element Modelling

Constructing system of equations (continued):

- Determine formulae for $\int_V (\nabla \times \mathbf{v}_i) \cdot (\nabla \times \mathbf{v}_j)\, dv$ and $\int_V \sigma\, \mathbf{v}_i \cdot \mathbf{v}_j\, dv$ (which depend only on i, j, σ and cell dimensions).

- Assemble the matrix equation:

$$\mathbf{M}\,\tilde{\mathbf{E}} = 0$$

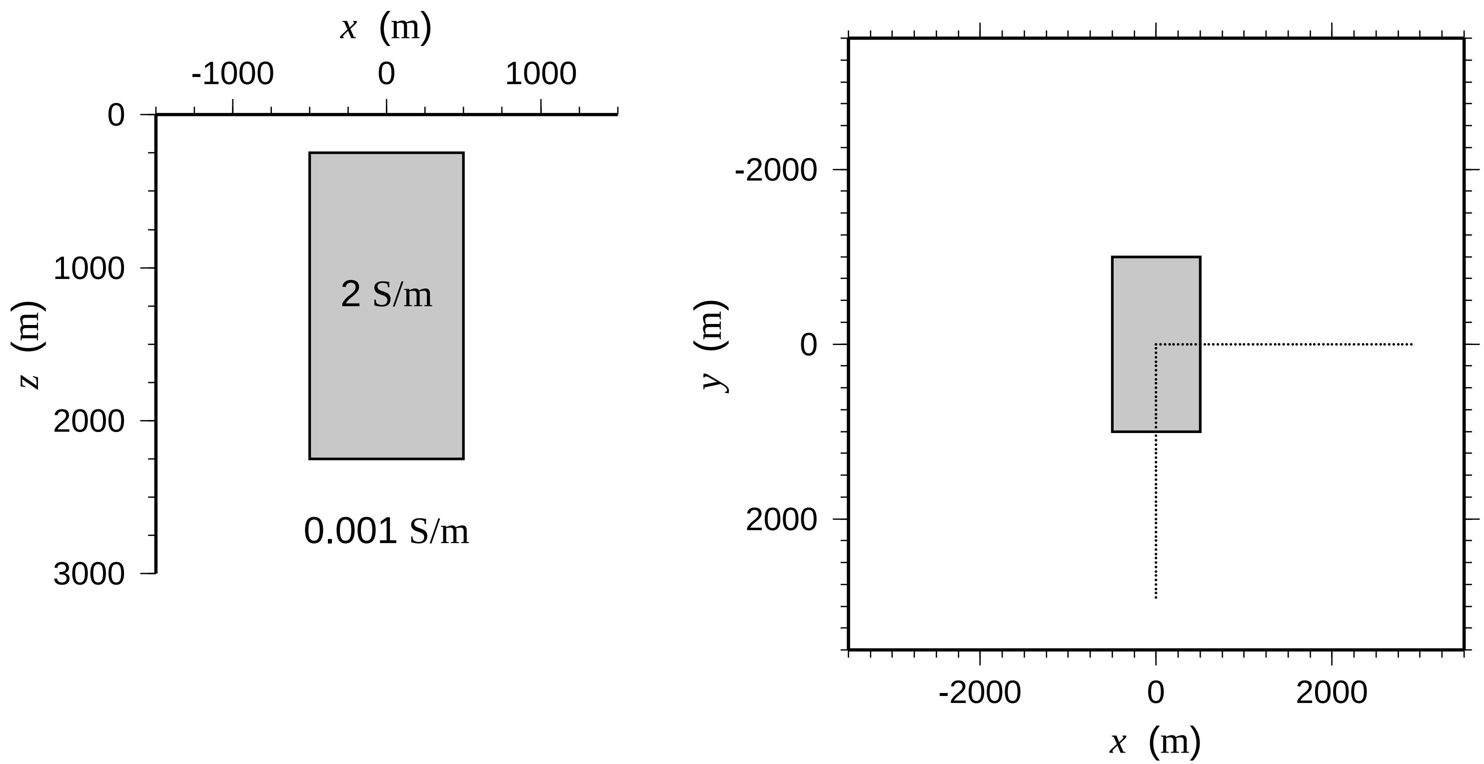
  (separating real and imaginary parts).

- Incorporate Dirichlet boundary conditions into the matrix equation.

- Solve using ILU preconditioned BCGSTAB, GMRES, etc. (Sparskit).

Standard Finite-Element Modelling: Results

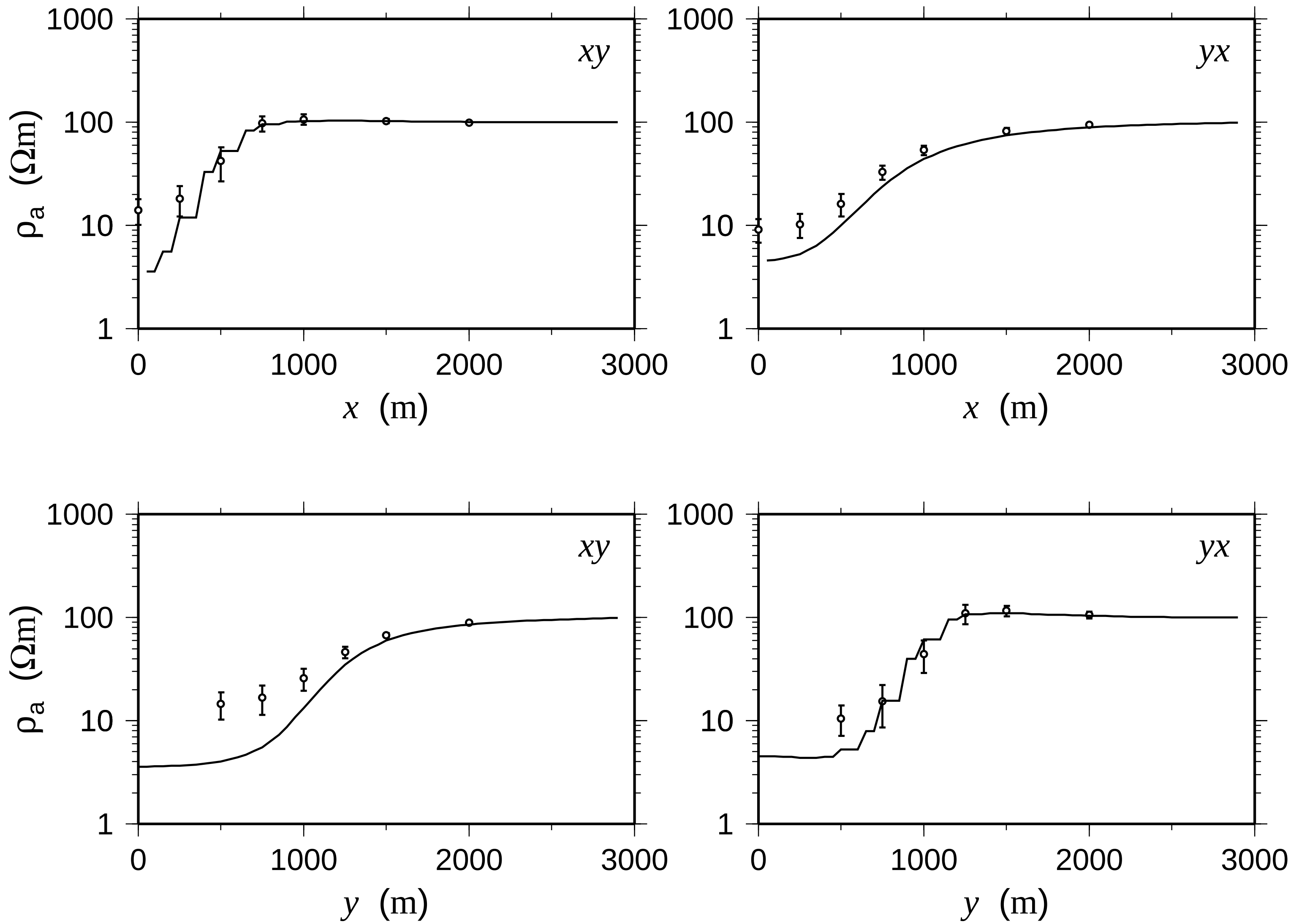
COMMEMI 3D-1A:



