Comparison of integral equation and physical scale modelling of the electromagnetic response of models with large conductivity contrasts

Colin G. Farquharson[1], Ken Duckworth[2] & Douglas W. Oldenburg[1]

[1] UBC – Geophysical Inversion Facility,
Department of Earth & Ocean Sciences,
University of British Columbia.

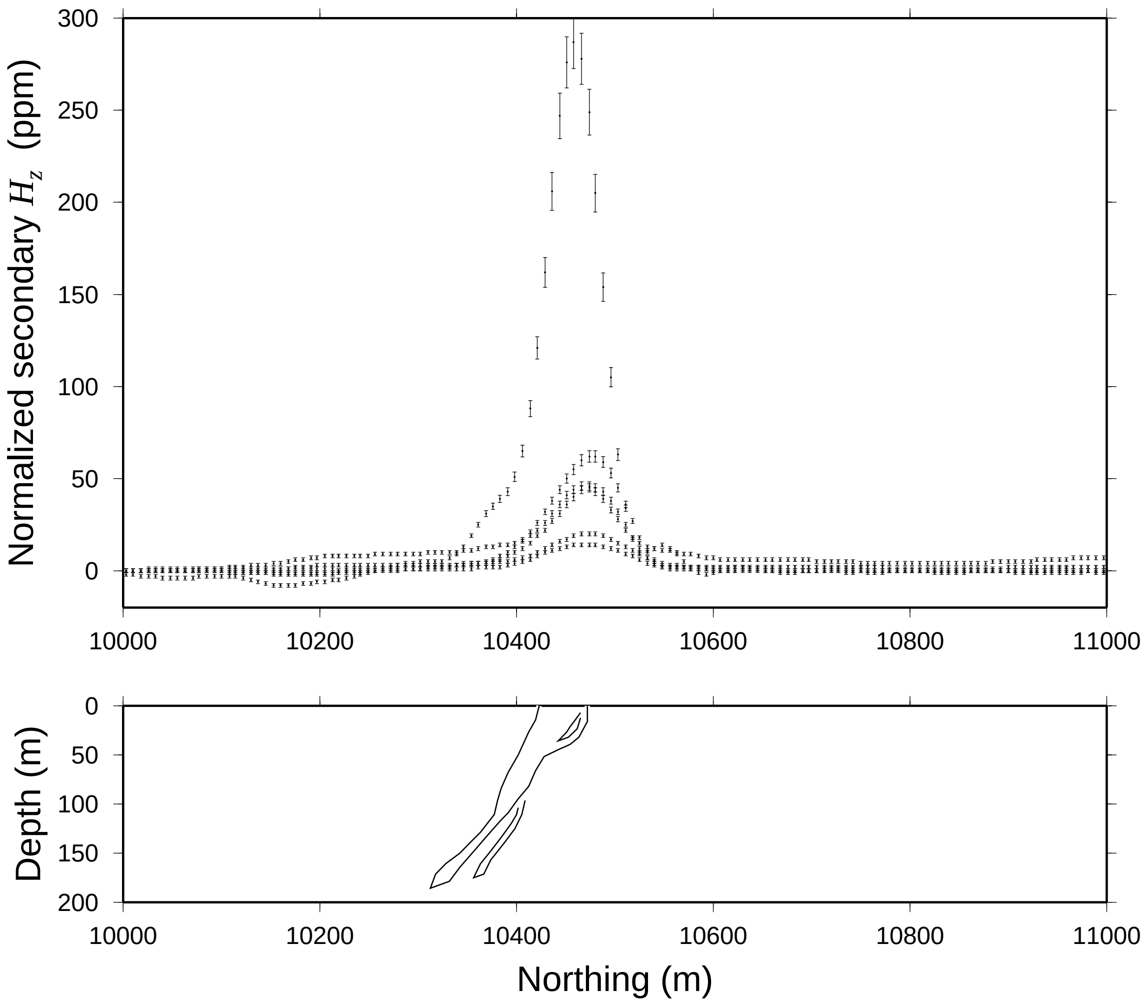
[2] Department of Geology & Geophysics,
University of Calgary.

Outline

- Introduction: a brief history of EM numerical modelling in geophysics.
- Another integral equation modelling program.
- Comparison with physical scale modelling results.

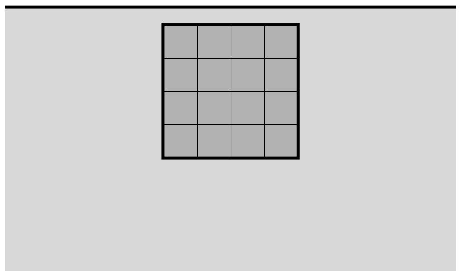
Introduction: a brief history

- The classic scenario, e.g., Heath Steele Stratmat, NB:

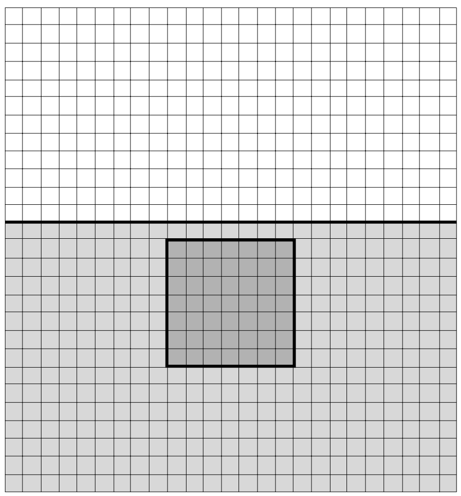


Introduction: a brief history

- Two main approaches to numerical modelling:
    - integral equation;



    - finite-difference/finite-element.

