Finite volume modelling of electromagnetic data using unstructured staggered grids

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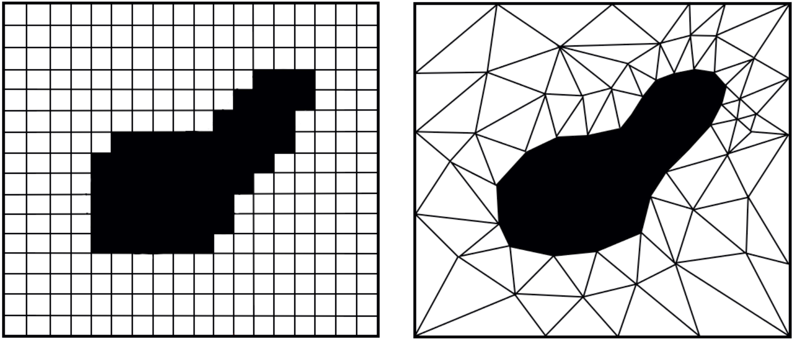
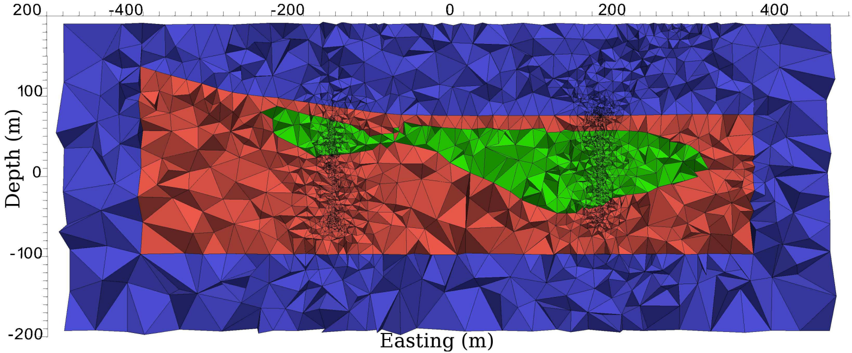
Unstructured grids

- model irregular structures



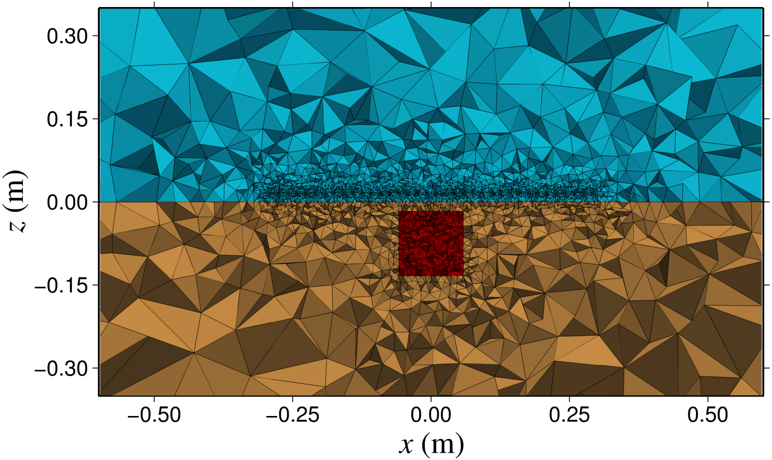
Unstructured grids

- topographical features
- geological interfaces



Unstructured grids

- local refinement (at observation points, sources, interfaces)



Maxwell's equations

Maxwell's equations: describe the behavior of electromagnetic fields.

$$\nabla \times \mathbf{E} = -i\omega\mu_0 \mathbf{H} - i\omega\mu_0 \mathbf{M}_p$$
$$\nabla \times \mathbf{H} = \sigma\, \mathbf{E} + \mathbf{J}_p$$

Helmholtz equation for electric field:

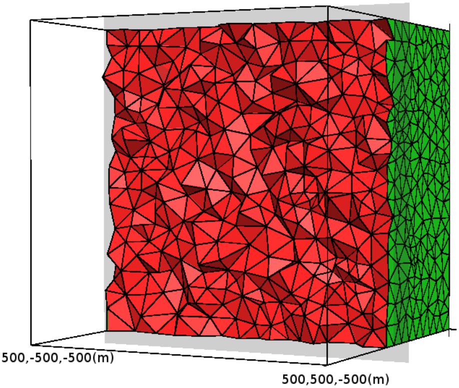
$$\nabla \times \nabla \times \mathbf{E} + i\omega\mu_0\sigma\mathbf{E} = -i\omega\mu_0 \mathbf{J}_p - i\omega\mu_0(\nabla \times \mathbf{M}_p)$$

Boundary condition:

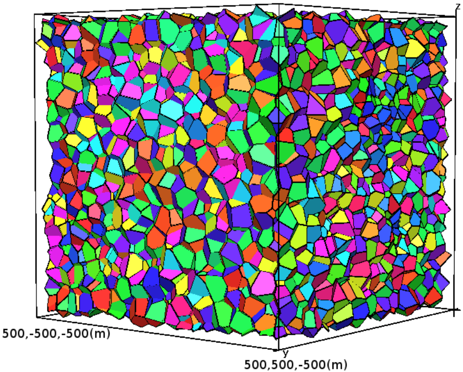
$$\mathbf{E} \cdot \tau = 0 \qquad on\, \Gamma$$

Staggered tetrahedral-Voronoï grids

tetrahedral grid



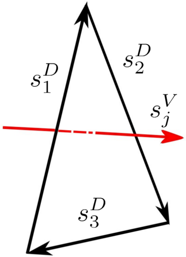
Voronoï grid

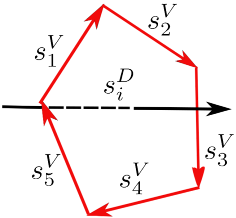


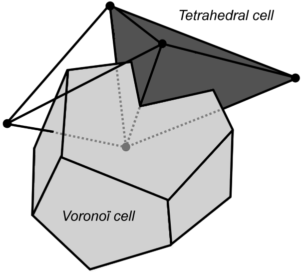
Integral form of Maxwell's equations:

$$\oint_{\partial A^D} \mathbf{E} \cdot d\mathbf{l}^D = -i\mu_0\omega \iint_{A^D} \mathbf{H} \cdot d\mathbf{A}^D - i\mu_0\omega \iint_{A^D} \mathbf{M}_p \cdot d\mathbf{A}^D$$

$$\oint_{\partial A^V} \mathbf{H} \cdot d\mathbf{l}^V = \sigma \iint_{A^V} \mathbf{E} \cdot d\mathbf{A}^V + \iint_{A^V} \mathbf{J}_p \cdot d\mathbf{A}^V$$







Discretized Helmholtz equation

Discretized form of Maxwell's equations:

$$\sum_{q=1}^{W_j^D} E_{i(j,q)}\, l_{i(j,q)}^D = -i\mu_0\omega H_j\, A_j^D - i\mu_0\omega M_{p_j}$$

$$\sum_{k=1}^{W_i^V} H_{j(i,k)} l_{j(i,k)}^V = \sigma E_i\, A_i^V + J_{p_i}$$

Discretized form of Helmholtz equation:

$$\sum_{k=1}^{W_i^V} \left(\left(\sum_{q=1}^{W_j^D} E_{i(j,q)}\, l_{i(j,q)}^D \right) \frac{l_{j(i,k)}^V}{A_{j(i,k)}^D} \right) + i\omega\mu_0\sigma E_i\, A_i^V$$

$$= -i\omega\mu_0 \sum_{k=1}^{W_i^V} M_{p_{j(i,k)}} \frac{l_{j(i,k)}^V}{A_{j(i,k)}^D} - i\omega\mu_0 J_{p_i}$$

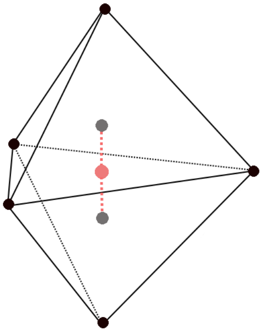
Sparse direct solver: MUMPS (Amestoy et al., 2006)
Interpolation inside tetrahedra: edge vector basis functions