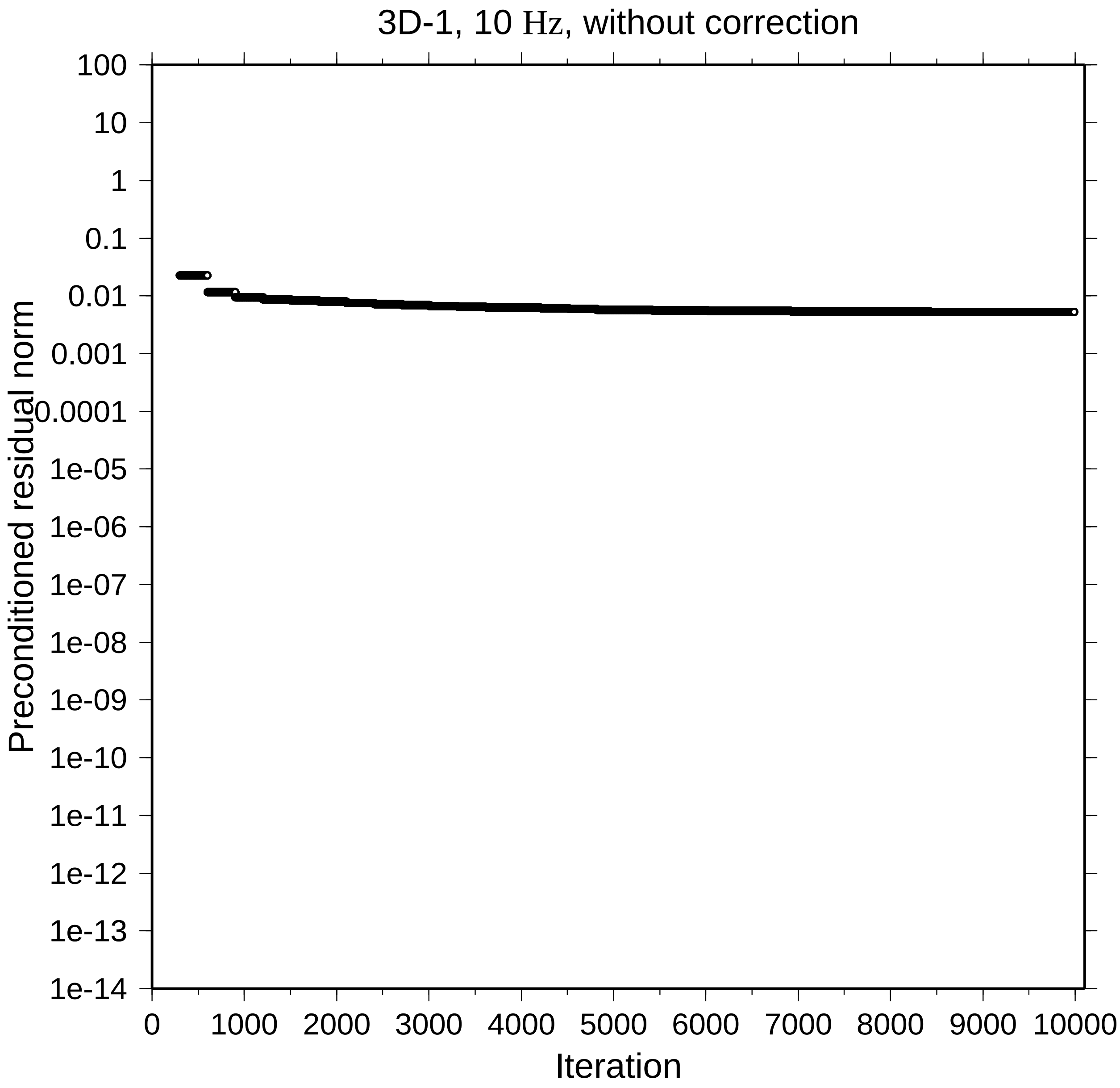
0.1 s, 10 s periods

Standard Finite-Element Modelling: Results

3D-1, 10 Hz, without correction

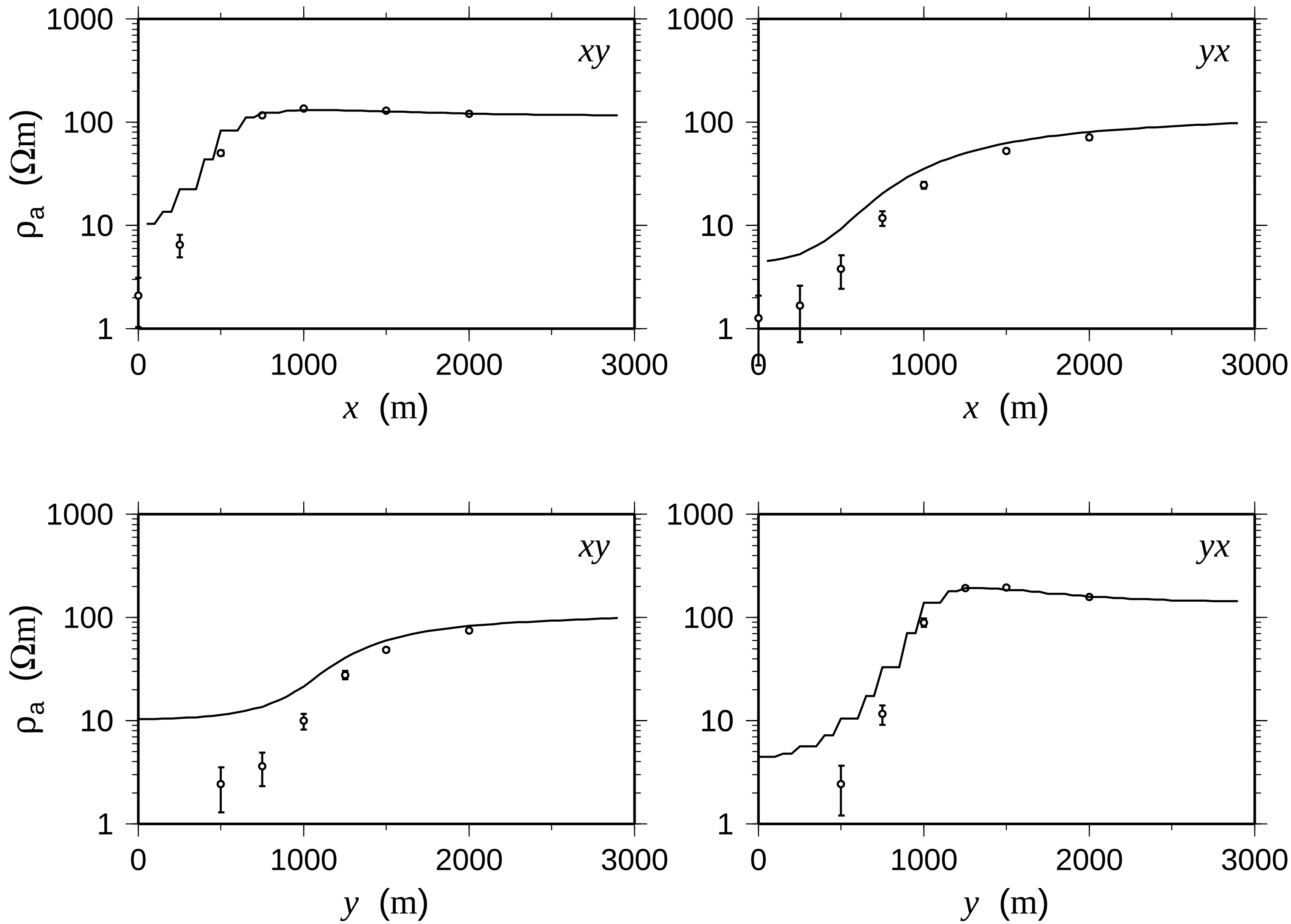


Standard Finite-Element Modelling: Results

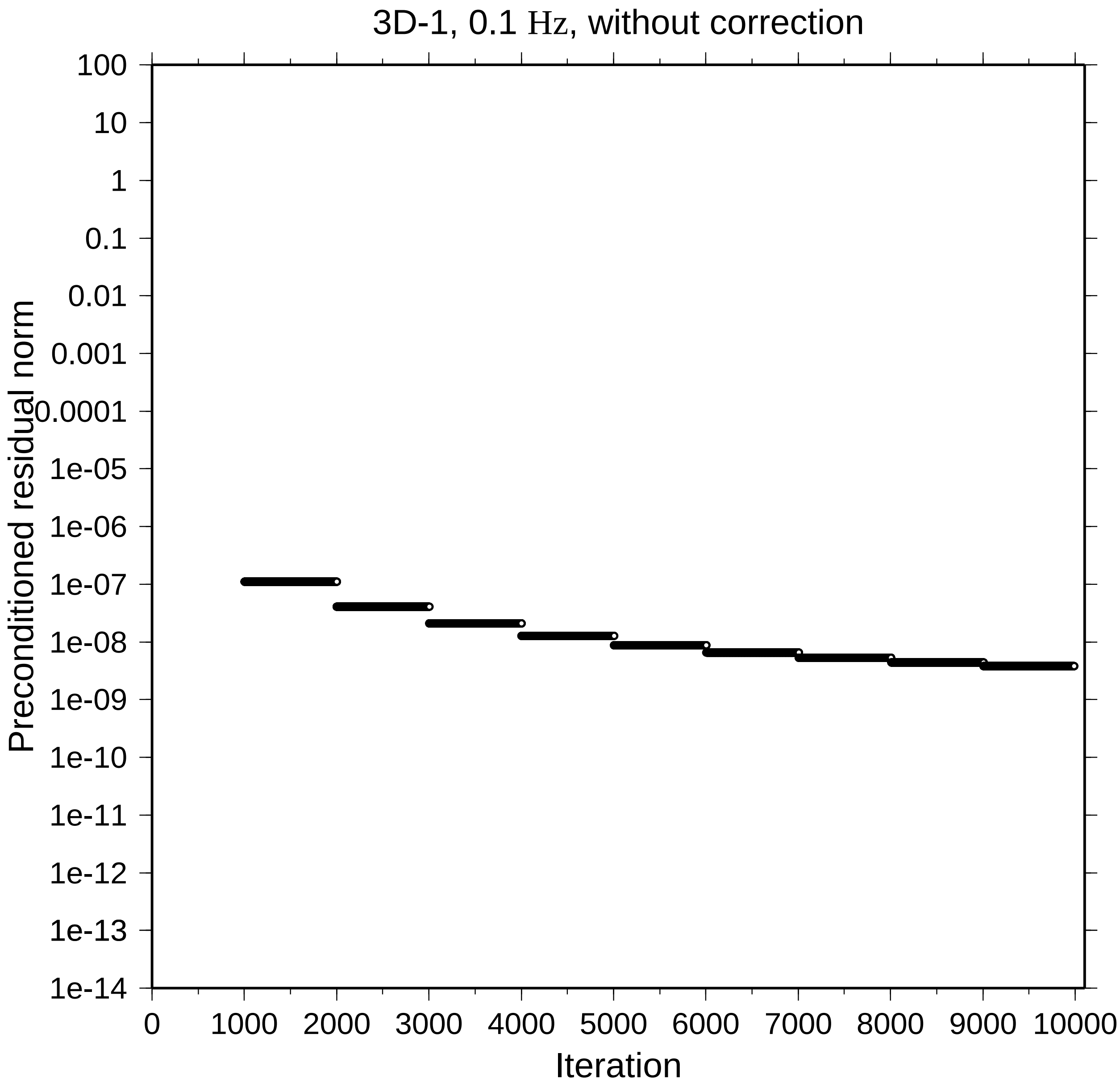


Standard Finite-Element Modelling: Results

3D-1, 0.1 Hz, without correction



Standard Finite-Element Modelling: Results



Finite-Element Modelling With Correction

- Electric field PDE:

$$\nabla \times \nabla \times \mathbf{E} - i\omega\mu_0\sigma\mathbf{E} = 0$$

- For low frequency, $\tilde{\mathbf{E}} + \nabla\phi$ for any ϕ is also effectively a solution of this PDE …

- … but not of $\nabla \cdot (\sigma\mathbf{E}) = 0$.

- Following Smith (1996), compute ϕ such that

$$\nabla \cdot (\sigma\nabla\phi) = -\nabla \cdot (\sigma\tilde{\mathbf{E}}),$$

  and 'correct' the approximate electric field by:

$$\tilde{\mathbf{E}} \leftarrow \tilde{\mathbf{E}} + \nabla\phi.$$

Finite-Element Modelling With Correction

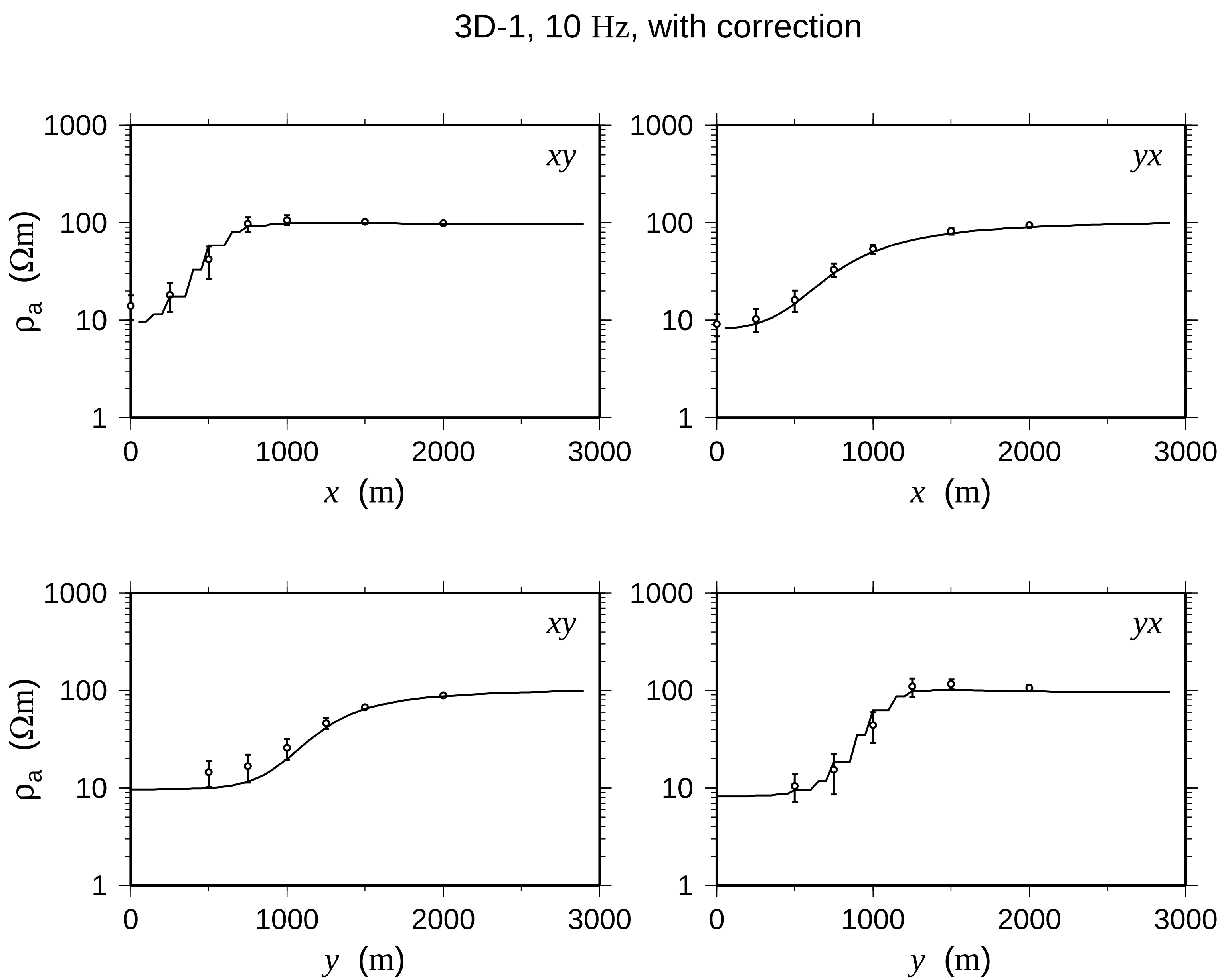
- Every so often in iterative solution process for edge-element solution of electric field PDE:
  - Standard finite-element solution of

$$\nabla \cdot (\sigma \nabla \phi) = -\nabla \cdot (\sigma \tilde{\mathbf{E}}),$$

    using nodal elements (tri-linear) for ϕ, Galerkin formulation, ILU preconditioned BCGSTAB.
  - Correct approximate electric field by:

$$\tilde{\mathbf{E}} \leftarrow \tilde{\mathbf{E}} + \nabla \phi.$$

Finite-Element Modelling With Correction: Results



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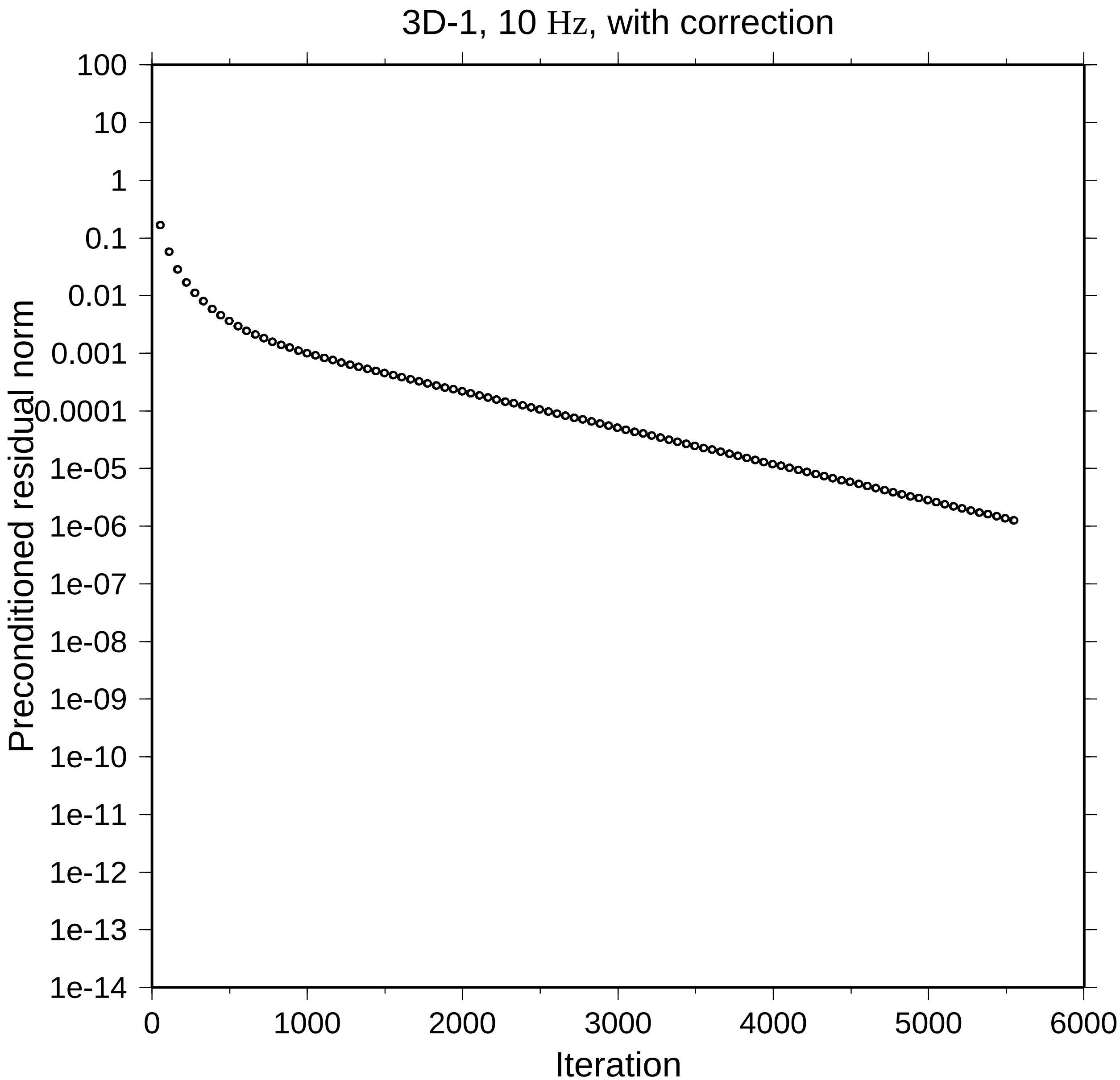

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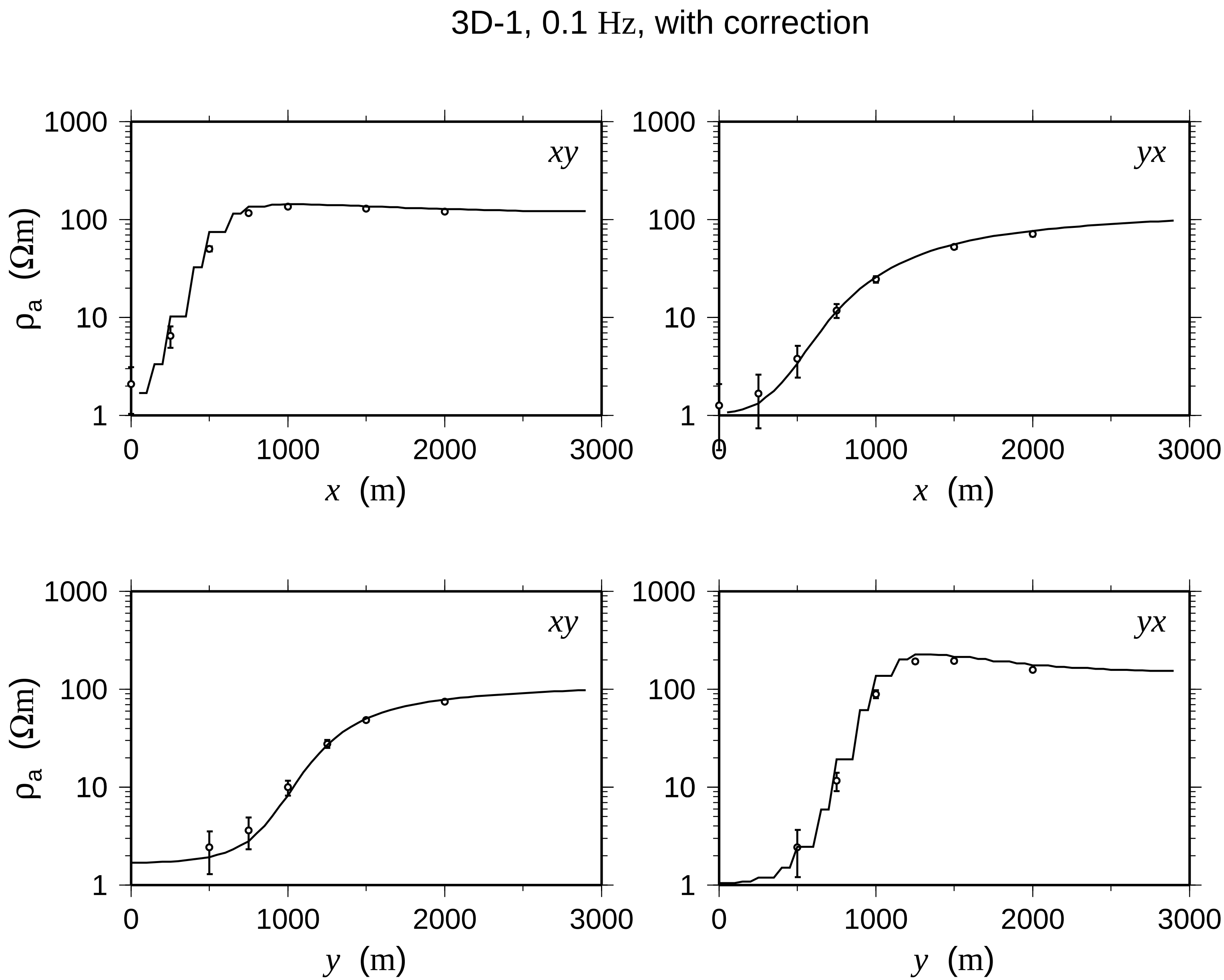