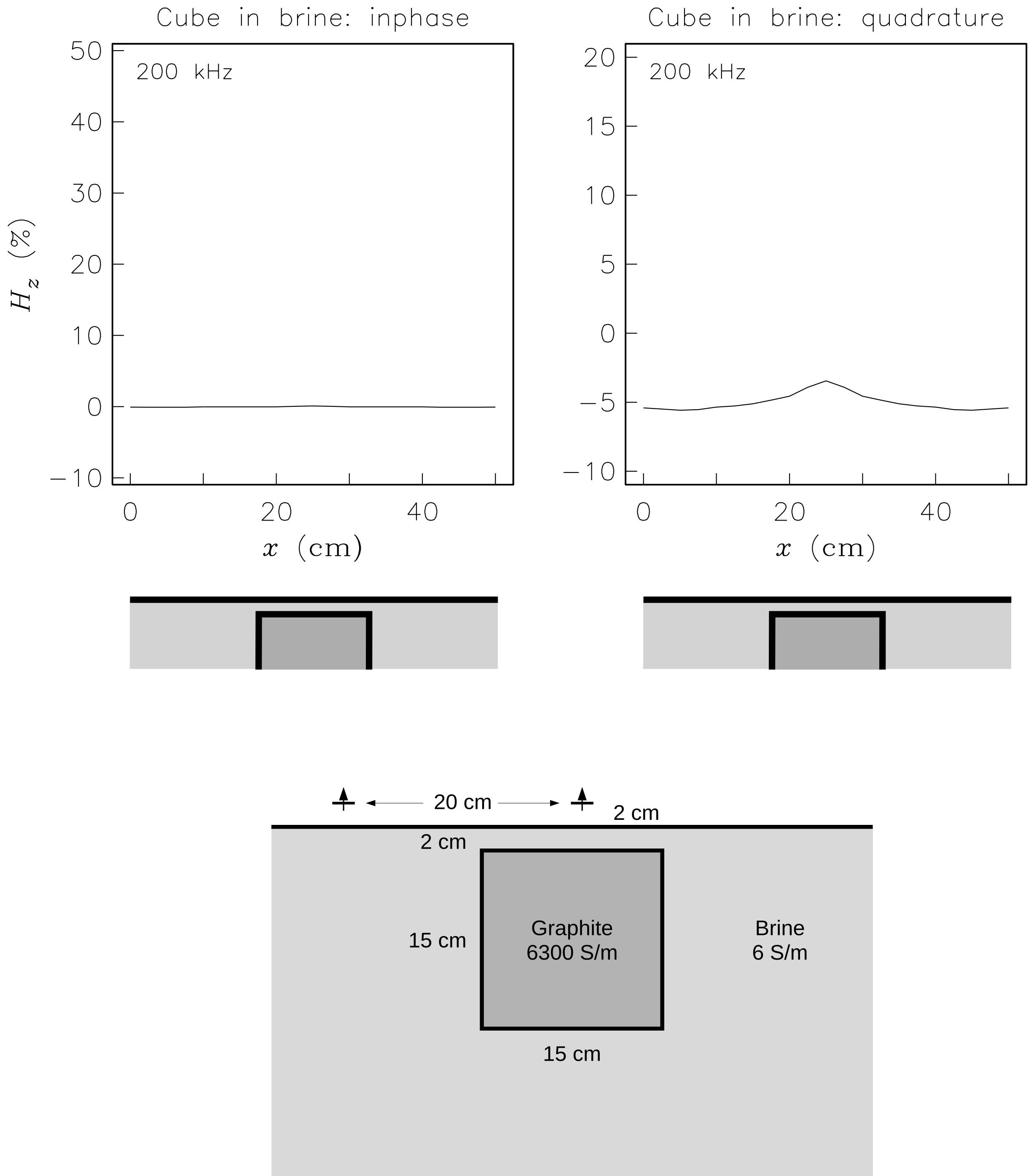


Introduction: a brief history

- Progression:
  - ◦ early interest in integral equation methods;
  - ◦ present interest in finite-difference & finite-element methods.

⋆ But implementations of integral equation methods failed for large contrasts.

For example . . .


Cube in brine: inphase
Cube in brine: quadrature
200 kHz
200 kHz
H_z (%)
x (cm)
x (cm)
20 cm
2 cm
2 cm
15 cm
Graphite
6300 S/m
Brine
6 S/m
15 cm

Another integral equation modelling program

- Maxwell's equations:

$$\nabla \times \mathbf{E} \;=\; i\omega\mu\,\mathbf{H}, \qquad \nabla \times \mathbf{H} \;=\; \sigma\,\mathbf{E} \,+\, \mathbf{J}^S;$$

  and conservation of charge:

$$\nabla \cdot \big(\sigma\,\mathbf{E}\big) \;=\; -\nabla \cdot \mathbf{J}^S.$$

- Differential equation for electric field:

$$-\nabla^2\mathbf{E} \,+\, \nabla\big(\nabla \cdot \mathbf{E}\big) \,-\, i\omega\mu\sigma\,\mathbf{E} \;=\; i\omega\mu\,\mathbf{J}^S.$$

Another integral equation modelling program

- Divide into background and anomalous:

$$\sigma = \sigma_b + \Delta\sigma, \qquad \mathbf{E} = \mathbf{E}_b + \mathbf{E}_s;$$

- such that

$$\nabla \times \mathbf{E}_b \;=\; i\omega\mu\,\mathbf{H}_b, \qquad \nabla \times \mathbf{H}_b \;=\; \sigma_b\,\mathbf{E}_b \;+\; \mathbf{J}^S;$$

and

$$\nabla \cdot \left(\sigma_b\,\mathbf{E}_b\right) \;=\; -\,\nabla \cdot \mathbf{J}^S.$$

- Hence, differential equation for secondary electric field:

$$-\,\nabla^2\mathbf{E}_s \;+\; \nabla\left(\nabla \cdot \mathbf{E}_s\right) \;-\; i\omega\mu\sigma_b\,\mathbf{E}_s \;=\; i\omega\mu\,\Delta\sigma\,\mathbf{E}.$$

Another integral equation modelling program

- Solution via Green's functions and integration over anomalous region:

$$\mathbf{E} \; = \; \mathbf{E}_b \; + \; i\omega\mu \int_{V_a} \underline{\mathbf{G}}^{(1)} \cdot \, \mathbf{E}\,\Delta\sigma\, dv \; + \; \int_{V_a} \mathbf{G}^{(2)}\, \nabla \cdot \mathbf{E}\, dv.$$