Grid generator: TetGen (Si, 2004)

Inclusion of EM sources

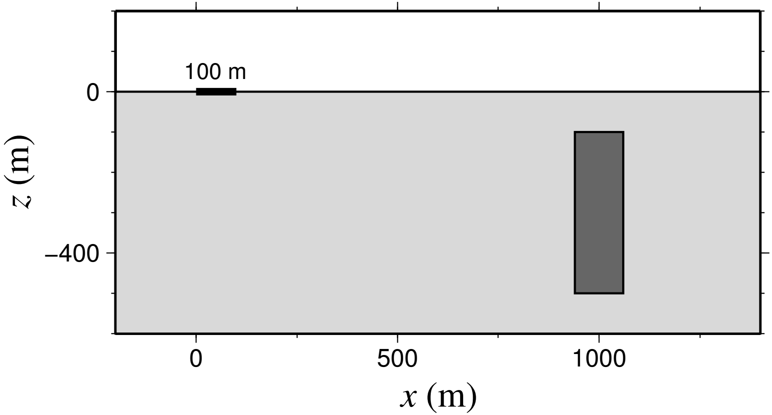
- grounded wire:

- point vertical magnetic dipole:



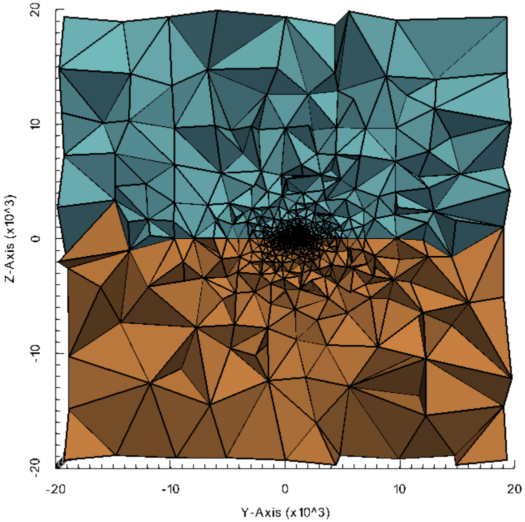
Example 1: long grounded wire

- 100 m wire along the x axis operating at 3 Hz
- dimensions of the prism: $120 \times 200 \times 400\ m$
- $\sigma_{ground} = 0.02\ S/m$; $\sigma_{prism} = 0.2\ S/m$
- observation points along the x axis



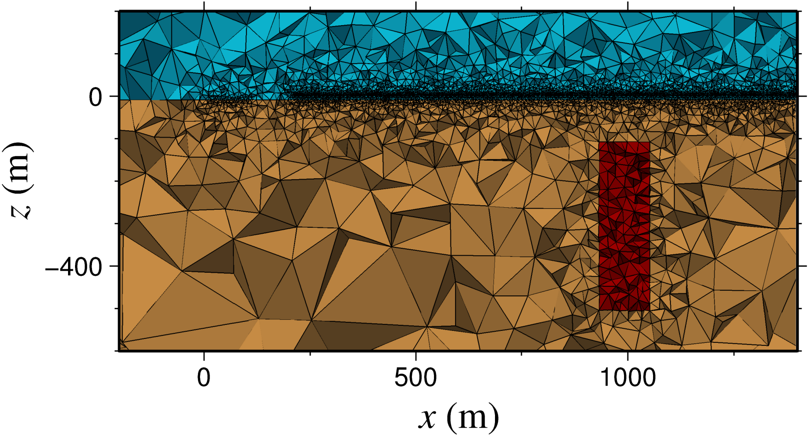
Example 1: long grounded wire

- dimensions of the domain: $40 \times 40 \times 40$ *km*
- number of tetrahedra: 162,689 ; number of unknowns: 189,105