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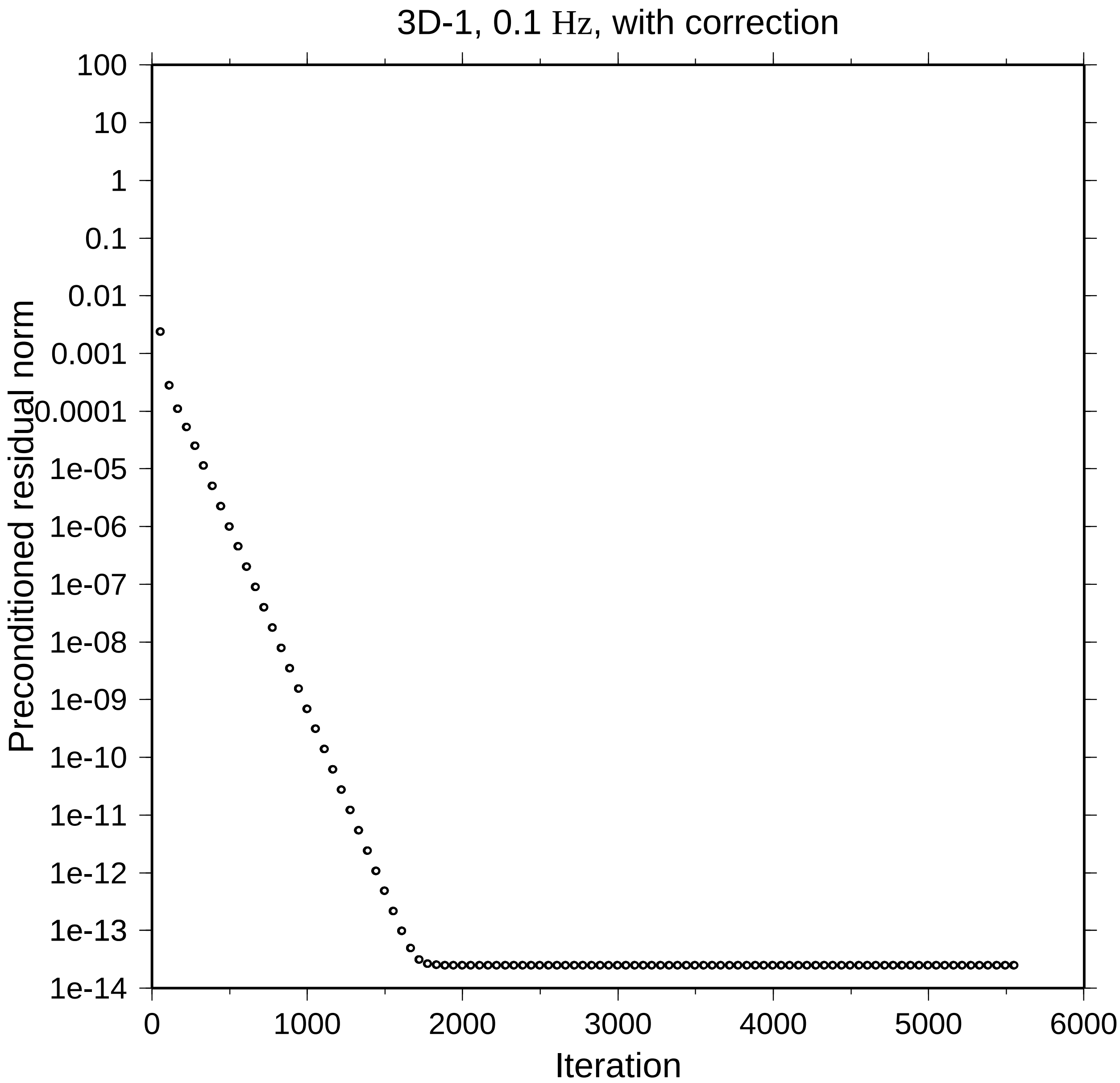