- The Green's functions:

$$\underline{\mathbf{G}}^{(1)} \; = \; \begin{pmatrix} g^w & 0 & 0 \\ 0 & g^w & 0 \\ 0 & 0 & g^w \end{pmatrix}, \qquad \mathbf{G}^{(2)} \; = \; \nabla g^w,$$

$$g^w(\mathbf{r};\mathbf{r}') \; = \; \frac{1}{4\pi} \frac{\exp(ik_b|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}, \qquad k_b^2 = i\omega\mu\sigma_b.$$

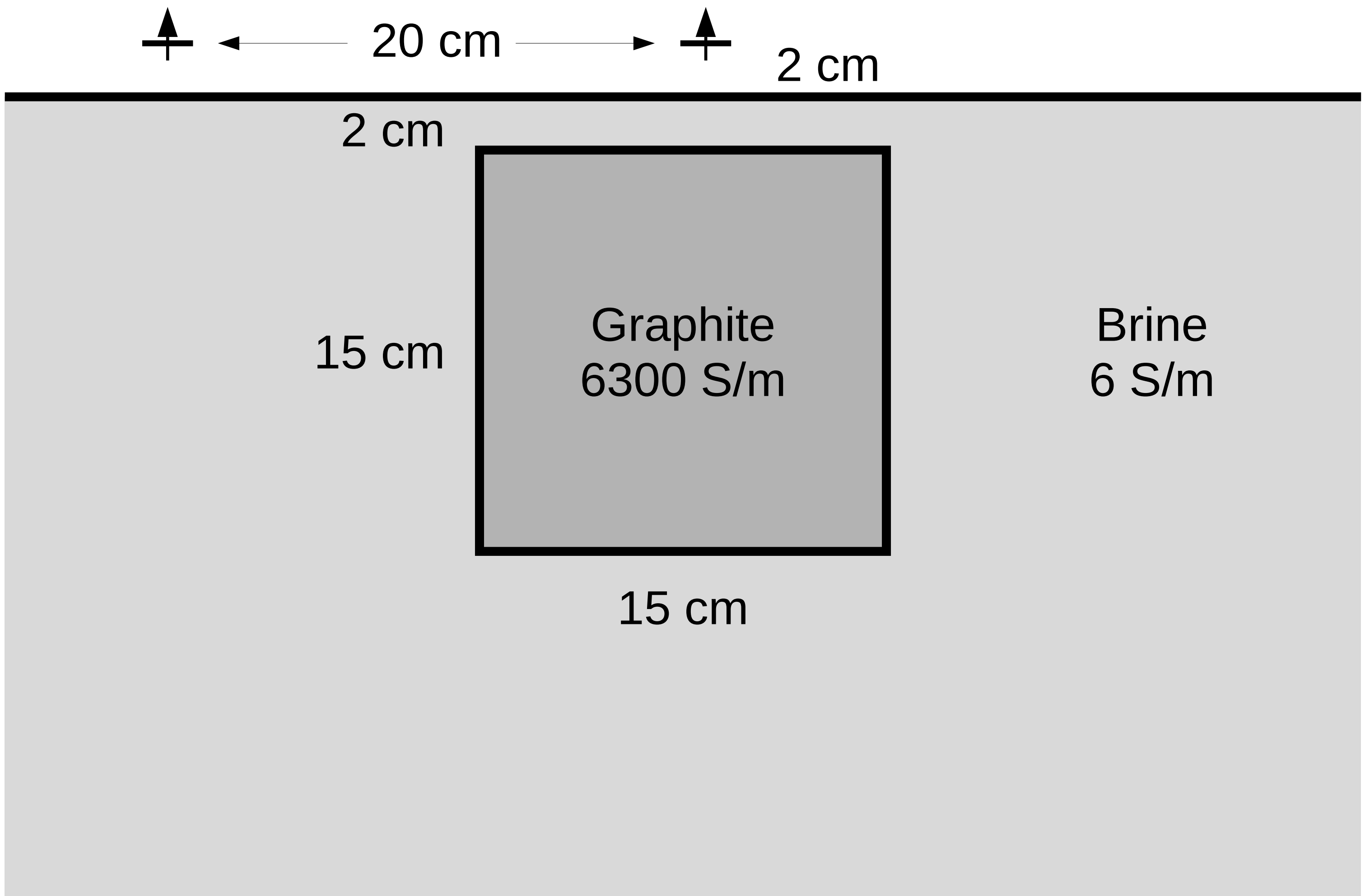
Another integral equation modelling program

- Traditional implementation:
    - pulse basis functions,
    - approximation of both integrals by integrations over equivalent spheres.