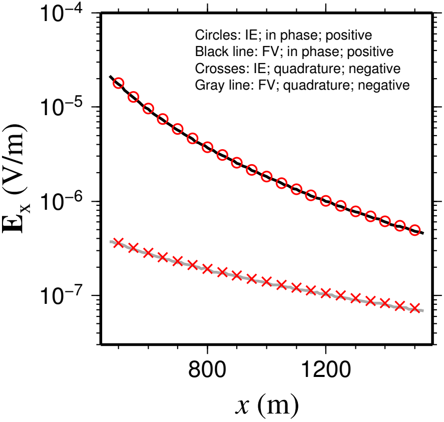
Example 1: long grounded wire

- grid refined at the source, observation points and the prism
- computation time: 40 *s* ; memory: 4 *Gbytes* (on Apple Mac Pro; 2.26 GHz Quad-Core Intel Xeon processor)

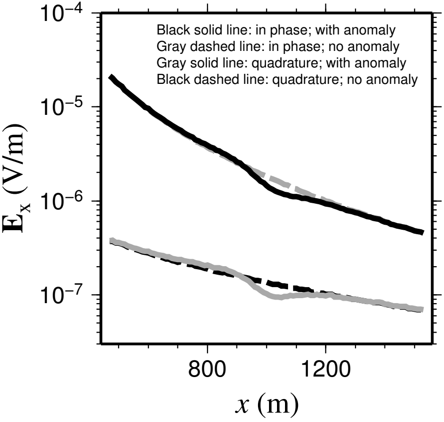


Example 1: long grounded wire

- without prism (homogeneous halfspace)
- total field
- FV vs IE (Farquharson and Oldenburg, 2002)

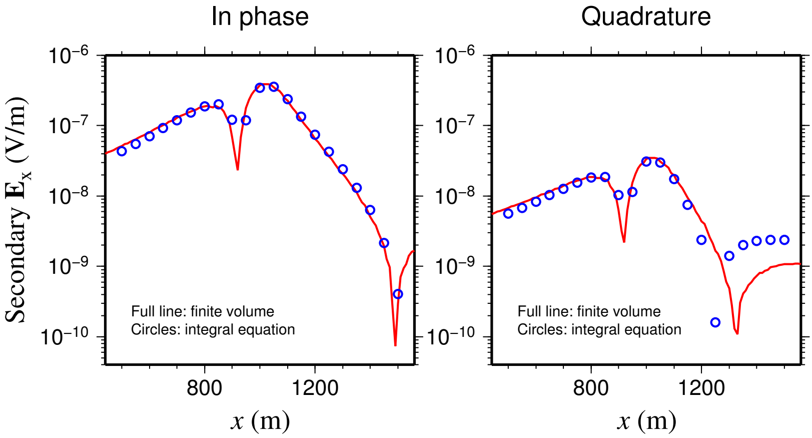


- with and without prism
- total field
- FV only



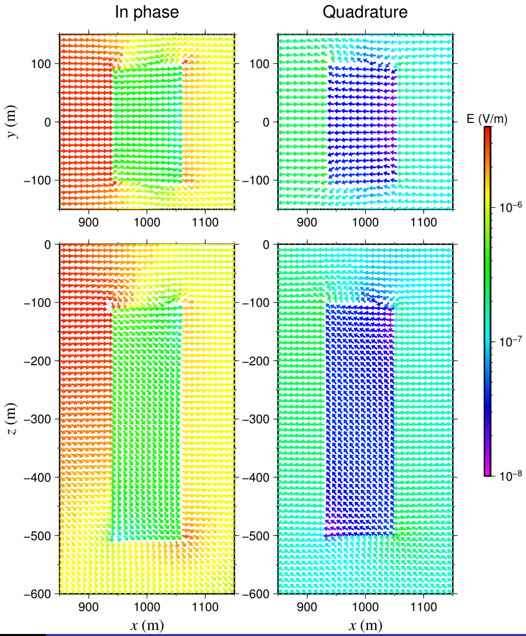
Example 1: long grounded wire

- scattered field
- FV vs IE



Example 1: long grounded wire

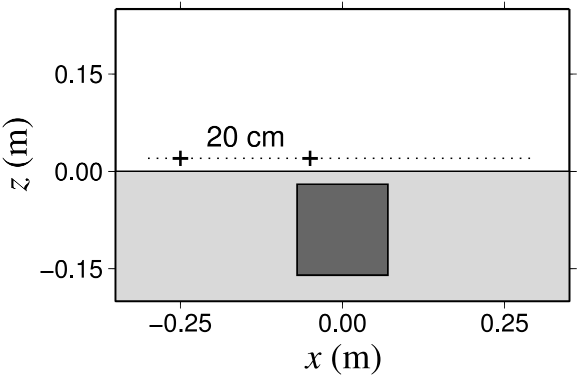
- horizontal section ($z = -150\ m$)
- total electric field (in phase and quadrature)