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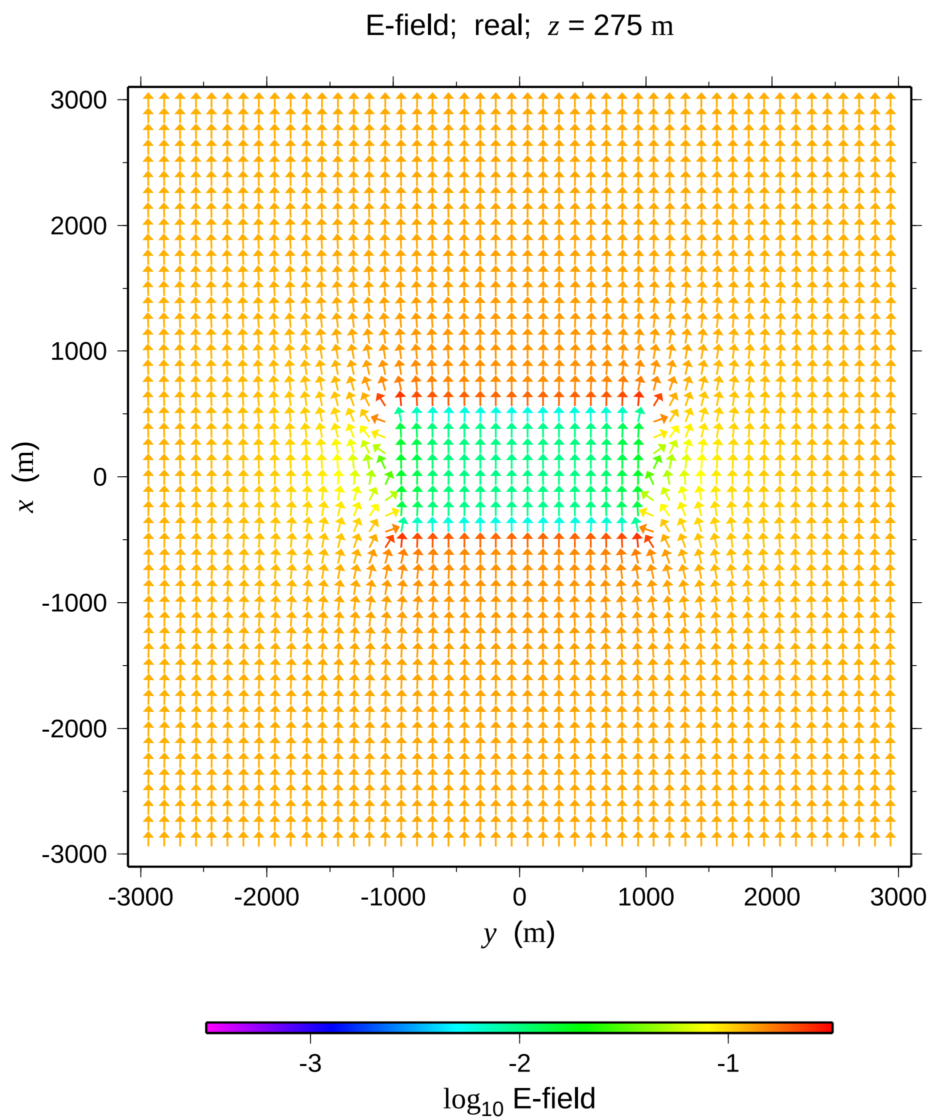
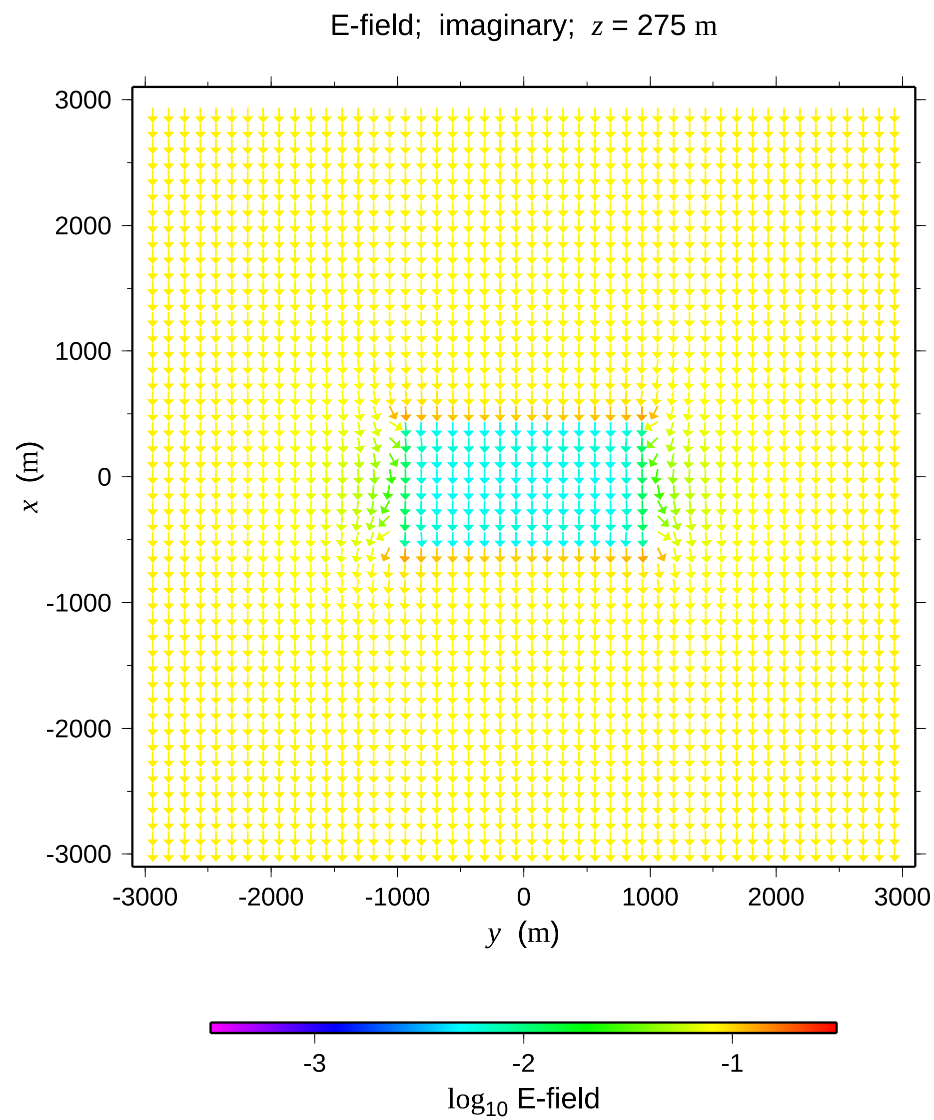
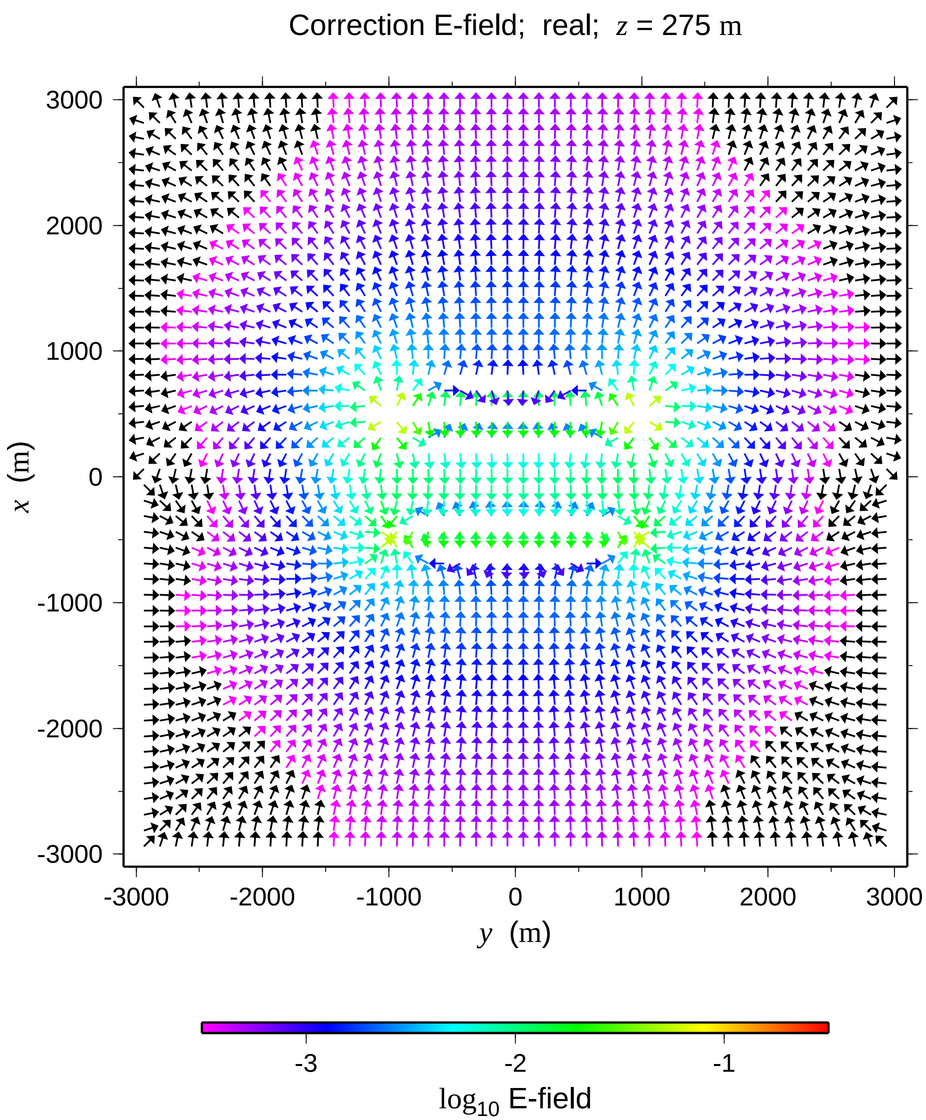
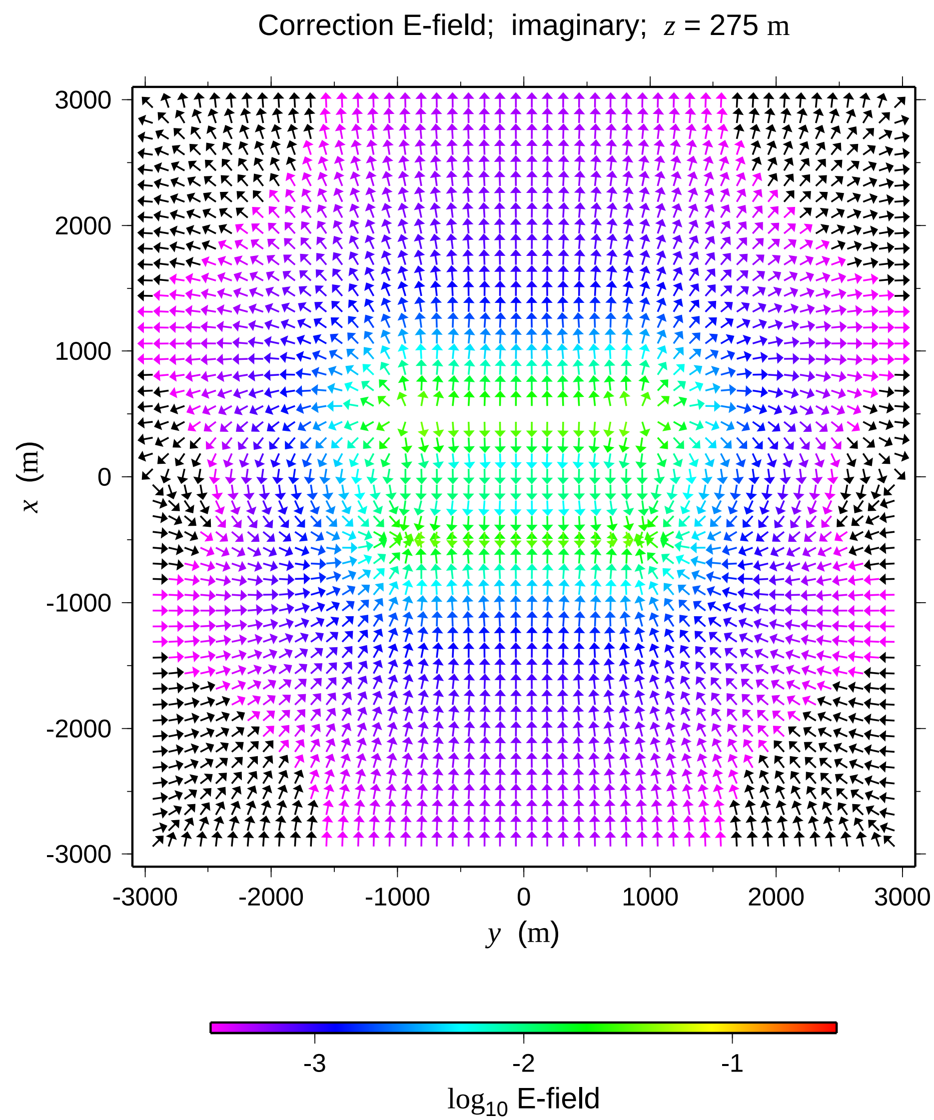




Finite-Element Modelling With Correction: Results





Conclusions

* A static divergence correction term, as introduced by Smith (1996) for a finite-difference solution, improves the performance of an edge-element finite-element solution for the electric field PDE.

  (Edge elements were not thought to require such a correction term.)