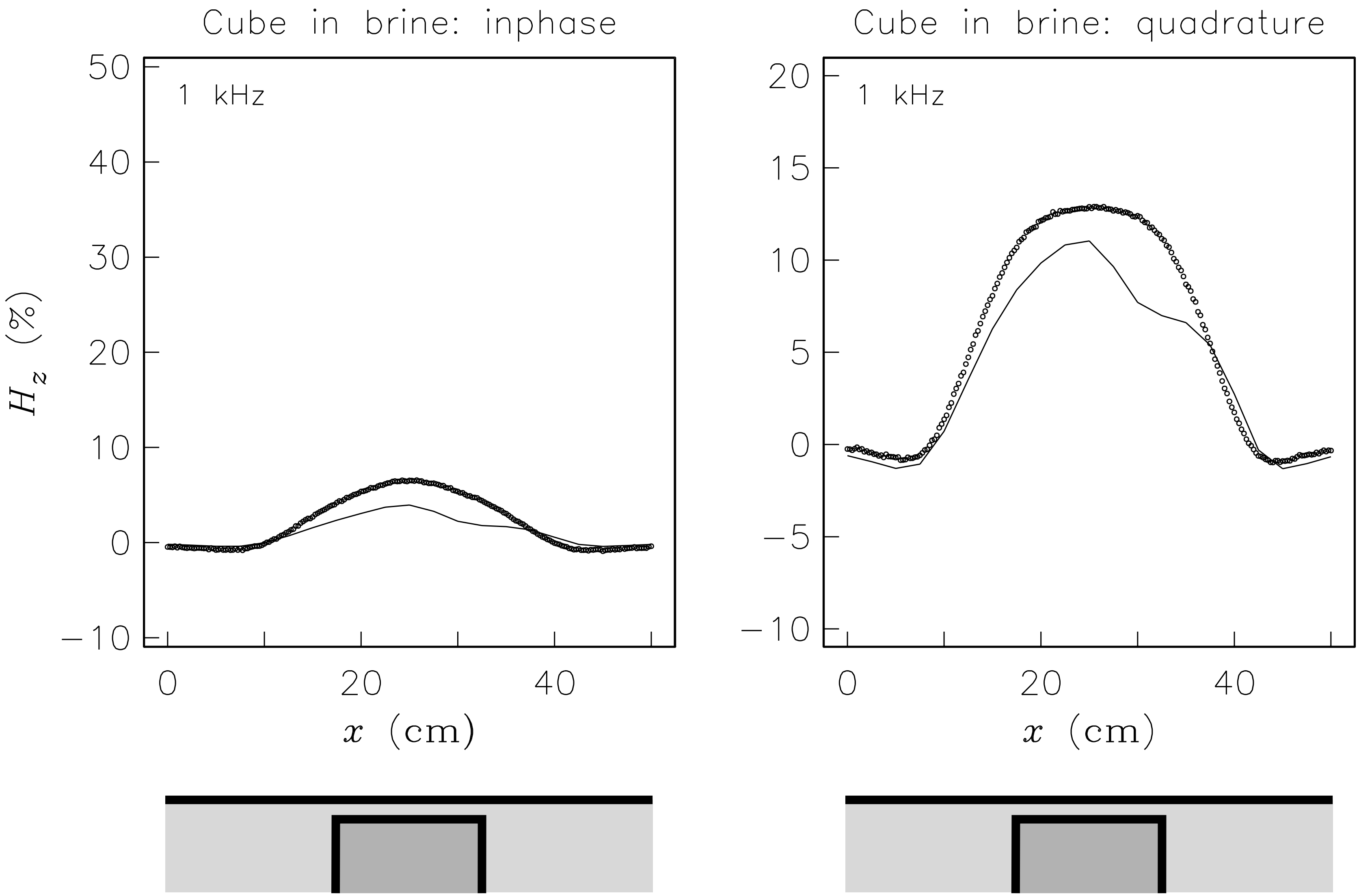
- New approach:
    - edge element basis functions,
    - volume & surface integrations kept separate,
    - Gaussian quadrature evaluation of integrals,
    - contrast (or lack thereof) between neighbouring cells kept explicit.

Comparison with physical scale modelling results

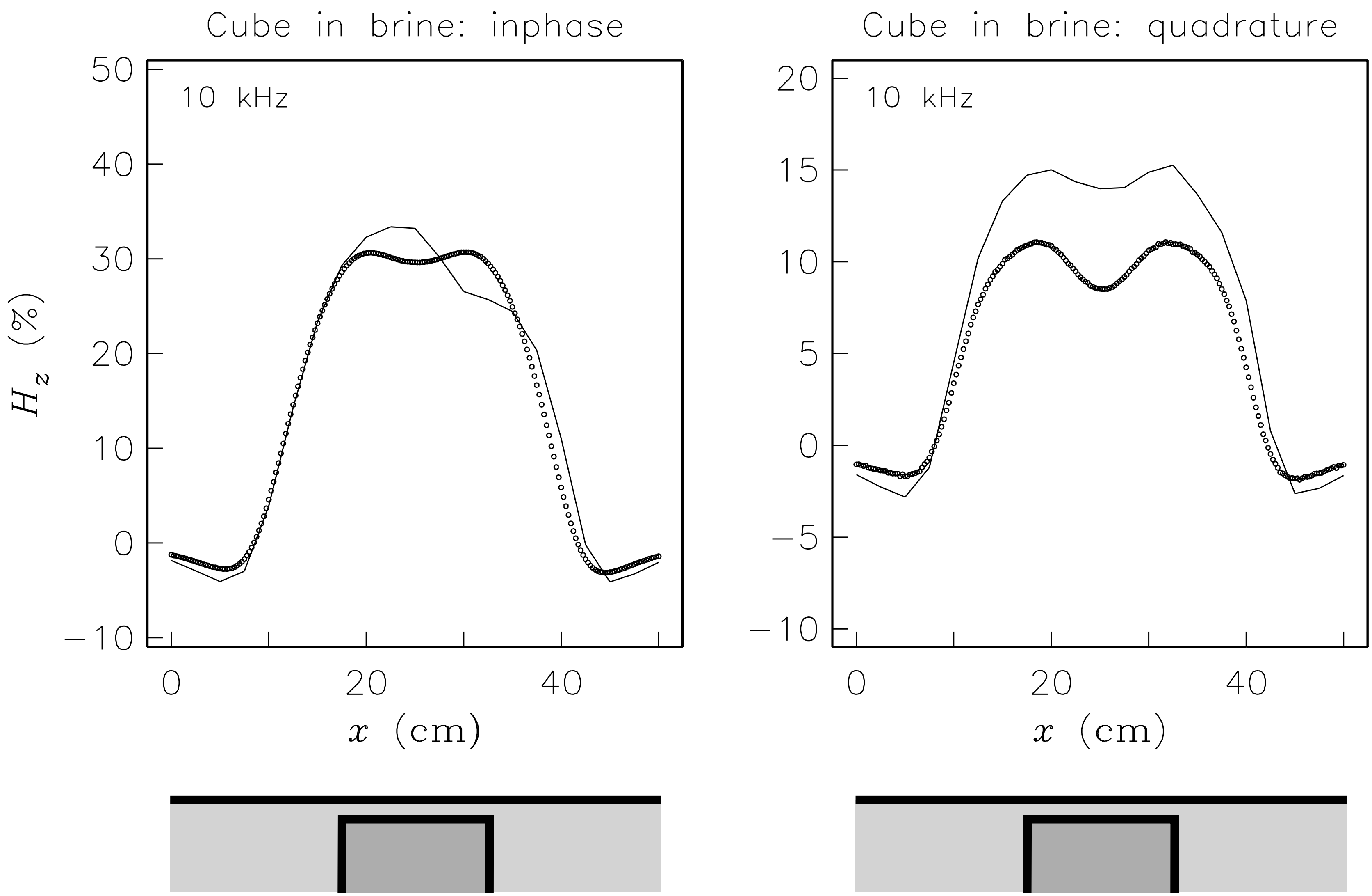
Comparison with physical scale modelling results