- vertical section ($y = 0\ m$)
- total electric field (in phase and quadrature)

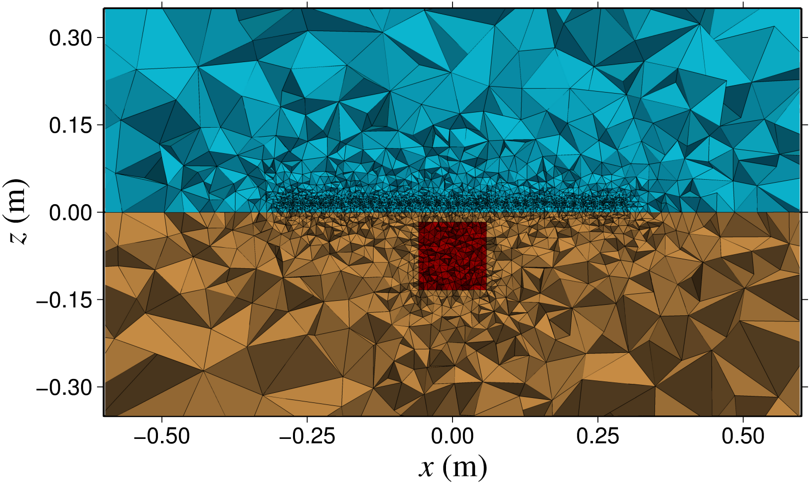
Example 2: magnetic dipole transmitter-receiver pairs

- graphite cube in brine (physical scale modelling measurements)
- transmitter-receiver pairs along the x axis at $z = 2\ cm$
- dimensions of the cubic graphite: $14 \times 14 \times 14\ cm$
- $\sigma_{brine} = 7.3\ S/m$; $\sigma_{prism} = 63,000\ S/m$
- frequencies: 1, 10, 100, 200, 400 kHz



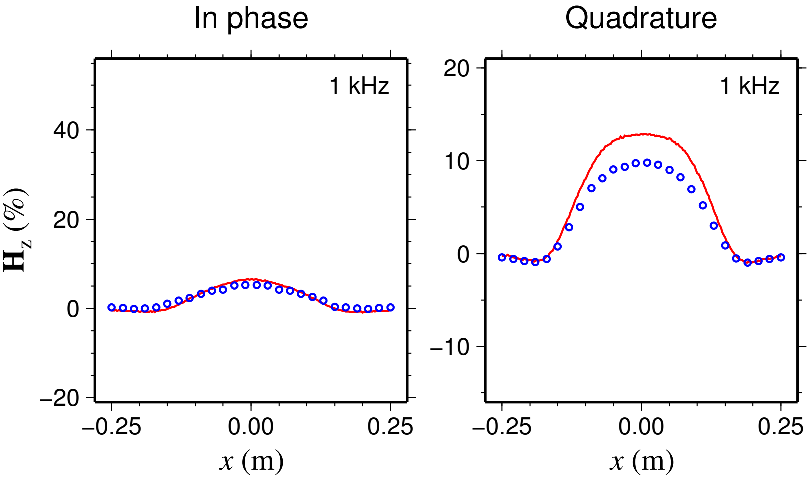
Example 2: magnetic dipole transmitter-receiver pairs

- grid refined at the sources, observation points and the prism



Example 2: magnetic dipole transmitter-receiver pairs

- scattered H-field: (total−free-space)/free-space
- FV vs PSM (Farquharson et al., 2006)