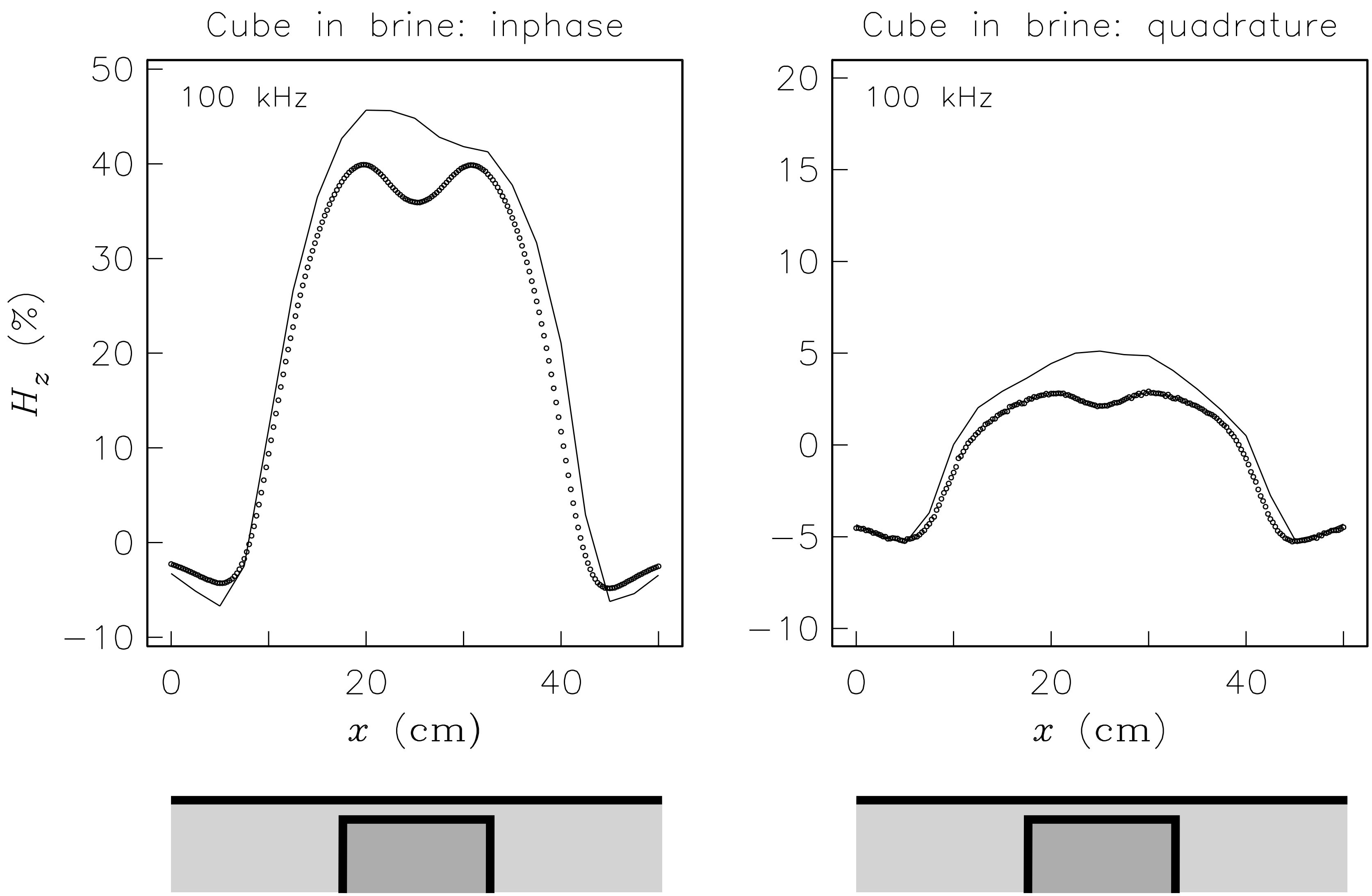
Comparison with physical scale modelling results



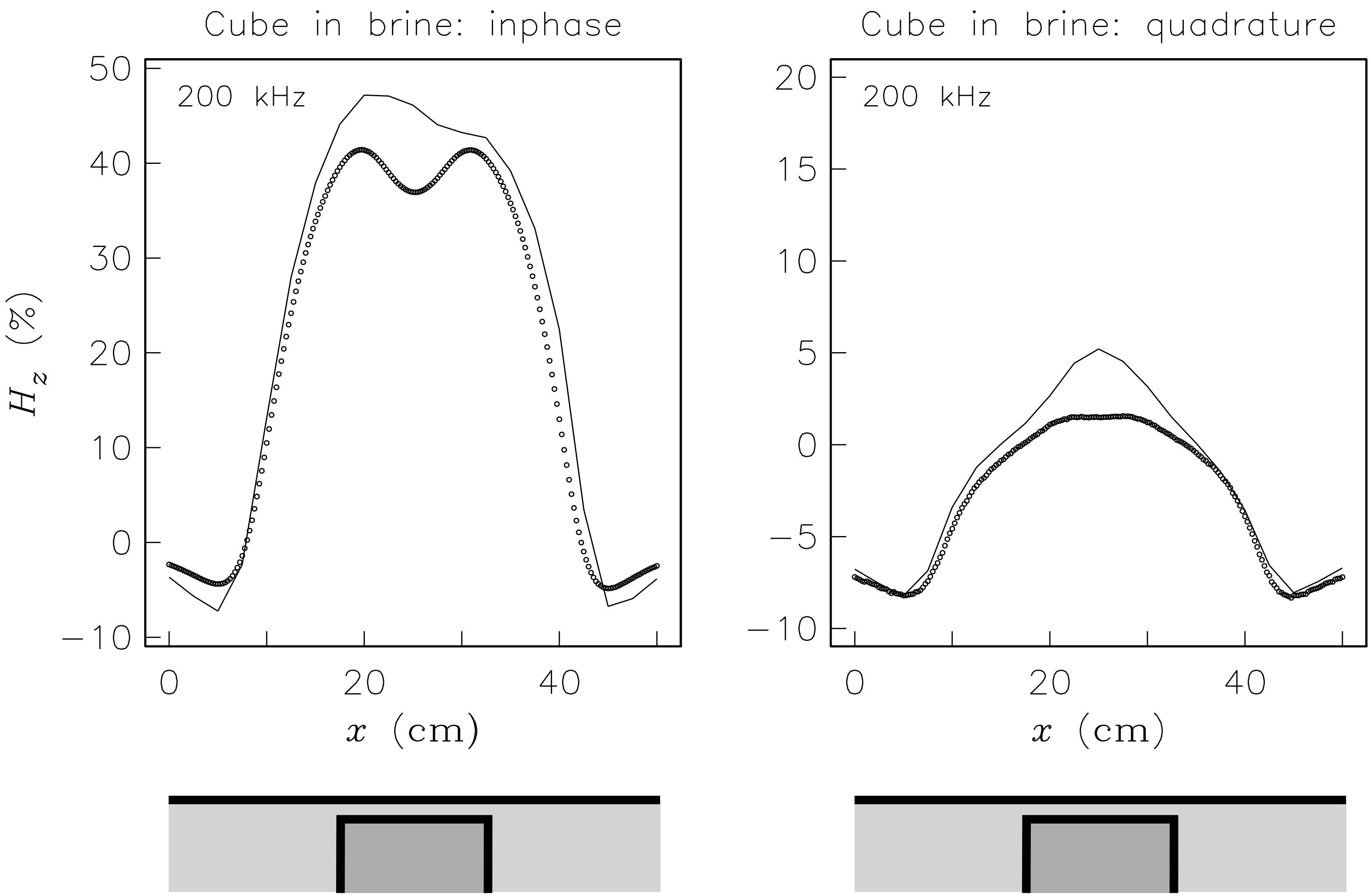
Comparison with physical scale modelling results



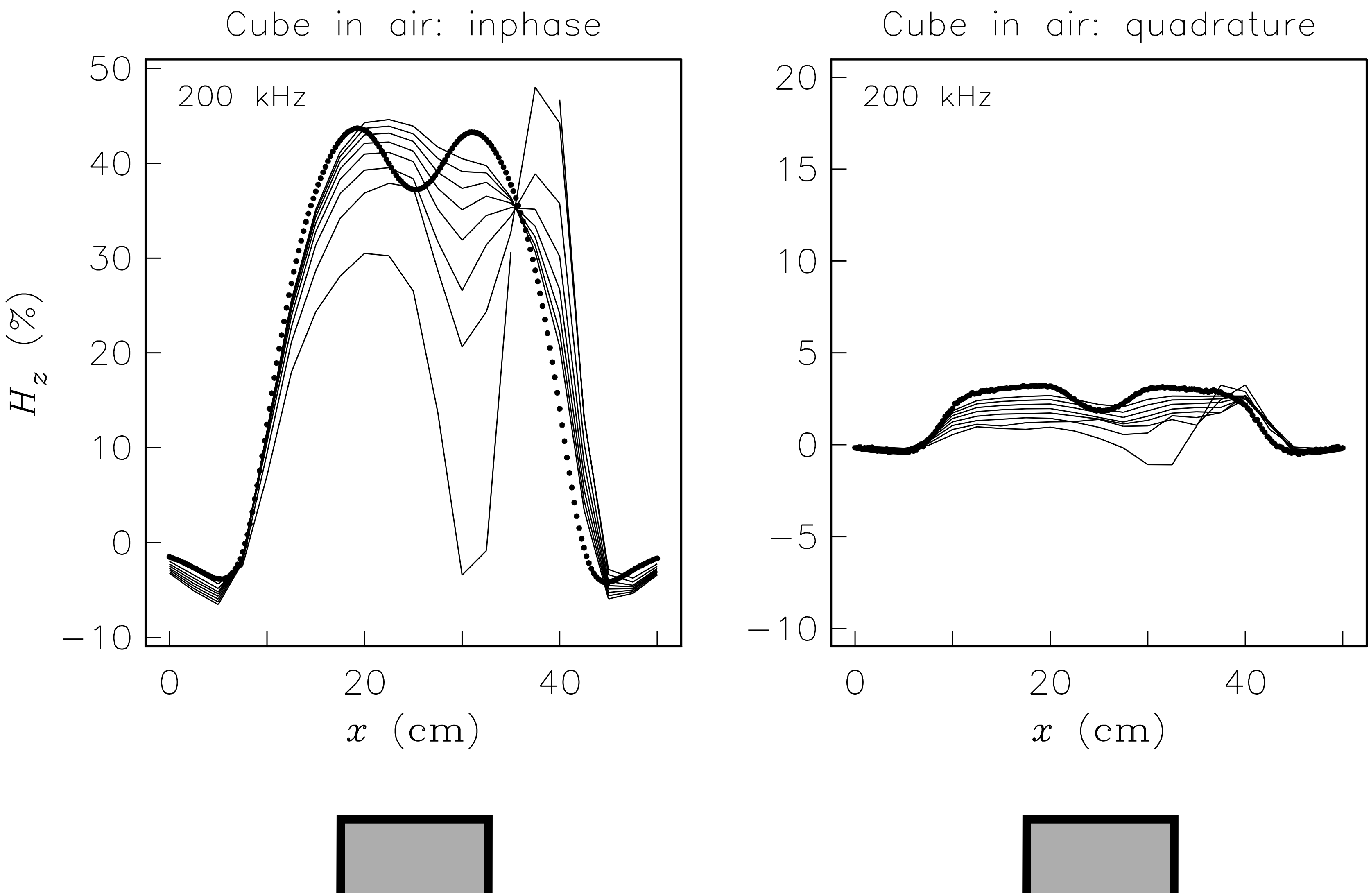
Comparison with physical scale modelling results