Example 2: magnetic dipole transmitter-receiver pairs

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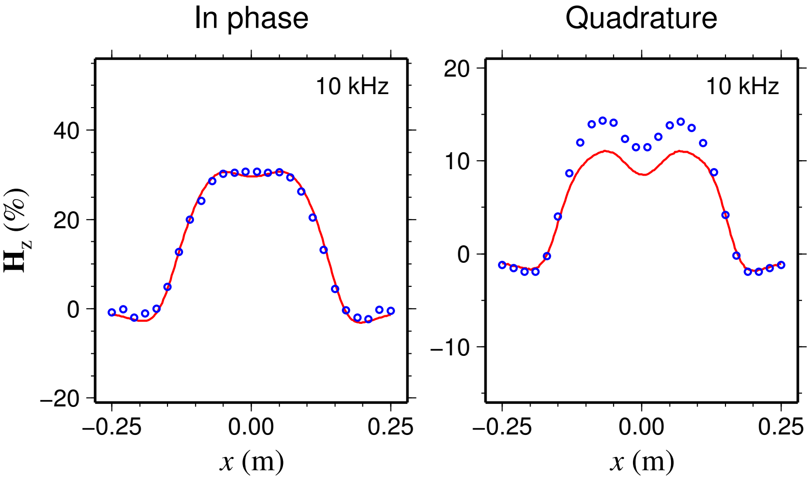
- FV vs PSM



- scattered H-field: (total−free-space)/free-space
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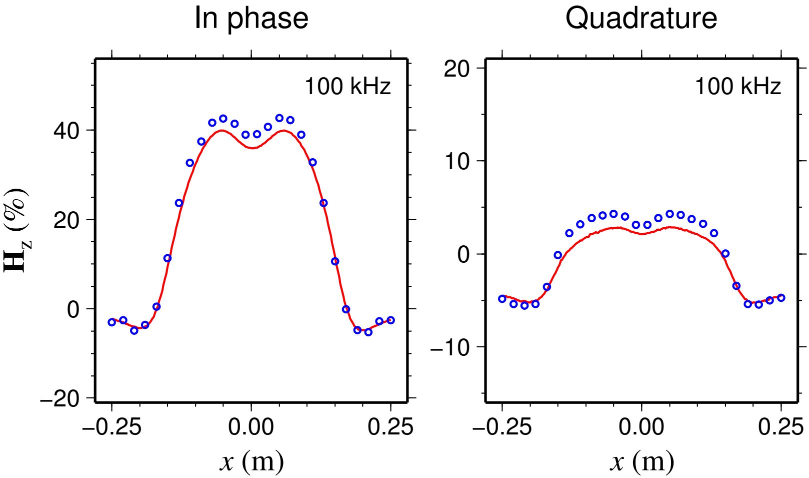

Example 2: magnetic dipole transmitter-receiver pairs

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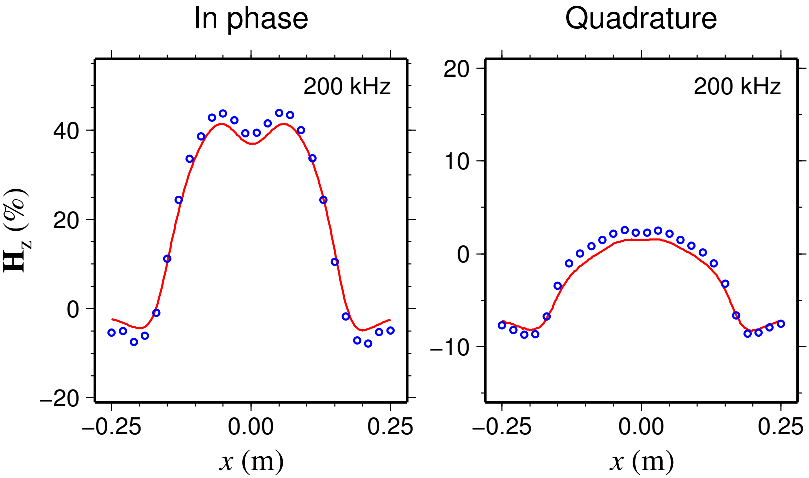
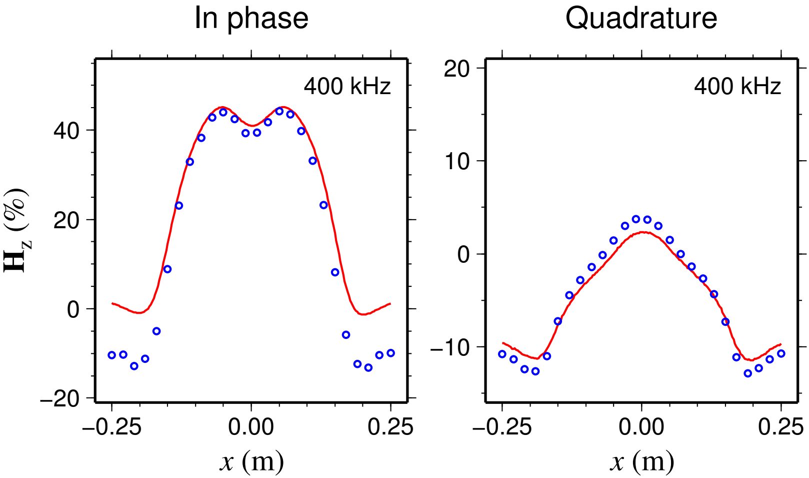
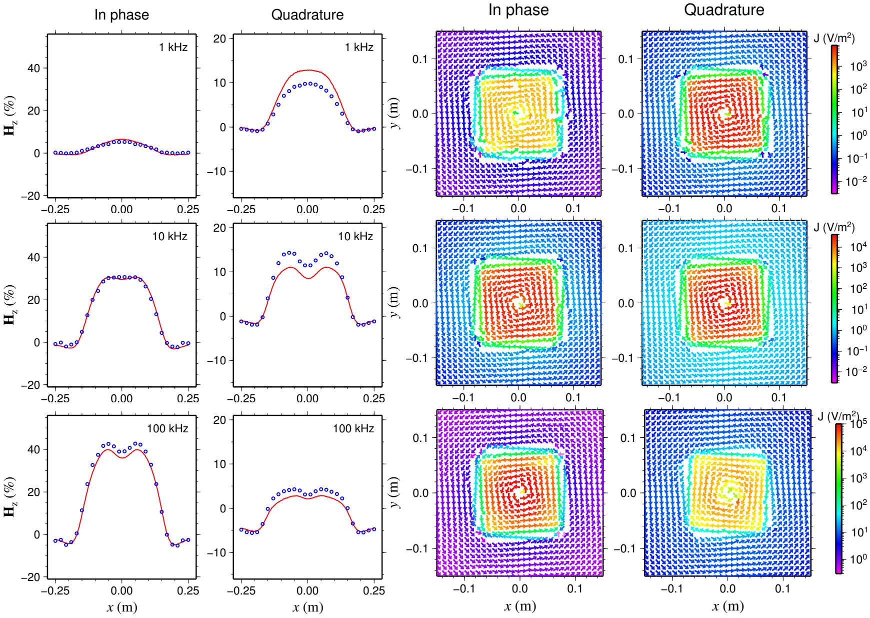


Example 2: magnetic dipole transmitter-receiver pairs

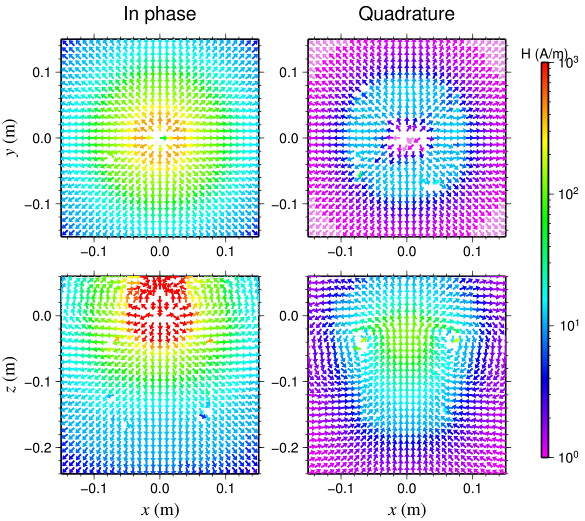
- scattered H-field: (total−free-space)/free-space
- FV vs PSM