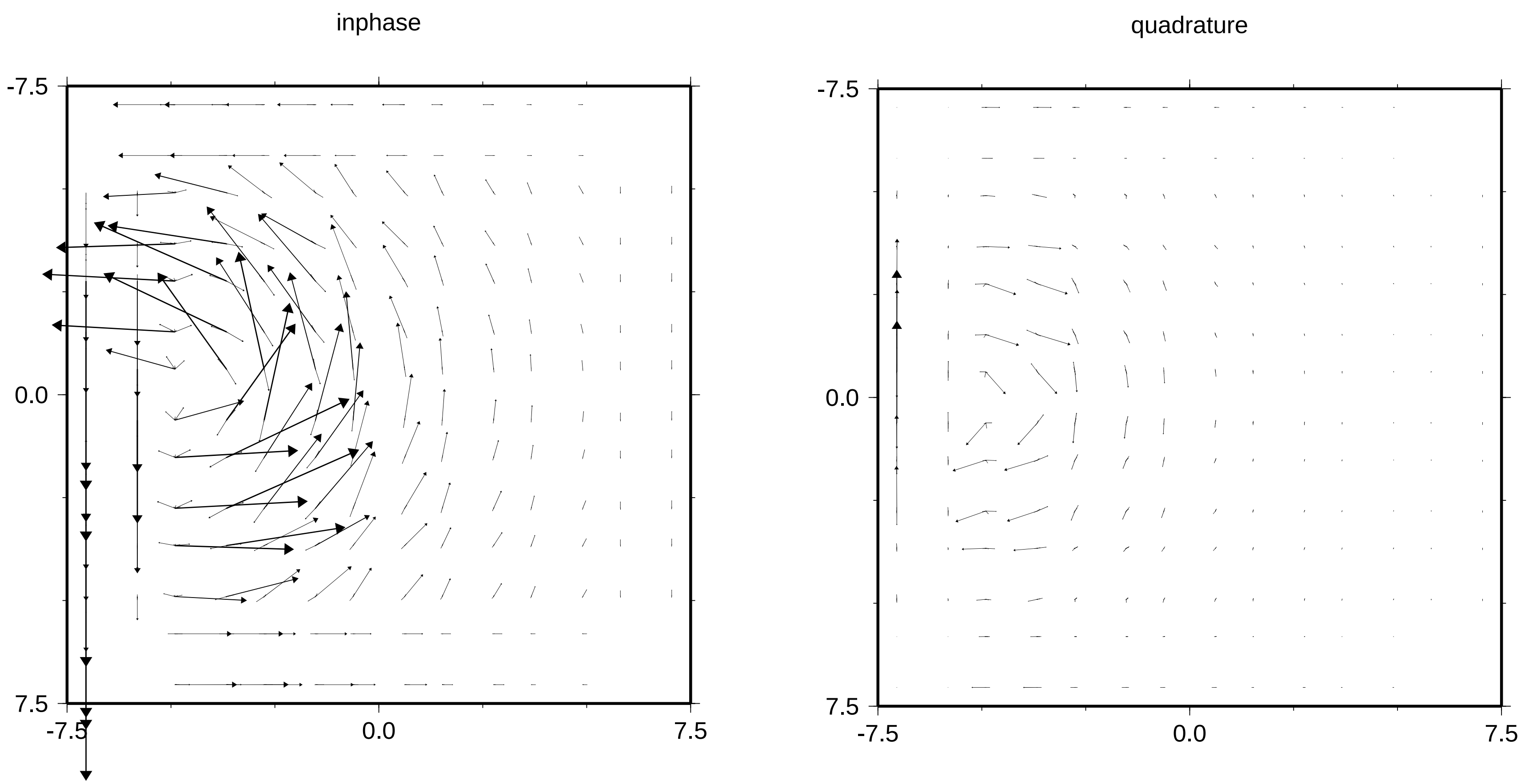
Comparison with physical scale modelling results



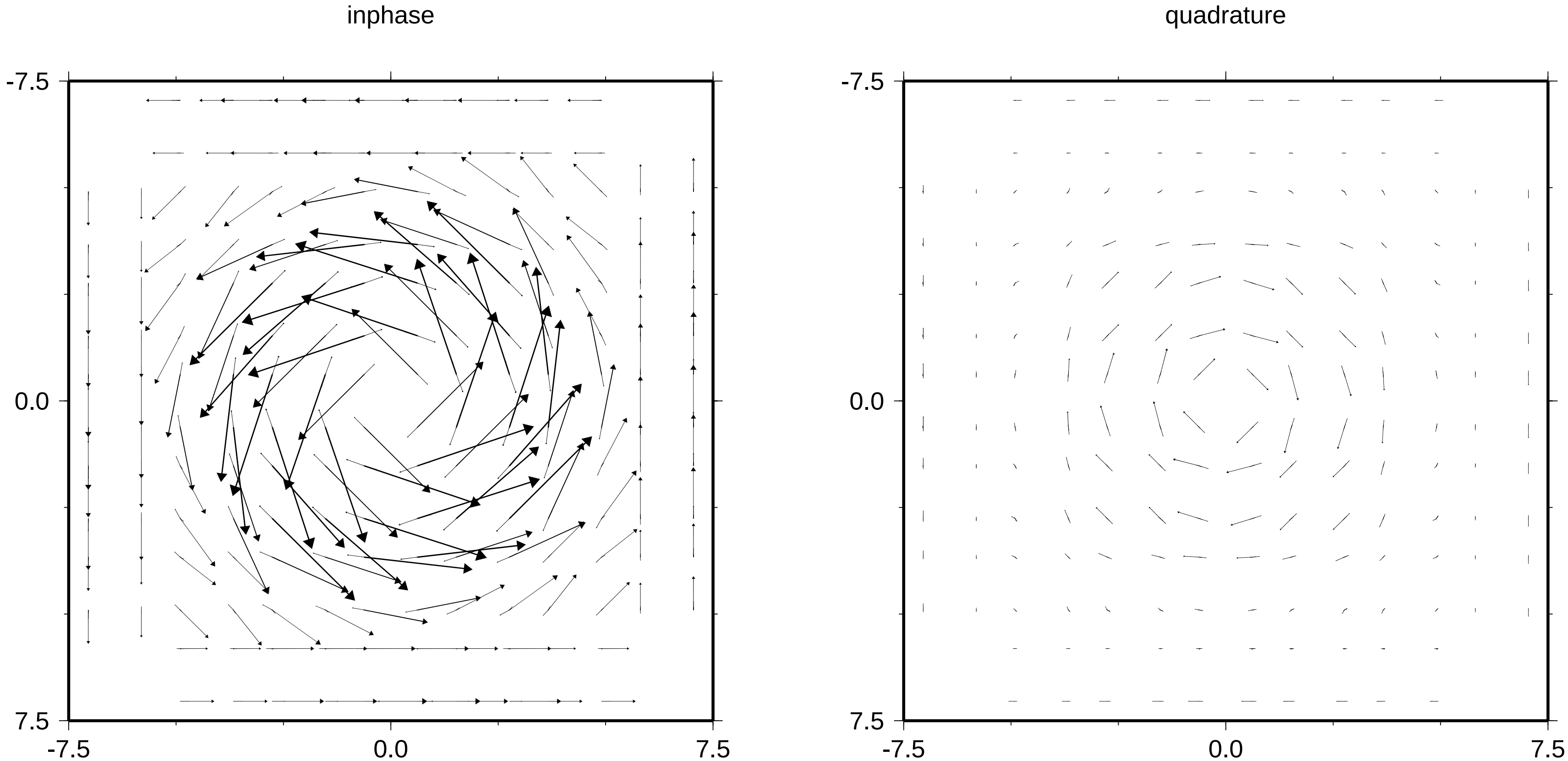
Comparison with physical scale modelling results



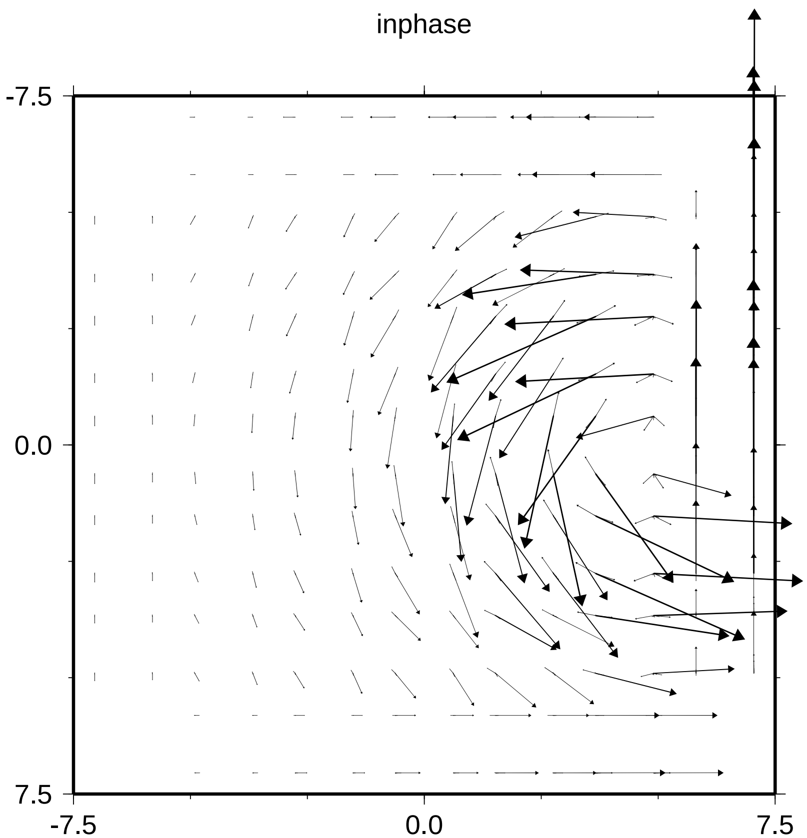
Comparison with physical scale modelling results

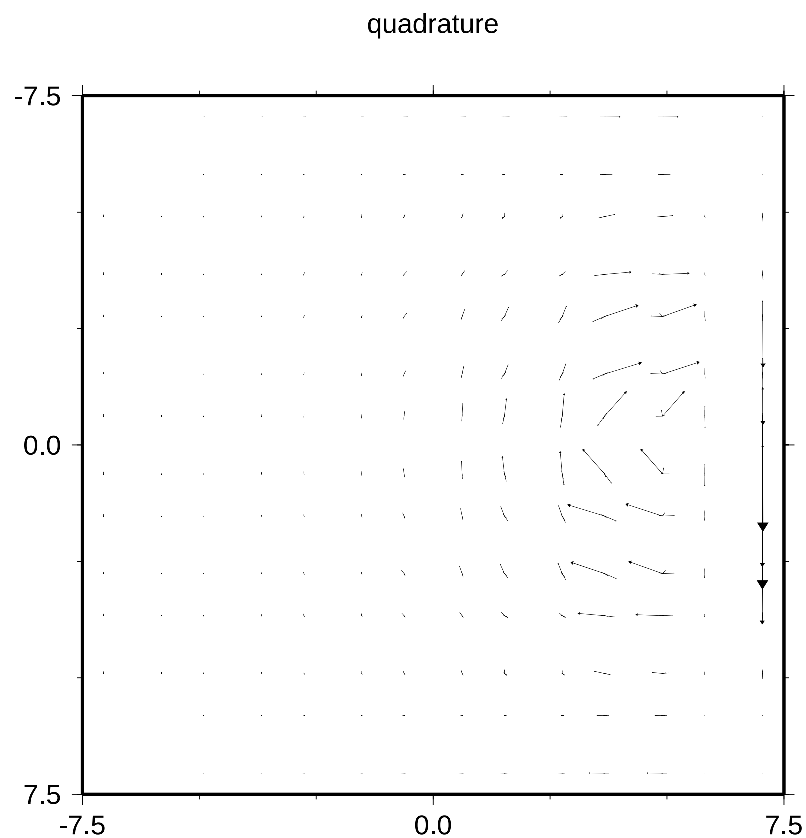


Comparison with physical scale modelling results



Comparison with physical scale modelling results





Comparison with physical scale modelling results