In phase
Quadrature
In phase
Quadrature
1 kHz
10 kHz
100 kHz
H_z (%)
x (m)
y (m)
J (V/m^2)

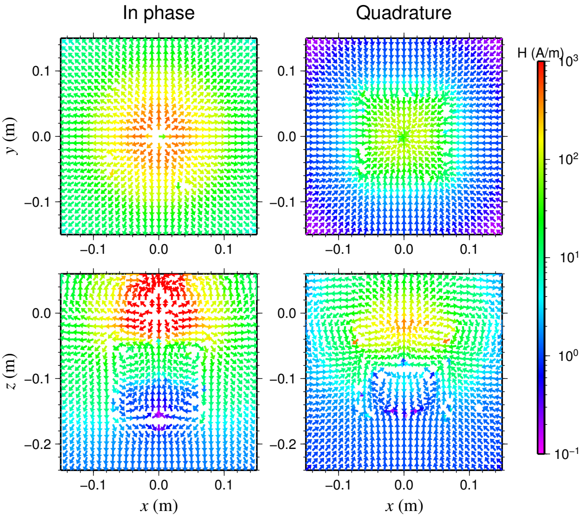
Example 2: magnetic dipole transmitter-receiver pairs

- total H-field (1 *kHz*)
- horizontal and vertical sections



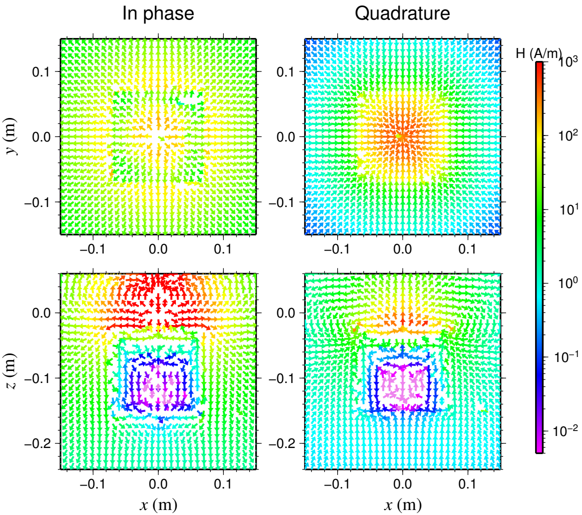
Example 2: magnetic dipole transmitter-receiver pairs

- total H-field (10 *kHz*)
- horizontal and vertical sections



Example 2: magnetic dipole transmitter-receiver pairs

- total H-field (100 *kHz*)
- horizontal and vertical sections



Conclusions

- A finite-volume technique is used for modelling the electromagnetic data. This technique uses the staggered tetrahedral-Voronoï grid.
- The Helmholtz equation is discretized and solved using a sparse direct solver (MUMPS).
- The scheme has been tested for two models: one with a long grounded wire source; another one for magnetic source-receiver pairs with large conductivity contrasts.
- For the both examples, the results from the FV scheme are in good agreement with those from the IE method (example 1) and the physical scale modelling measurements (example 2).

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- Vale

Atlantic Canada Opportunities Agency

NSERC CRSNG

VALE

References

- Amestoy, P. R., Guermouche, A., L'Excellent, J. -Y. and Pralet, S., 2006. Hybrid scheduling for the parallel solution of linear systems, Parallel Computing, 32, 136156.
- Farquharson, C. G., and D. W. Oldenburg, 2002, An integral-equation solution to the geophysical electromagnetic forward-modelling problem, in Three-Dimensional Electromagnetics: Proceedings of the Second International Symposium: Elsevier, 319.
- Farquharson, C. G., Duckworth, K. and D. W. Oldenburg, 2006. Comparison of integral equation and physical scale modeling of the electromagnetic responses of models with large conductivity contrasts, Geophysics, 71, G169-G177.
- Si, H., 2004. TetGen, a quality tetrahedral mesh generator and three-dimensional delaunay triangulator, v1.3 (Technical Report No. 9). Weierstrass Institute for Applied Analysis and Stochastics.